7. Experimental implementations of quantum information processing

The key requirements that need to be fulfilled by any physical implementation of quantum information processing have been summarized by <u>David DiVincenzo</u> ("The physical implementation of quantum computation" Fortschr. Phys. <u>48</u> (2000), 771–783)

(i)	a scalable system of well defined qubits
(ii)	the ability to prepare the state of a qubit
(iii)	sufficiently long coherence times
(iv)	implementation of a minimal set of universal quantum gates
(v)	the ability to measure the state of a qubit in the computational basis

In addition there are two more requirements to build quantum <u>networks</u>

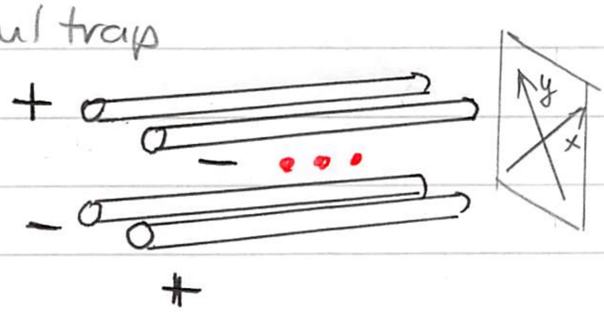
(vi)	the ability to interconvert "stationary" into "flying" qubits and vice versa
(vii)	the ability to faithfully transmit flying qubits between different locations

We will discuss here only a few important examples.

7.1 ion traps

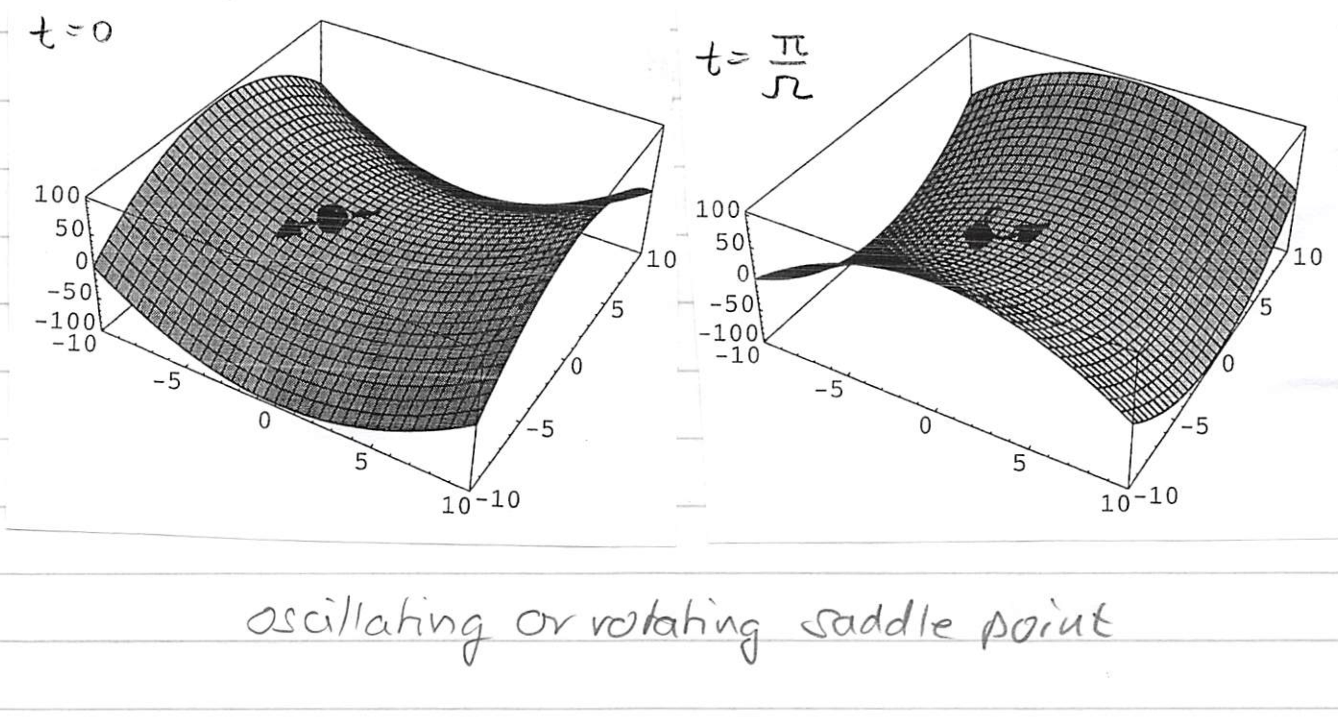
One of the first and currently the most precise (highest fidelities) implementations make use of the strong Coulomb interaction between trapped ions. Scalability is however an issue, J.I. Cirac and P. Zoller (Phys. Rev. Lett. 74, 4091 (1995))

linear Paul trap



quadrupole field (dynamic! static field cannot produce extremum in center)

$$\Phi = \frac{U + V\cos(\Omega t)}{2 r_0^2}\,(x^2 - y^2) \qquad (1)$$

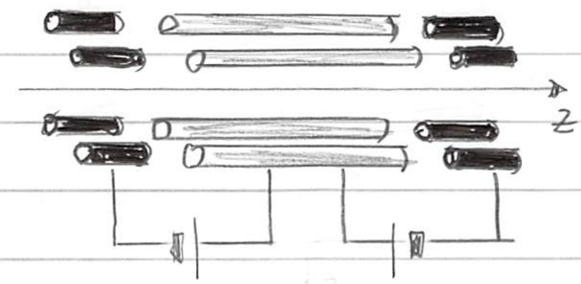


oscillating or rotating saddle point

for certain parameter regime of U, V, Ω
stable dynamics with time-averaged harmonic potential

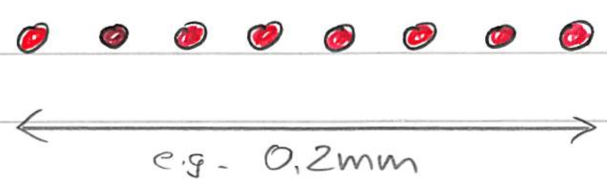
(2) $$q\phi_{eff} = \frac{1}{2} m \omega_r^2 (x^2+y^2) \qquad \omega_r \simeq \frac{qV}{\sqrt{2} m \Omega r_0^2}$$

in addition: micromotion (on short timescales).
Additional electrodes at end caps yield confinement in z-direction



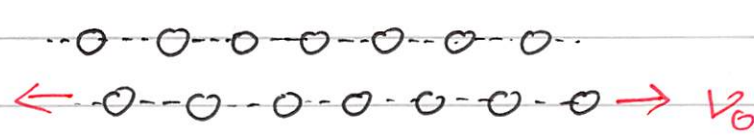
potential in z-direction
$\omega_z < \omega_r$

Coulomb repulsion of ions $\Rightarrow$ small linear crystals of ions (phase transition to other configurations if # of ions increased or parameter modified)



elementary excitations: oscillations about equilibrium position.

lowest mode: in-phase oscillation of all ions

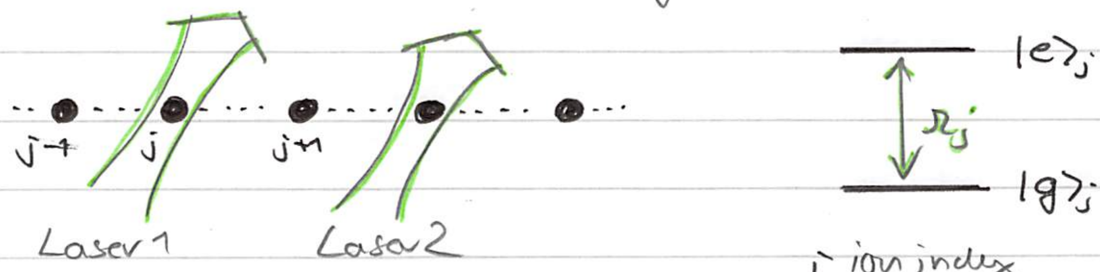


e.g. $\nu_0 = 2\pi \times 50\,\text{kHz}$
$\nu_1 = \sqrt{3}\,\nu_0$

for $k_B T \ll \nu_1 - \nu_0 \Rightarrow$ only lowest oscillation mode relevant

qubit: two internal states $|g\rangle, |e\rangle$
with optical transition $|g\rangle \leftrightarrow |e\rangle$

distance of ions $\gg$ optical wavelength $\Rightarrow$ individual addressing



j ion index

Ω_j Rabi-frequency of optical drive

$\Omega_j = dE_j/\hbar$

Hamiltonian

$$(3) \qquad H = H_0^{ions} + \underbrace{\hbar \nu_0 a^+ a}_{\text{harmonic oscillator of common mode}} + \sum_{j=1}^{N} \frac{\hbar \Omega_j}{2} \sin(\vec{k}_j \vec{r}_j - \omega t)(\sigma_j^+ + \sigma_j^-)$$

$\sigma_j^+ = |e\rangle_{j\,j}\langle g|$ $\qquad$ $\sigma_j^- = |g_j\rangle_j\langle e|$

For small oscillation amplitudes $\qquad m = N m_0 \qquad m_0$ mass of one ion

$$(4) \qquad \hat{\vec{r}}_j = \hat{\vec{r}}_{j0} + \underbrace{\sqrt{\frac{\hbar}{2m\nu_0}}(\hat{a} + \hat{a}^+)}_{= \hat{z}} \vec{e}_z$$

$$(5) \qquad \vec{k}_j \vec{r}_j = \vec{k}_j \vec{r}_{j0} + k_{jz} \hat{z}$$

$$(6) \qquad k_{jz} \hat{z} = \eta (\hat{a}^+ + \hat{a})$$

$$(7) \qquad \eta = \frac{\pi}{\lambda} \sqrt{\frac{2\hbar}{m_0 \nu}} \frac{1}{\sqrt{N}} \qquad \text{Lamb-Dicke parameter}$$

(8) $$\eta^2 = \frac{1}{N}\frac{\hbar}{2m_0}\left(\frac{2\pi}{\lambda}\right)^2\frac{1}{\nu_0} = \frac{\omega_{rec}}{N\nu_0}$$

ω_{rec} = recoil frequency (recoil energy = kinetic energy which an atom exchanges by momentum transfer when absorbing/emitting a photon of momentum $\hbar k$)
ω_{rec} is very small $\sim$ η small

expansion of eq. (3) (Lamb-Dicke regime)

(9) $$e^{i\vec{k}_j \vec{r}_j} \simeq 1 + i\eta(\hat{a}^+ + \hat{a})$$

then

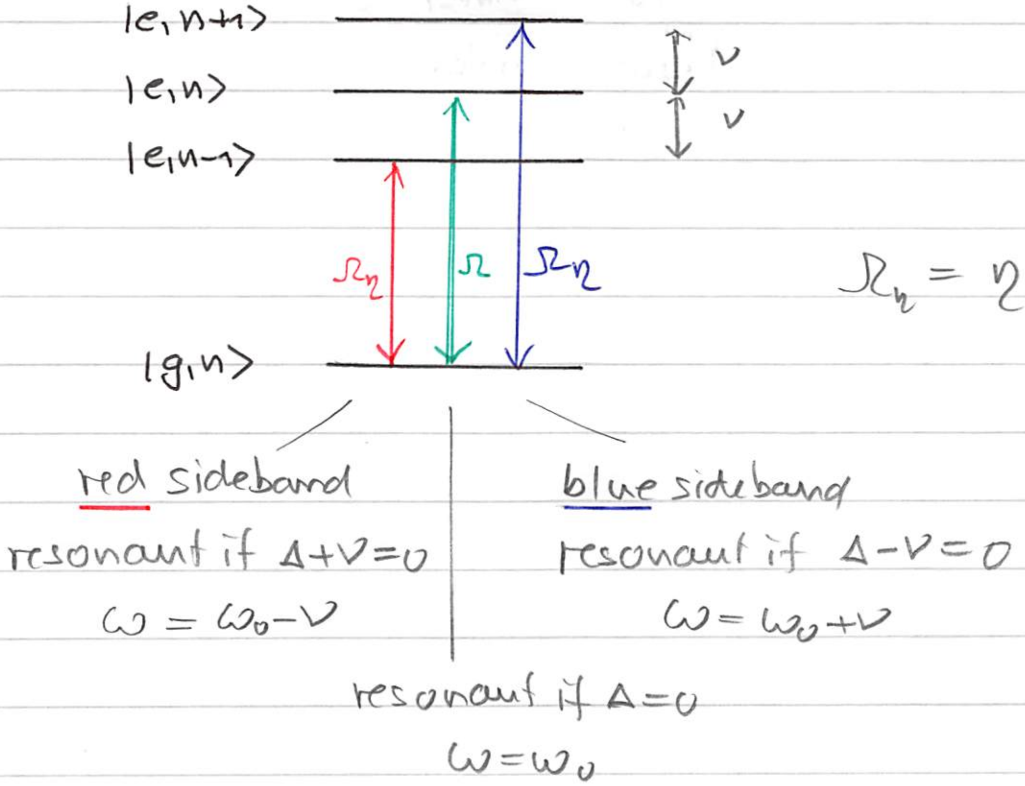
(10) $$H = H_0^{ion} - \sum_{j=1}^{N} \frac{\hbar\Omega_j}{2}(\sigma_j^+ + \sigma_j^-)\sin\omega t$$
$$+ \sum_{j=1}^{N} \frac{\hbar\Omega_j}{2}\eta(\hat{a}+\hat{a}^+)(\sigma_j^+ + \sigma_j^-)\cos\omega t$$

In the interaction picture $\sigma^+ \to \sigma^+ e^{i\omega_0 t}$, $\hat{a} \to \hat{a}e^{-i\nu t}$
and rotating wave approximation (RWA)

(11) $$e^{\pm i(\omega_0+\omega)t} \approx 0$$ averages to zero

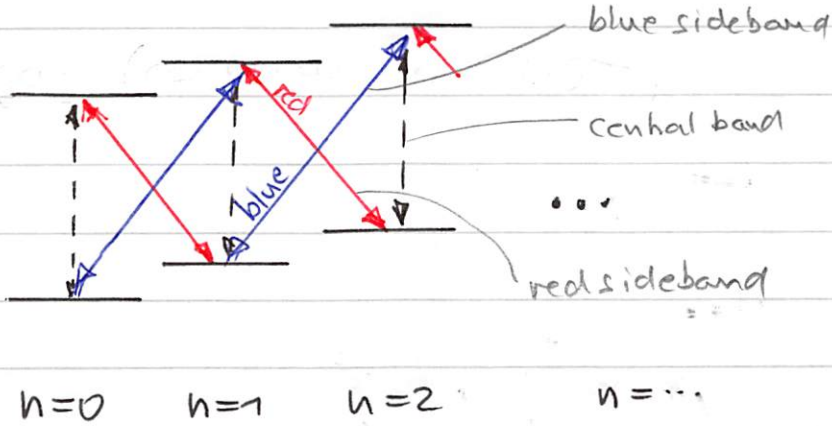
(12) $$H = H_0^{ion} + \sum_{j=1}^{N} i\frac{\hbar\Omega_j}{2}\left(e^{-i\Delta t}\sigma_j^+ - h.a.\right)$$
$$+ \sum_{j=1}^{N}\frac{\hbar\Omega_j}{2}\eta\left[\left(\hat{a}^+ e^{-i(\Delta-\nu)t} + \hat{a}e^{i(\Delta+\nu)t}\right)\sigma_j^+ + h.a.\right]$$

where $\Delta = \omega - \omega_0$. Thus an individual ion has the following transitions



$$\Omega_\eta = \eta\,\Omega \ll \Omega$$

optical excitation of an ion on a sideband is associated with absorption / emission of a phonon of the collective oscillations of all ions $\Rightarrow$ coupling to other ions (qubits). Coupling on red sideband only possible if phonon mode is excited, i.e. if $n \geq 1$



If ν is larger than the decay rates and Rabi-frequencies the different bands can be spectroscopically resolved.

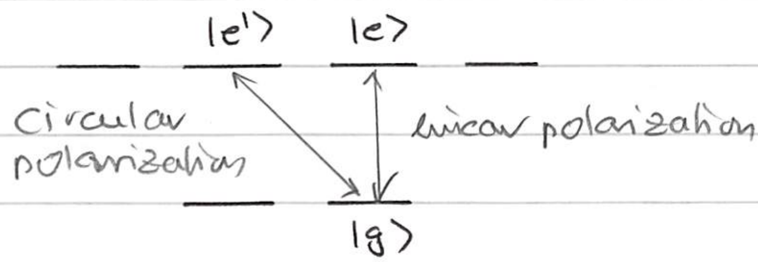
phase gate (equivalent to CNOT)

phonon mode in ground state, i.e. $n=0$
(cooling required!)

qubits: ion 1 $\{|g\rangle_1, |e\rangle_1\}$
ion 2 $\{|g\rangle_2, |e\rangle_2\}$

need degenerate excited state (auxiliary level)

e.g.



(i) π laserpulse at ion 1 on red sideband

$$\begin{aligned} &|g_1\rangle|g_2\rangle|0\rangle & & |g_1\rangle|g_2\rangle|0\rangle \\ &|g_1\rangle|e_2\rangle|0\rangle & \longrightarrow & |g_1\rangle|e_2\rangle|0\rangle \\ &|e\rangle_1|g_2\rangle|0\rangle & & -i\,|g_1\rangle|g_2\rangle|1\rangle \\ &|e\rangle_2|e\rangle_2|0\rangle & & -i\,|g_1\rangle|e_2\rangle|1\rangle \end{aligned}$$

(first two lines: red sideband and $n=0$)

(ii) 2π laserpulse at ion 2 ($g \leftrightarrow e'$) on red sideband

$$\begin{aligned} &|g_1\rangle|g_2\rangle|0\rangle & & |g_1\rangle|g_2\rangle|0\rangle \\ &|g_1\rangle|e_2\rangle|0\rangle & & |g_1\rangle|e_2\rangle|0\rangle \\ &-i\,|g_1\rangle|g_2\rangle|1\rangle & \longrightarrow & +i\,|g_1\rangle|g_2\rangle|1\rangle \\ &-i\,|g_1\rangle|e_2\rangle|1\rangle & & -i\,|g_1\rangle|e_2\rangle|1\rangle \end{aligned}$$

(first two lines: red sideband)

(iii) π laserpulse at ion 1 on red sideband

$$\begin{array}{rcl} |g_1\rangle|g_2\rangle|0\rangle & & |g_1\rangle|g_2\rangle|0\rangle \\ |g_1\rangle|e_2\rangle|0\rangle & \longrightarrow & |g_1\rangle|e_2\rangle|0\rangle \\ i\,|g_1\rangle|g_2\rangle|1\rangle & & |e_1\rangle|g_2\rangle|0\rangle \\ -i\,|g_1\rangle|e_2\rangle|1\rangle & & -|e_1\rangle|e_2\rangle|0\rangle \end{array}$$

red sideband

alltogether Phase-gate

$$\boxed{\begin{array}{rcl} |g\rangle|g\rangle & \longrightarrow & |g\rangle|g\rangle \\ |g\rangle|e\rangle & \longrightarrow & |g\rangle|e\rangle \\ |e\rangle|g\rangle & \longrightarrow & |e\rangle|g\rangle \\ |e\rangle|e\rangle & \longrightarrow & -|e\rangle|e\rangle \end{array}}$$

- first experimental implementations
  D. Wineland (Boulder, NIST) Nature 422, 412 (2003)
  R. Blatt (Innsbruck) Nature 421, 48 (2003)

- problem of Cirac-Zoller gate: cooling of ions to vibrational ground state ($n=0$). proposal without need of ground-state cooling: A. Sørensen, K. Mølmer Phys. Rev. Lett 82, 1971 (1999)

- scaling to large system using (state-preserving) transport of ions