
Problem 8. – *Lippmann-Schwinger-Equation in position space*
We know the Lippmann-Schwinger-Equation from the lecture:

$$|\psi_E\rangle = |\phi_E\rangle + \mathrm{G}_0^{(+)} V |\psi_E\rangle = |\phi_E\rangle + \frac{1}{E - H_0 + i\epsilon} V |\psi_E\rangle$$

(a) Find the Lippmann-Schwinger-Equation for a plane wave $\phi_E(r) = Ae^{ikr}$ and a general potential $V(r)$ in position space.

Hint: expand $G_0^{(+)}$ in it's Fourier-Series

(b) Now consider an attractive δ-potential $V(r) = \frac{-\gamma}{2m}\delta^3(\vec{r}) \quad ; \gamma > 0$. Solve the Lippmann-Schwinger- Equation explicitly for a plane wave.

Problem 9. – *Lorentz Transformation*
Consider two inertial frames Σ, Σ' with parallel axes and relative speed $\vec{v} = v\vec{e_x}$. The transformation between Σ and Σ' is given by

$$\begin{pmatrix} x^{0\prime} \\ x^{1\prime} \end{pmatrix} = A(v) \begin{pmatrix} x^0 \\ x^1 \end{pmatrix} \quad , \; x^{2\prime} = x^2 \quad , \; x^{3\prime} = x^3. \tag{1}$$

Find the matrix $A(v)$ by using the 2. postulate of relativity, the linearity of the transformation, and $A^{-1}(v) = A(-v)$.

Problem 10. – *Relativistic Momentum- and Positon operator*
The contravariant four-vectors of position and momentum are: $x^\mu : \{ct, x, y, z\} = \{ct, \vec{r}\}$
$p^\mu : \{\frac{E}{c}, p_x, p_y, p_z\} = \{\frac{E}{c}, \vec{p}\}$. Find the form of the corresponding operators in position space and momentum space, that satisfy the commutation relation

$$[x^\mu, p^\nu] = -i\hbar g^{\mu\nu}.$$

Problem 11. – *Klein-Gordon-Equation*
Show, that the Klein-Gordon-Equation

$$-\hbar^2 \frac{\partial^2}{\partial t^2}\psi = (-\hbar^2 c^2 \Delta + m_0^2 c^4)\psi$$

remains invariant under to Lorentz transformations.