Problem 1. – *Foldy-Wouthuysen Transformation*
Consider a free Dirac particle in momentum space under Foldy-Wouthuysen Transformation $\Phi = \widehat{U}\Psi$, $U = e^{i\widehat{S}}$, where

$$\widehat{S} = -\frac{i}{2m_0 c}\beta\vec{\alpha}\vec{p} f\left(\frac{p}{m_0 c}\right), \quad p = |\vec{p}|$$

(a) Show: $\beta e^{-i\widehat{S}} = e^{i\widehat{S}}\beta$
(b) Show by expansion of $e^{i\widehat{S}}$ that the Hamiltonian of a free Dirac particle transforms under $\widehat{H}_\Phi = e^{i\widehat{S}}\widehat{H}e^{-i\widehat{S}} = e^{2i\widehat{S}}\left(c\vec{\alpha}\vec{p} + \beta m_0 c^2\right)$ to

$$\widehat{H}_\Phi = \beta\left[m_0 c^2 \cos\left(yf\left(y\right)\right) + cp\sin\left(yf\left(y\right)\right)\right] + \frac{\vec{\alpha}\vec{p}}{p}\left[cp\cos\left(yf\left(y\right)\right) - m_0 c^2 \sin\left(yf\left(y\right)\right)\right], \qquad (1)$$

where $y = \frac{p}{m_0 c}$.
(c) Show, that the second term in (1) vanishes for $f\left(y\right) = \frac{1}{y}\arctan\left(y\right)$.

Problem 2. – *Lie Group*
Consider a group of unitary operators (Lie group)

$$\widehat{U}\left(\alpha_\mu\right) = e^{-i\widehat{L}_\mu \alpha_\mu}$$

with hermitian operators $\widehat{L}_\mu$ (called generatores). Show explicitly by performing successive infinitesimal operations

$$\widehat{U}^{-1}\left(\delta\beta_\mu\right)\widehat{U}^{-1}\left(\delta\alpha_\mu\right)\widehat{U}\left(\delta\beta_\mu\right)\widehat{U}\left(\delta\alpha_\mu\right)$$

that the following (Lie) algebra holds for the generators $\widehat{L}_\mu$

$$\left[\widehat{L}_\mu, \widehat{L}_\nu\right] = C_{\mu\upsilon\lambda}\widehat{L}_\lambda$$