Problem 1. – excitation with static electric field
A Hydrogen atom with spin-less electron, treated as an electron in a Coulomb potential centered at the origin, is initially prepared in its ground state. At $t = 0$ it is exposed to a homogeneous electric field

$$
E = E(t) e_z = E_0 e_z e^{-\Gamma t} \Theta(t)
$$

where $\Theta(t)$ is the Heaviside step function. The interaction Hamiltonian in dipole approximation reads

$$
H = e z E(t)
$$

What is the probability to find the atom at $t \to \infty$ in the excited state with $n = 2$?

Problem 2. – laser excitation of $1D$ harmonic oscillator
Consider a one-dimensional harmonic oscillator in an external laser field (oscillating electric field)

$$
H = H_0 + H_1,
$$

$$
H_0 = \frac{\hat{p}^2}{2m} + \frac{1}{2} m \omega_0^2 \hat{x}^2,
$$

$$
H_1 = \frac{e \hat{p}}{2m \omega} E_0 \sin(\omega t) - \frac{e \hat{x}}{2} E_0 \cos(\omega t).
$$

Initially the oscillator is in its ground state $|0\rangle$. What is the probability to find the oscillator in an excited state $|n\rangle$ at a time $t$, if the interaction $H_1$ with the laser field is switched on at $t = 0$?

Problem 3. – spontaneous and thermal induced transitions
Consider a Hydrogen atom excited to the $2p$ state and exposed to thermal radiation at temperature $T$. From there is can undergo either spontaneous or thermally induced emission to the ground state $1s$. Estimate at what temperature $T$ the transition probability per unit time for spontaneous and thermally induced emission are equal.

Problem 4. – momentum of the quantized electromagnetic field
The momentum of the quantized electromagnetic field is given by the integral over the momentum denisty

$$
\hat{P} = \varepsilon_0 \int d^3r \hat{E} \times \hat{B}.
$$

Show that $\hat{P}$ can be expressed in the following form

$$
\hat{P} = \sum_{k,\lambda} \hbar \hat{a}_{k,\lambda}^{\dagger} \hat{a}_{k,\lambda},
$$

which can be interpreted as the total momentum of photons each with momentum $\hbar k$. 