7.2 superconducting qubits

Scalable quantum computing most likely means to go to the solid state. A promising avenue are superconducting qubits and Josephson elements.

- Solid-state devices are typically <u>dissipative</u> (Ohmic losses)

  $\Longrightarrow$ superconducting circuits

- transition frequencies in superconducting qubits (SCQ) are in the range of 5-20 GHz; to avoid thermal <u>fluctuations</u> and <u>noise</u>

  $\Longrightarrow$ low temperatures

- to create systems with two relevant states (qubits) we need nonlinear elements

  $\Longrightarrow$ Josephson junctions

two types of qubits

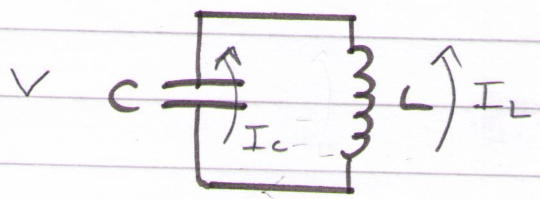
(i) charge qubit
(ii) flux qubit

literature:

- Y. Makhlin, G. Schön, A. Shnirman Rev. Mod. Phys. 73, 357 (2001)
- G. Wendin, V.S. Shumeiko, arxiv 0508729
- M. Devoret, A. Wallraff, J. Martinis, arxiv 0411174

(A) quantum LC oscillator

consider quantum description of an LC circuit



V voltage at capacitor
I current

Q charge on capacitor
Φ magn. flux through inductor

elements of circuit

(13) $I_c = \frac{d}{dt} Q = C \frac{dV_c}{dt}$ $\qquad I_L = \frac{\Phi}{L}$

(14) $V_L = \frac{d}{dt}\Phi$

kirchhoffs laws: $V_L = V_c \qquad I_c + I_L = 0$

(15) $$\boxed{\frac{d^2}{dt^2}\Phi + \frac{\Phi}{LC} = 0}$$

harmonic oscillator

(16) $$\omega_{LC} = \frac{1}{\sqrt{LC}}$$

energy of capacitor and of inductor

(17) $$H_C = \frac{1}{2C} Q^2 \qquad H_L = \frac{1}{2L} \Phi^2$$

Hamiltonian

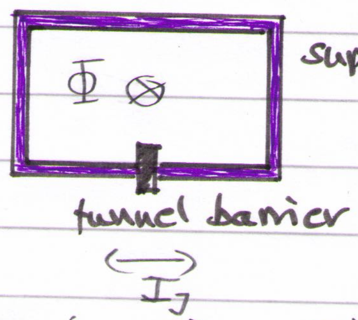
(18) $$\boxed{H = \frac{1}{2C} \hat{Q}^2 + \frac{1}{2L} \hat{\Phi}^2} \qquad \boxed{E_n = \hbar \omega_{LC} (n + \tfrac{1}{2})}$$

canonical coordinate: Φ momentum: Q

(19) $$[\hat{\Phi}, \hat{Q}] = i\hbar$$

(B) Josephson element

harmonic oscillator has aequidistant spectrum. In order to obtain a qubit with two relevant states we need a nonlinear element $\Rightarrow$ Josephson tunnel junction



Φ flux through ring
I_J current through barrier

Analogously to a Bose-Einstein condensate the Cooper pairs of a superconductor are described by a complex wave function

(20) $$\Psi = |\Psi| e^{i\varphi}$$

If a magnetic flux Φ penetrates the loop the cooper pairs pick up a phase proportional to the flux when going around the loop

$$(21) \qquad \theta = \Delta\varphi = \frac{2e}{\hbar}\Phi = 2\pi\frac{\Phi}{\Phi_0}$$

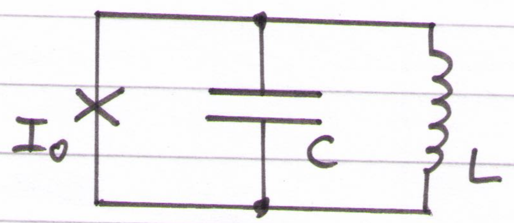
$\Phi_0 = h/2e$ flux quantums

The tunnel current increases with increasing flux but must be periodic in θ mod 2π

$$(22) \qquad \boxed{I_J = I_0 \sin\theta}$$

I_0 critical Josephson current

The loop acts as a primitive coil and has a capacity. The equivalent circuit diagram is thus



If there is in addition to the flux Φ created by the oscillating current an external magnetic flux Φ_e (used for tuning purposes)

$$(23) \qquad I_L = \frac{(\Phi - \Phi_e)}{L} = \frac{\hbar}{2eL}(\theta - \theta_e)$$

Kirchhoff yields than $I_C + I_L + I_J = 0$

$$(24) \qquad C\frac{d^2}{dt^2}\Phi + I_0 \sin(2\pi\Phi/\Phi_0) + \frac{\Phi - \Phi_e}{L} = 0$$

or equivalently

$$\frac{\hbar}{2e} C \ddot{\theta} + I_0 \sin\theta + \frac{\hbar}{2eL}(\theta - \theta_e) = 0 \tag{25}$$

It is easy to see that this is the Euler-Lagrange equation

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\theta}} - \frac{\partial \mathcal{L}}{\partial \theta} = 0$$

for

$$\boxed{\mathcal{L} = \frac{\hbar^2}{4E_C}\dot{\theta}^2 - E_J(1-\cos\theta) - E_L\frac{(\theta-\theta_e)^2}{2}} \tag{26}$$

where

$$\boxed{E_C = \frac{(2e)^2}{2C}} \quad \underline{\text{charging energy}} \tag{27}$$

= energy of capacitor C with charge difference of one Cooperpair 2e

and

$$\boxed{E_J = \frac{\hbar}{2e} I_0} \quad \underline{\text{Josephson energy}} \tag{28}$$

and

$$\boxed{E_L = \frac{\Phi_0^2}{4\pi^2 L}} \quad \underline{\text{induction energy}} \tag{29}$$

(one flux quantum)

Hamiltonian $\quad H = p\dot{\theta} - \mathcal{L}$

with

$$p = \frac{\partial \mathcal{L}}{\partial \dot{\theta}} = \left(\frac{\hbar}{2e}\right)^2 C\dot{\theta} \tag{30}$$

Since $\dot{\theta} = \frac{2e}{\hbar}\dot{\Phi}$ (see eq. (21)) and $\dot{\Phi} = V_L$ one finds

$$(31) \qquad p = \frac{\hbar}{2e} C V = \hbar \frac{Q}{2e} = \hbar n$$

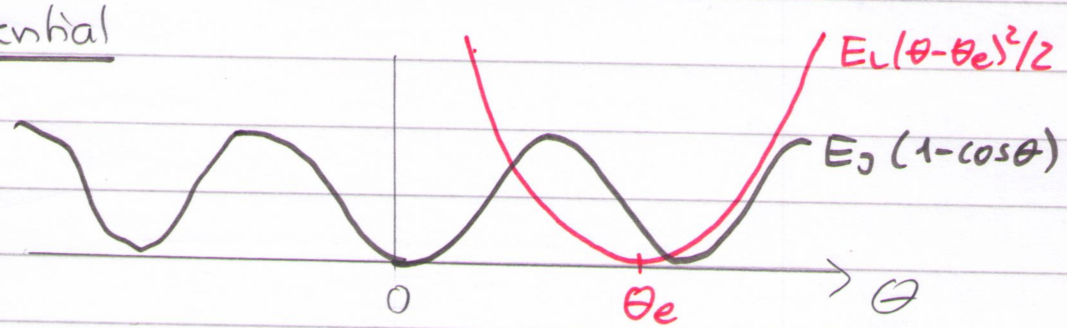
where n is the <u>number of Cooper pairs</u>

$$(32) \qquad \boxed{H = E_C \hat{n}^2 + E_J(1-\cos\hat{\theta}) + E_L \frac{(\hat{\theta}-\theta_e)^2}{2}}$$

fundamental commutator

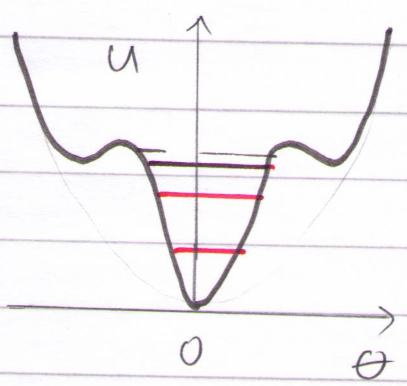
$$(33) \qquad \boxed{[\hat{\theta}, \hat{n}] = i}$$

<u>potential</u>

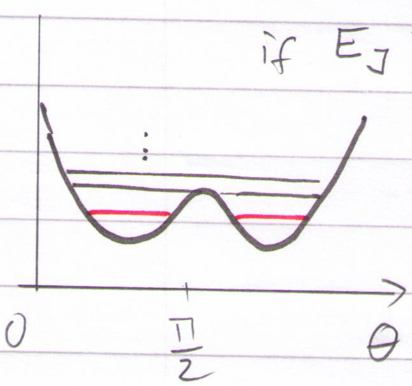


$$U = E_J(1-\cos\theta) + E_L(\theta-\theta_e)^2/2$$

$\boxed{\theta_e = 0}$ $\qquad$ $\boxed{\theta_e = \pi/2}$

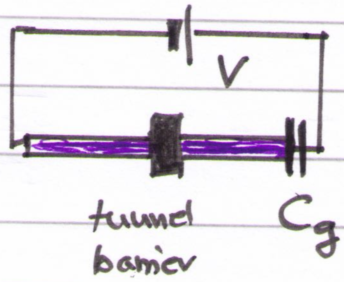


if $E_J > E_L$



(c) single cooper-pair box (SCB): charge qubit

- qubit



application of external voltage V via gate capacitance allows to change number of cooper pairs ; $E_L \approx 0$

(34) $$\boxed{H = E_C(\hat{n} - n_g)^2 - E_J \cos\hat{\theta}}$$

now $E_C = \dfrac{(2e)^2}{2(C_g + C_J)}$ $\qquad \underbrace{2e\, n_g}_{\text{charge}} = C_g V$

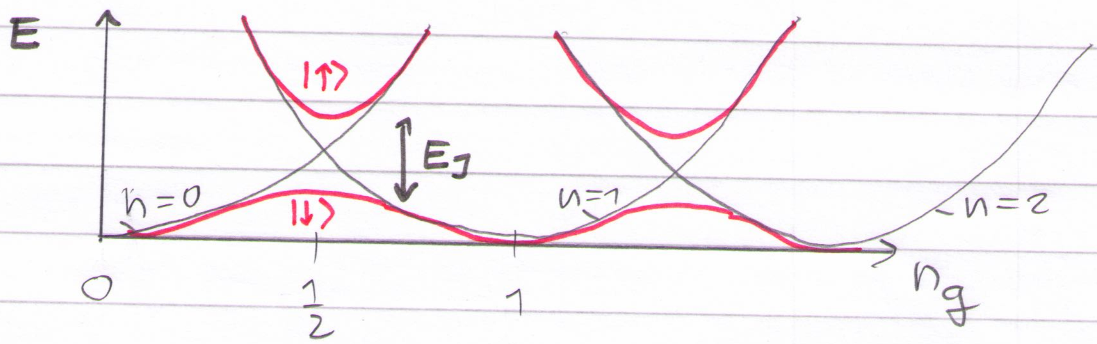
$\boxed{E_C \gg E_J}$

introduce basis states of number of cooper pairs $\hat{n}$

(35) $$H = \sum_n E_C (n - n_g)^2 |n\rangle\langle n| - \frac{1}{2} E_J \big(|n\rangle\langle n+1| + |n+1\rangle\langle n| \big)$$

as $\langle n| \cos\hat{\theta} |n'\rangle = \delta_{n,n'\pm 1}$

Energy sequence of parabolas as function of n_g plus small perturbation $\sim E_J$



for $n_g \simeq \frac{1}{2}$ effective two-level system

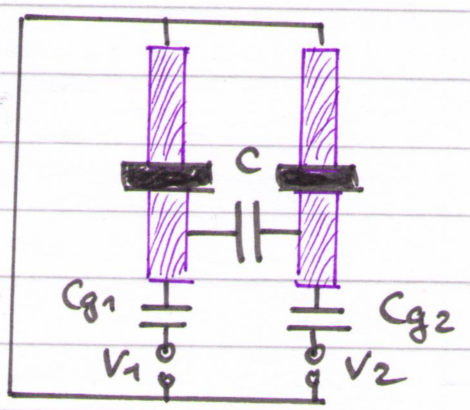
(36) $$\boxed{H = -\frac{1}{2} B_z \sigma_z - \frac{1}{2} B_x \sigma_x}$$

(37) $$B_z = E_C (1 - 2n_g) \qquad B_x = E_J$$

varying n_g through the applied voltage allows to perform single-bit operations

- quantum gates

parallel tunnel junctions with capacitive coupling

(38) 

$$E_C' = \frac{(2e)^2}{2C}$$

charging energy of coupling capacitor

(39) $$H_{int} = \sum_{n_1, n_2} E_C' (n_1 - n_2)^2 \, |n_1, n_2\rangle \langle n_1, n_2|$$

for $n_{g1} \approx n_{g2} \approx \frac{1}{2}$ $\qquad |n_1, n_2\rangle \in \{ |00\rangle, |01\rangle, |10\rangle, |11\rangle \}$

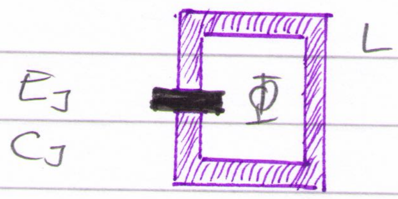
$$H_{int} = E_C' \left(|01\rangle\langle 01| + |10\rangle\langle 10| \right)$$

$$= E_C' \left((1+\sigma_z^1)(1-\sigma_z^2) + (1-\sigma_z^1)(1+\sigma_z^2) \right)$$

(40) $\boxed{H_{int} = E_C' \left(2 - \sigma_z^1 \sigma_z^2 \right)}$ Ising interaction

(D) Josephson flux qubit

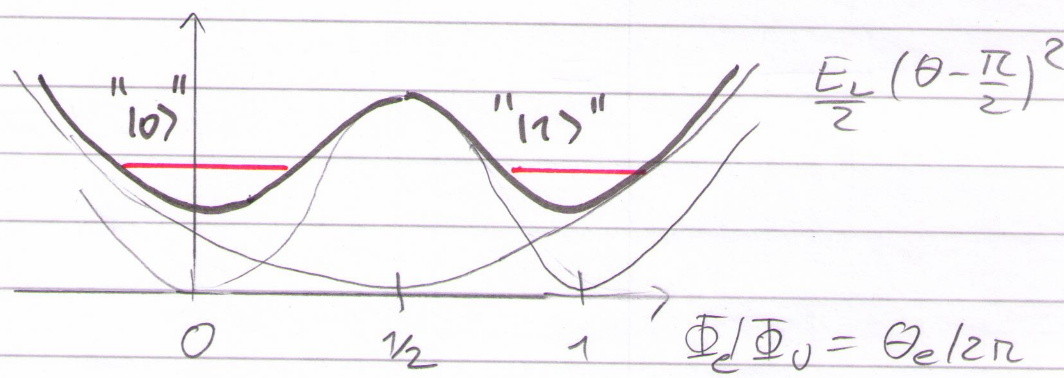
$\boxed{E_J \gg E_C}$



now E_L cannot be ignored

(41) $H = E_C \hat{n}^2 + E_J (1 - \cos\hat{\theta}) + E_L \dfrac{(\hat{\theta} - \theta_e)^2}{2}$
$\hat{=}$ (32)

Choose $\theta_e \simeq \pi/2$ i.e. $\Phi_e \simeq \Phi_0/2$



single-bit hamiltonian

$$H = -\frac{1}{2} B_z \sigma_z - \frac{1}{2} B_x \sigma_x \tag{42}$$

here

$$B_z = 4\pi \sqrt{6(\beta_L - 1)}\, E_J \left(\frac{\Phi_e}{\Phi_0} - 1 \right) \tag{43}$$

$$\beta_L = E_J / (\Phi_0^2 / 4\pi^2 L)$$

Two qubit gates can be realized via an inductive coupling of

flux qubit 1 flux qubit 2

= the end =