
Problem 1. – *excitation with static electric field*
A Hydrogen atom with spin-less electron, treated as an electron in a Coulomb potential centered at the origin, is initially prepared in its ground state. At $t = 0$ it is exposed to a homogeneous electric field

$$\mathbf{E} \;=\; E(t)\,\mathbf{e}_z = E_0\,\mathbf{e}_z\,e^{-\Gamma t}\Theta(t)$$

where $\Theta(t)$ is the Heaviside step function. The interaction Hamiltonian in dipole approximation reads

$$H \;=\; e\,z\,E(t)$$

What is the probability to find the atom at $t \to \infty$ in the excited state with $n = 2$?

Problem 2. – *laser excitation of 1D harmonic oscillator*
Consider a one-dimensional harmonic oscillator in an external laser field (oscillating electric field)

$$\begin{aligned} H &= H_0 + H_1, \\ H_0 &= \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega_0^2\hat{x}^2, \\ H_1 &= \frac{e\hat{p}}{2m\omega}E_0\sin(\omega t) - \frac{e\hat{x}}{2}E_0\cos(\omega t). \end{aligned}$$

Initially the oscillator is in its ground state $|0\rangle$. What is the probability to find the oscillator in an excited state $|n\rangle$ at a time t, if the interaction H_1 with the laser field is switched on at $t = 0$.

Problem 3. – *spontaneous and thermal induced transitions*
Consider a Hydrogen atom excited to the $2p$ state and exposed to thermal radiation at temperature T. From there is can undergo either spontaneous or thermally induced emission to the ground state $1s$. Estimate at what temperature T the transition probability per unit time for spontaneous and thermally induced emission are equal.

Problem 4. – *momentum of the quantized electromagnetic field*
The momentum of the quantized electromagnetic field is given by the integral over the momentum denisty

$$\hat{\mathbf{P}} = \varepsilon_0 \int d^3r\,\hat{\mathbf{E}} \times \hat{\mathbf{B}}.$$

Show that $\hat{\vec{P}}$ can be expressed in the following form

$$\hat{\mathbf{P}} = \sum_{\mathbf{k},\lambda} \hbar\,\mathbf{k}\,\hat{a}^\dagger_{\mathbf{k},\lambda}\hat{a}_{\mathbf{k},\lambda},$$

which can be interpreted as the total momentum of photons each with momentum $\hbar\,\mathbf{k}$.