The key requirements that need to be fulfilled by any physical implementation of quantum information processing have been summarized by David DiVincenzo ("The physical implementation of quantum computing," Found. Phys. 38 (2000), 771–783).

(i) A scalable system of well defined qubits
(ii) The ability to prepare the state of a qubit
(iii) Sufficiently long coherence times
(iv) Implementation of a minimal set of universal quantum gates
(v) The ability to measure the state of a qubit in the computational basis

In addition there are two more requirements to build quantum networks

(vi) The ability to interconvert "stationary" into "flying" qubits and vice versa
(vii) The ability to faithfully transmit flying qubits between different locations

We will discuss here only a few important examples.
7.1 Ion traps

One of the first and currently the most precise (highest fidelity) implementations make use of the strong Coulomb interaction between trapped ions. Scalability is however an issue, J. I. Cirac and P. Zoller (Phys. Rev. Lett. 74, 4091 (1995))

Linear Paul trap

Quadrapole field (dynamic! Static field cannot produce extremum in center)

\[ \phi = \frac{U + V \cos(\omega t)}{2\nu^2} (x^2 - y^2) \]

\[ t = 0 \]

\[ t = \frac{\pi}{\nu} \]

Oscillating or rotating saddle point
for certain parameter regime of \( U, V, \Omega \),

stable dynamics with time-averaged harmonic potential

\[
\phi_{\text{eff}} = \frac{1}{2} m \omega^2 (x^2 + y^2)
\]

\[
\omega^2 = \frac{9 V}{12 \pi \rho \text{C}_0^2}
\]

in addition: micromotion (on short time scales).

Additional electrodes at end caps yield confinement in \( z \)-direction.

\[
\omega_z < \omega
\]

Coulomb repulsion of ions \( \Rightarrow \) small linear.

Crystals of ions (phase transition to other configuration if \# of ions increased or parameter modified).

elementary excitations: oscillations about equilibrium position.

lowest mode: in-phase oscillation of all ions

\[
\cdots \rightarrow V_0 \rightarrow \cdots
\]

\[
\begin{align*}
&\ll 0.2 \text{mm} \\
&\text{for } k_B T \ll V_1 - V_0 \Rightarrow \text{only lowest oscillation mode relevant}
\end{align*}
\]
gubit: two internal states $|g\rangle, |e\rangle$
(with optical transition $|g\rangle \leftrightarrow |e\rangle$)

distance of ions $\rightarrow$ optical wavelength $\rightarrow$ individual addressing

$|e\rangle_j, |g\rangle_j$  
$j$ - ion index

$J_{2j} = \frac{dE_j}{\hbar}$

$J_{2j}$ - Rabi frequency of optical drive

Hamiltonian

\begin{equation}
H = H_0^{\text{ions}} + \hbar v_0 a^+ a + \sum_{j=1}^{N} \frac{\hbar J_{2j}}{2} \sin (\vec{r}_j \cdot \vec{r}_j - \omega t) (a_j^+ + a_j)
\end{equation}

$|e\rangle_j = \langle g_j | \langle g_j |$  
$|g\rangle_j = \langle e_j|$  

For small oscillation amplitudes $m = Nm_0$  
$m_0$ mass of ion

\begin{equation}
\hat{1}_j = \frac{\hat{1}_j}{\omega_0} + \sqrt{\frac{\hbar}{2m_{25}}} \left( \hat{a}^+ \hat{a} - \hat{a}^+ \hat{a} \right)
\end{equation}

(5)  
$\vec{b}_j \cdot \vec{r}_j = \vec{b}_j \cdot \vec{r}_j + k_{j2} \hat{z}$

(6)  
$k_{j2} \hat{z} = \gamma (\hat{a}^+ \hat{a})$

Lamb-Dicke parameter
\( v^2 = \frac{1}{N} \frac{\hbar}{2m_0} \left( \frac{2\pi}{\gamma} \right)^2 \frac{1}{v_0} = \frac{\omega_{\text{rec}}}{v_0^2} \)

\( \omega_{\text{rec}} = \) recoil frequency (recoil energy = kinetic energy which an atom exchanges by momentum transfer when absorbing/ emitting a photon of momentum \( k \)).

\( \omega_{\text{rec}} \) is very small \( \Rightarrow \) \( v \) small

Expansion of eq. (3) (Lamb–Dicke regime)

\( e^{i \bar{r}_{ij}} \approx 1 + \frac{i}{2} \eta (\hat{a}^+ + \hat{a}^-) \)

then

\[
H = H_0^{\text{ion}} - \sum_{j=0}^{N} \frac{\hbar R_j}{2} (\hat{d}_j^+ + \hat{d}_j^-) \sin \omega t \]

\[
+ \sum_{j=0}^{N} \frac{\hbar R_j}{2} \eta (\hat{a}^+ \hat{a}^-) (\hat{d}_j^+ + \hat{d}_j^-) \cos \omega t
\]

In the interaction picture \( \hat{a}^+ \rightarrow \hat{a}^+ e^{i\omega t}, \hat{d}^+ \rightarrow \hat{d}^+ e^{-i\omega t} \)

and rotating wave approximation (RWA)

\( e^{\pm i(\omega_0 + \omega)t} \approx 0 \) average to zero

\[
H = H_0^{\text{ion}} + \sum_{j=0}^{N} \frac{i \hbar R_j}{2} (e^{-i \omega t} \hat{d}_j^+ - \text{h.a.})
\]

\[
+ \sum_{j=0}^{N} \frac{\hbar R_j}{2} \eta \left[ (\hat{a}^+ e^{-i(\Delta + \omega)t} + \hat{a}^- e^{i(\Delta + \omega)t}) \hat{d}_j^+ \right.
\]

\[
+ \text{h.a.}
\]

where \( \Delta = \omega - \omega_0 \). Thus an individual ion has the following transitions
Optical excitation of an ion on a sideband is associated with absorption / emission of a phonon of the collective oscillations of all ions \( \Rightarrow \) coupling to other ions (qubit). Coupling on red sideband only possible if phonon mode is excited, i.e., if \( n \geq 1 \).

If \( \nu \) is larger than the decay rate and Rabi-frequency, the different bands can be spectroscopically resolved.
phase gate (equivalent to CNOT)

phonon mode in ground state, i.e. \( n = 0 \)
(cooling required!)

qubits: ion 1 \( \{ |g\rangle, |e\rangle \} \)
ion 2 \( \{ |g\rangle, |e\rangle \} \)

need degenerate excited state (auxiliary level)

e.g.

\[
\begin{array}{c}
|e\rangle \\
\text{Circular polarization} \\
\text{Linear polarization} \\
|g\rangle
\end{array}
\]

\( \text{\textcolor{red}{(i)}} \) \( \pi \) laser pulse on ion 1 on red sideband

\[
\begin{align*}
|g\rangle |g\rangle |e\rangle |e\rangle |e\rangle & \to |g\rangle |g\rangle |g\rangle \text{ red sideband and } n = 0 \\
|g\rangle |g\rangle |e\rangle |e\rangle |e\rangle & \to -i |g\rangle |g\rangle |g\rangle |g\rangle |e\rangle \\
|g\rangle |g\rangle |e\rangle |e\rangle |e\rangle & \to -i |g\rangle |g\rangle |g\rangle |e\rangle |e\rangle
\end{align*}
\]

\( \text{\textcolor{red}{(ii)}} \) \( 2\pi \) laser pulse on ion 2 (\( g \leftrightarrow e' \)) on red sideband

\[
\begin{align*}
|g\rangle |g\rangle |e\rangle |e\rangle |e\rangle & \to |g\rangle |g\rangle |g\rangle |g\rangle |e\rangle \text{ red sideband} \\
|g\rangle |g\rangle |e\rangle |e\rangle |e\rangle & \to -i |g\rangle |g\rangle |g\rangle |g\rangle |e\rangle \\
|g\rangle |g\rangle |e\rangle |e\rangle |e\rangle & \to -i |g\rangle |g\rangle |g\rangle |e\rangle |e\rangle
\end{align*}
\]
[(iii)] \[ \pi \text{ laser pulse at ion 1 on red sideband} \]

\[ \begin{align*}
|g_1\rangle|g_2\rangle|10\rangle & \rightarrow |g_1\rangle|g_2\rangle|10\rangle \\
|g_1\rangle|e_2\rangle|10\rangle & \rightarrow |g_1\rangle|e_2\rangle|10\rangle \\
|g_1\rangle|q_2\rangle|11\rangle & \rightarrow |e_1\rangle|g_2\rangle|10\rangle \\
|e_1\rangle|e_2\rangle|11\rangle & \rightarrow -|e_1\rangle|e_2\rangle|10\rangle
\end{align*} \]

called together, Phase gate

\[ \begin{align*}
|g\rangle|g\rangle & \rightarrow |g\rangle|g\rangle \\
|g\rangle|e\rangle & \rightarrow |g\rangle|e\rangle \\
|e\rangle|g\rangle & \rightarrow |e\rangle|g\rangle \\
|e\rangle|e\rangle & \rightarrow -|e\rangle|e\rangle
\end{align*} \]

- First experimental implementation:

- Problem of Cirac-Zoller gate: cooling of ions to vibrational ground state \((n=0)\), proposal without need of ground-state cooling: A. Sørensen, K. Mølmer

- Scaling to large system using (state-preserving) transport of ions

\[ \text{register/memory} \]

\[ \text{operation region} \]

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