Problem 31. *–Trotter decompositions*
Show, that while

$$e^{-i\epsilon\frac{T+V}{\hbar}} = e^{-i\epsilon\frac{V}{\hbar}} e^{-i\epsilon\frac{T}{\hbar}} (1 + \frac{\epsilon^2}{2\hbar^2}[V,T]) \tag{1}$$

it holds

$$e^{-i\epsilon\frac{T+V}{\hbar}} = e^{-i\epsilon\frac{V}{2\hbar}} e^{-i\epsilon\frac{T}{\hbar}} e^{-i\epsilon\frac{V}{2\hbar}} + \mathcal{O}(\epsilon^3) \tag{2}$$

Problem 32. *–Feynman path integral*
In the lecture we used expression (2) to construct a path integral. Using (1) show, that alternatively one finds

$$\langle x_b, t | x_a, t=0 \rangle = \int \mathcal{D}x \int \mathcal{D}p \; e^{i\int \mathrm{d}t \frac{1}{\hbar}(p\dot{x} - H)} \tag{3}$$

where

$$\mathcal{D}x = \prod_{n=1}^{N} \mathrm{d}x_n \qquad \mathcal{D}p = \prod_{n=1}^{N+1} \frac{\mathrm{d}p_n}{2\pi\hbar}$$

Show first, that up to second order in ϵ

$$\langle x_n | \, e^{-i\epsilon\frac{H}{\hbar}} \, | x_{n-1} \rangle \approx \int \frac{\mathrm{d}p_y}{2\pi\hbar} e^{\frac{i}{\hbar}p_n(x_n - x_{n-1}) - \frac{i}{\hbar}\epsilon[T(p_n) + V(x_n)]}$$