
Problem 16. –*Many particle operators*
Show, that the momentum operator $\hat{\vec{p}} = -i\hbar\vec{\nabla}$ is a single particle operator, but the position operator $\hat{\vec{x}} = \vec{x}$ isn't. Use the states from problem 13:

$$\Psi_{(+)} = \int \mathrm{d}^3p\ A(\vec{p}) \begin{pmatrix} m_0c^2 + \hbar\omega_p \\ m_0c^2 - \hbar\omega_p \end{pmatrix} e^{i(\frac{\vec{p}\cdot\vec{x}}{\hbar} - \omega_p t)}$$

$$\Psi_{(-)} = \int \mathrm{d}^3p\ \tilde{A}(\vec{p}) \begin{pmatrix} m_0c^2 - \hbar\omega_p \\ m_0c^2 + \hbar\omega_p \end{pmatrix} e^{i(\frac{\vec{p}\cdot\vec{x}}{\hbar} + \omega_p t)}$$

where $\hbar\omega_p = c\sqrt{p^2 + m_0^2c^4}$.

Problem 17. – *Some calculations*
Proof the following statement:

$$(\vec{\sigma}\cdot\vec{A})(\vec{\sigma}\cdot\vec{B}) = \vec{A}\cdot\vec{B}\mathbb{1}_2 + i\vec{\sigma}\cdot(\vec{A}\times\vec{B})$$

Problem 18. –*Free Dirac equation*
Find the stationary solutions of the free Dirac equation:

$$i\hbar\frac{\partial}{\partial t}\Psi = (c\vec{\alpha}\cdot\vec{p} + \beta m_0c^2)\Psi$$

Use the ansatz: $\Psi = \begin{pmatrix} \phi_0 \\ \chi_0 \end{pmatrix} e^{\frac{i}{\hbar}(\vec{p}\vec{r} - Et)}$.
What is the non-relativistic limit?

Problem 19. – *Dirac algebra*
Show, that the solutions α_i, β of the Dirac algebra:

$$\{\alpha_m, \alpha_n\} = \alpha_m\alpha_n + \alpha_n\alpha_m = 2\delta_{mn}\mathbb{1}_4$$
$$\{\alpha_m, \beta\} = \alpha_m\beta + \beta\alpha_m = 0$$
$$\alpha_m^2\beta^2 = \mathbb{1}_4$$

are determined up to an unitary transformation.