7. Experimental implementation of quantum information processing

The key requirements that need to be fulfilled by any physical implementation of quantum information processing have been summarized by David DiVincenzo ("The physical implementation of quantum computation," Fort. Phys. 48 (2000), 771-783):

(i) a scalable system of well-defined qubits
(ii) the ability to prepare the state of a qubit
(iii) sufficiently long coherence times
(iv) implementation of a minimal set of universal quantum gates
(v) the ability to measure the state of a qubit in the computational basis

In addition there are two more requirements to build quantum networks:

(vi) the ability to interconvert "stationary" into "flying" qubits and vice versa
(vii) the ability to faithfully transmit flying qubits between different locations

We will discuss here only a few important examples.
7.1 ion traps

One of the first and currently the most precise (highest fidelities) implementations make use of the strong Coulomb interactions between trapped ions. Scalability is however an issue. J.I. Cirac and P. Zoller (Phys. Rev. Lett. 74, 4091 (1995))

linear Paul trap

\[ \phi = \frac{U + V \cos(2\pi t)}{2r_0^2} (x^2 - y^2) \]

oscillating or rotating saddle point
for certain parameter regime of $U, V, R$

stable dynamics with time-averaged harmonic potential

$$\phi_{\text{eff}} = \frac{4}{3} m \omega_r^2 (x^2 + y^2) \quad \omega_r = \frac{g V}{12 m J_2 r_0}$$

in addition: microphonon (on short timescales)
additional electrodes on end caps yield confinement in $z$-direction

\[ \omega_z < \omega_r \]

Coulomb repulsion of ions $\Rightarrow$ small linear
crystals of ions (phase transition to other configuration if # of ions increased or parameter modified)

elementary excitations: oscillations about equilibrium position.

lowest mode: in-phase oscillation of all ions

$$\cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \rightarrow V_0$$

$$\leftarrow \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \rightarrow V_0$$

for $k_B T \ll V_1 - V_0 \Rightarrow$ only lowest oscillation mode relevant
gubit: two internal states 
\( |g\rangle \), \( |e\rangle \)

(with optical transition 
\( |g\rangle \leftrightarrow |e\rangle \)

distance of ions \( \Rightarrow \) optical wavelength \( \Rightarrow \) individual addressing

\( 1e_j \)

\( j \) ion index

\( 2_j \) Rabi frequency of optical drive

\( J_{2j} = d E_j / \hbar \)

Hamiltonian

\[
H = H_0 \text{ ions} + \hbar \nu_0 a^+ a + \sum_{j=1}^{N} \frac{\hbar R_j \sin (b_j \varphi_j - \omega t)}{2} (a_j^+ a_j)
\]

\( a_j^+ = |e\rangle_{j} \langle g| \quad a_j = |g\rangle_{j} \langle e| \)

For small oscillation amplitudes \( m = N m_0 \)

\[
\hat{r}_j = \hat{r}_{j0} + \sqrt{\frac{\hbar}{2 m_0 \nu}} (\hat{a} + \hat{a}^+) \hat{z}
\]

\[
\hat{b}_j \hat{r}_j = \frac{\hbar^2}{2} \hat{r}_{j0} + k_{j2} \hat{z}
\]

\[
k_{j2} \hat{z} = \gamma \left( \hat{a}^+ + \hat{a} \right)
\]

\[
\gamma = \frac{\pi}{\sqrt{2}} \sqrt{\frac{2 \hbar}{m_0 \nu}} \frac{1}{\sqrt{N}} \text{ Lamb-Dicke parameter}
\]
(8) \[ \gamma^2 = \frac{1}{N} \frac{b \hbar}{2m_0} \left( \frac{2 \pi}{\lambda} \right)^2 \frac{1}{v_0} = \frac{\omega_{\text{rec}}}{N \nu_0} \]

\( \omega_{\text{rec}} \) = recoil frequency (recoil energy = kinetic energy which an atom exchanges by momentum transfer when absorbing/emanating a photon of momentum \( \hbar k \))

\( \omega_{\text{rec}} \) is very small \( \ll \eta \) small

Expansion of eq. (3) (Lamb-Dicke regime)

(9) \[ e^{i \hbar \omega t} \sim 1 + i \eta (\hat{a}^+ + \hat{a}) \]

Then

\[ H = H_0 \hbar \omega - \sum_{j=0}^{N} \frac{\hbar \omega_j}{2} (\hat{d}_j^+ + \hat{d}_j^-) \sin \omega t \]

(10) \[ + \sum_{j=0}^{N} \frac{\hbar \omega_j}{2} \eta (\hat{a}^+ \hat{d}_j^+ + \hat{a} \hat{d}_j^+) \cos \omega t \]

In the interaction picture \( \hat{a}^+ \rightarrow \hat{a}^+ e^{i \omega t}, \hat{a} \rightarrow \hat{a} e^{-i \omega t} \) and rotating wave approximation (RWA)

(11) \[ e^{i (\omega \omega_0 + \omega^2) t} \approx 0 \] averaged to zero

\[ H = H_0 \hbar \omega + \sum_{j=0}^{N} \frac{\hbar \omega_j}{2} (\hat{d}_j^+ - \h.o.) \]

(12) \[ + \sum_{j=0}^{N} \frac{\hbar \omega_j}{2} \eta \left[ \hat{a}^+ e^{-i (\Delta + \nu) t} + \hat{a} e^{i (\Delta + \nu) t} \right] \hat{d}_j^+ \]

[ + h.o. ]

where \( \Delta = \omega - \omega_0 \). Thus an individual ion has the following transition...
Optical excitation of an ion on a sideband is associated with absorption/emission of a phonon of the collective oscillations of all ions \( \Rightarrow \) coupling to other ions (qubits). Coupling on red sideband only possible if phonon mode is excited, i.e., if \( n \geq 1 \).
phase gate (equivalent to CNOT)

phonon mode in ground state, i.e. \( n = 0 \)
(cooling required!)

qubits: ion 1 \( \{ |g\rangle_1, |e\rangle_2 \} \)
ion 2 \( \{ |g\rangle_2, |e\rangle_2 \} \)

need degenerate excited state (auxiliary level)

e.g.

\[
\begin{array}{c}
|e\rangle_1 & \quad |e\rangle_2 \\
\text{Circular} & \quad \text{Linear polarization}
\end{array}
\]

\( 3 \)
i \pi \) laser pulse on ion 1 on red sideband

\[
\begin{array}{c}
|g_1\rangle|g_2\rangle|10\rangle & \quad |g_1\rangle|g_2\rangle|10\rangle \\
|g_1\rangle|e_2\rangle|10\rangle & \quad |g_1\rangle|e_2\rangle|10\rangle \\
|e_1\rangle|g_2\rangle|10\rangle & \quad -i|g_1\rangle|g_2\rangle|1\rangle \\
|e_1\rangle|e_2\rangle|10\rangle & \quad -i|g_1\rangle|e_2\rangle|1\rangle
\end{array}
\]

\( 2 \pi \) laser pulse on ion 2 \( (g \leftrightarrow e') \) on red sideband

\[
\begin{array}{c}
|g_1\rangle|g_2\rangle|10\rangle & \quad |g_1\rangle|g_2\rangle|10\rangle \\
|g_1\rangle|e_2\rangle|10\rangle & \quad |g_1\rangle|e_2\rangle|10\rangle \\
-i|g_1\rangle|g_2\rangle|1\rangle & \quad +i|g_1\rangle|g_2\rangle|1\rangle \\
-i|g_1\rangle|e_2\rangle|1\rangle & \quad -i|g_1\rangle|e_2\rangle|1\rangle
\end{array}
\]
\textbf{ciii) \textit{π} laser pulse at ion 1 on red sideband}

\[
\begin{align*}
|g_1\rangle|g_2\rangle|0\rangle & \rightarrow |g_1\rangle|g_2\rangle|0\rangle \quad \text{red sideband} \\
|g_1\rangle|e_2\rangle|0\rangle & \rightarrow |g_1\rangle|e_2\rangle|0\rangle \\
-i|g_1\rangle|g_2\rangle|1\rangle & \rightarrow |e_1\rangle|g_2\rangle|1\rangle \\
-i|g_1\rangle|e_2\rangle|1\rangle & \rightarrow -|e_2\rangle|e_2\rangle|1\rangle
\end{align*}
\]

all together Phase gate

\[
\begin{align*}
|g\rangle|g\rangle & \rightarrow |g\rangle|g\rangle \\
|g\rangle|e\rangle & \rightarrow |g\rangle|e\rangle \\
|e\rangle|g\rangle & \rightarrow |e\rangle|g\rangle \\
|e\rangle|e\rangle & \rightarrow -|e\rangle|e\rangle
\end{align*}
\]

* first experimental implementations


* problem of Cincir-roller gate: cooling of ions to vibrational ground state \(n=0\), proposal without need of ground-state cooling: A. Sørensen, K. Mølmer,

* scaling to large system using (state-preserving) transport of ions

\[
\begin{array}{c}
\text{register/memory} \\
\text{operation region}
\end{array}
\]
7.2 Superconducting qubits

Scalable quantum computing most likely means to go to the solid state. A promising avenue are superconducting qubits and Josephson elements.

- Solid-state devices are typically dissipative (チリック lasers)

  $\Rightarrow$ Superconducting circuit

- Transition frequencies in superconducting qubits (SCQ) are in the range of 5-20 GHz; to avoid thermal fluctuations and noise

  $\Rightarrow$ low temperatures

- To create systems with two relevant states (qubit), we need non-linear elements

  $\Rightarrow$ Josephson junctions

Two types of qubits

(i) Charge qubit
(ii) Flux qubit
(A) quantum LC oscillator

Consider quantum description of an LC circuit

\[ V \quad C \quad L \quad I_c \quad I_L \quad V \text{ voltage at capacitor} \quad I \text{ current} \]

elements of circuit:

- \( Q \) change on capacitor
- \( \Phi \) mag. flux through inductor

(13) \[ I_c = \frac{dQ}{dt} = \frac{C}{dt} V_c \quad I_L = \frac{\Phi}{L} \]

(14) \[ V_L = \frac{d\Phi}{dt} \]

Kirchhoff's laws:

\[ V_L = V_c \quad I_c + I_L = 0 \]

(15) \[ \frac{d^2 \Phi}{dt^2} + \frac{\Phi}{LC} = 0 \]

harmonic oscillator

(16) \[ \omega_{lc} = \frac{1}{\sqrt{L/C}} \]
energy of capacitor and of inductor

\[ H_C = \frac{1}{2C} Q^2 \quad H_L = \frac{1}{2L} \Phi^2 \]

Hamiltonian

\[ H = \frac{1}{2C} Q^2 + \frac{1}{2L} \Phi^2 \quad E_n = \hbar \omega_L (n+\frac{1}{2}) \]

canonical coordinate: \( \Phi \)  
momentum: \( Q \)

\[ [\hat{\Phi}, \hat{Q}] = i\hbar \]

Josephson element

A harmonic oscillator has equidistant spectrum. In order to obtain a qubit with two relevant states, we need a nonlinear element \( \Rightarrow \) Josephson tunnel junction.

Supercurrenting ring

\[ \Phi \text{ flux through ring} \quad I_j \text{ current through barrier} \]

Analogously to a Bose-Einstein condensate, the Cooper pairs of a superconductor are described by a complex wave function

\[ \Psi = 12\Phi_1 e^{i\Phi} \]
If a magnetic flux $\Phi$ penetrates the loop, the Cooper pairs pick up a phase proportional to the flux when going around the loop:

$$\Theta = \Delta \Phi = \frac{2e}{h} \Phi = 2\pi \frac{\Phi}{\Phi_0}$$

$\Phi_0 = \hbar/2e$ flux quantum.

The tunnel current increases with increasing flux, but must be periodic in $\Theta$ modulo $2\pi$.

$$I_J = I_0 \sin \Theta$$

$I_0$ critical Josephson current.

The loop acts as a primitive coil and has a capacity. The equivalent circuit diagram is thus:

\[ \begin{array}{c}
I_0 \\
\text{C} \\
\text{L}
\end{array} \]

If there is an addition to the flux $\Phi$ created by the oscillating current, an external magnetic flux $\Phi_e$ (used for tuning purposes):

$$I_L = \frac{\Phi - \Phi_e}{L} = \frac{5}{2eL} (\Theta - \Theta_e)$$

Kircchoff yields then $I_C + I_L + I_J = 0$.

$$C \frac{d^2 \Phi}{dt^2} + I_0 \sin (2\pi \frac{\Phi}{\Phi_0}) + \Phi - \Phi_e = 0$$

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or equivalently

\[
\frac{5}{2e} C \dot{\theta} + I_0 \sin \theta + \frac{5}{2eL} (\theta - \theta_e) = 0
\]

It is easy to see that this is the Euler–Lagrange equation

\[
\frac{\partial}{\partial \theta} \frac{\partial L}{\partial \dot{\theta}} - \frac{\partial L}{\partial \theta} = 0
\]

for

\[
L = \frac{5}{4E_c} \dot{\theta}^2 - E_j (\mu - \cos \theta) - E_L (\theta - \theta_e)^2
\]

where

\[
E_c = \frac{(2e)^2}{2C}
\]

charging energy

= energy of capacitor C with charge difference of one Cooper pair 2e

and

\[
E_j = \frac{5}{2e} I_0
\]

Josephson energy

and

\[
E_L = \frac{\Phi_0^2}{4\pi^2 L}
\]

induction energy (one flux quantum)

Hamiltonian

\[
H = p \dot{\theta} - L
\]

with

\[
p = \frac{\partial L}{\partial \dot{\theta}} = \left( \frac{5}{2e} \right)^2 C \dot{\theta}
\]
Since $\theta = \frac{2e}{h} \Phi$ (see eq. (21)) and $\dot{\Phi} = V_L$ one finds

$$p = \frac{q}{2e} C V = \frac{q}{2e} \frac{q}{2e} = \frac{q}{2e}$$

where $n$ is the number of Cooper pairs

$$H = E_c n^2 + E_g (1 - \cos \Theta) + E_L \frac{(\hat{\Theta} - \Theta_e)^2}{2}$$

fundamental commutator

$$[\hat{\Theta}, \hat{n}] = i$$

potential

$$U = E_0 (1 - \cos \Theta) + E_L (\Theta - \Theta_e)^2/2$$

$\Theta_e = 0$

$\Theta_e = \pi/2$

if $E_J > E_L$
(c) single cooper-pair box (SCB): charge qubit

\[ H = E_c (\hat{n} - n_g)^2 - E_j \cos \Theta \]

Now \[ E_c = \frac{(2e)^2}{2(C_g + C_j)} \quad 2e n_g = C_g V \]

\[ E_c \gg E_j \]

Introduce basis states of number of cooper pairs \( \hat{n} \)

\[ H = \sum_n E_c (n - n_g)^2 |n\rangle \langle n| \]

(35) \[ - \frac{1}{2} E_j (|n\rangle \langle n+1| + |n+1\rangle \langle n|) \]

as \[ \langle n\| \cos \Theta \| n'\rangle = \delta_{n+n'} \]

Energy sequence of parabolas as function of \( n_g \) plus small perturbation \( \sim E_j \)
for \( n_q \approx \frac{1}{2} \) effective two-level system

\[
H = -\frac{1}{2} B_z \sigma_z - \frac{1}{2} B_x \sigma_x
\]

\[
B_z = E_c (1 - 2n_q) \quad B_x = E_j
\]

Changing \( n_q \) through the applied voltage allows to perform single-bit operations

- **quantum gates**
- Parallel tunnel junctions with capacitive coupling

\[
E_c' = \frac{(2e)^2}{2C}
\]

changing energy of coupling capacitor

\[
H_{int} = \sum_{n_1, n_2} E_c' (n_1 - n_2)^2 \langle n_1, n_2 \rangle \langle n_1, n_2 \rangle
\]
for $n_1, n_2 \approx \frac{1}{2}$

$|n_1, n_2\rangle \in \{ |10\rangle, |01\rangle, |11\rangle, |10\rangle\}$

$$H_{int} = E_c' \left( |10\rangle, \langle 01| + |10\rangle, \langle 10| \right)$$

$$= E_c' \left( (1 + d_2^2)(1 - d_2^2) + (1 - d_2^2)(1 + d_2^2) \right)$$

(40) $H_{int} = E_c' \left( 2 - d_2^2 \right)$

Using interactions

(D) Josephson flux qubit

$E_J \gg E_c$

$n_1, n_2 \approx \frac{1}{2}$

$E_L = E_c \hat{n}^2 + E_J \left( n_1 - n_2 \hat{s} \right) + E_L \left( \hat{\Theta} - \Theta_e \right)$

$\frac{1}{2} (32)$

Choose $\Theta_e = \frac{\pi}{2}$ i.e. $\Phi_e = \Phi_0 / 2$

Diagram with $E_L \left( \hat{\Theta} - \frac{\pi}{2} \right)^2$ axis and $\Phi_e / \Phi_0 = \Theta_e / 2\pi$
Single-bit Hamiltonian

\[ H = -\frac{1}{2} B_2 \mathcal{G}_2 - \frac{1}{2} B_x \mathcal{G}_x \]

where

\[ B_2 = 4\pi \sqrt{6 (\beta_L - 1)} E_J \left( \frac{\Phi_e}{\Phi_0} - 1 \right) \]

\[ \beta_L = \frac{E_J}{(\Phi_0^2/4\pi^2 L)} \]

Two qubit gates can be realized via an inductive coupling of

\[ \text{flux qubit 1} \quad \text{flux qubit 2} \]

= the end =