7. Experimental implementations of quantum information processing

The key requirements that need to be fulfilled by any physical implementation of quantum information processing have been summarized by David DiVincenzo ("The physical implementation of quantum computation" Fortschr. Phys. 48 (2000), 771–783)

(i) a scalable system of well defined qubits
(ii) the ability to prepare the state of a qubit
(iii) sufficiently long coherence times
(iv) implementation of a minimal set of universal quantum gates
(v) the ability to measure the state of a qubit in the computational basis

In addition there are two more requirements to build quantum networks

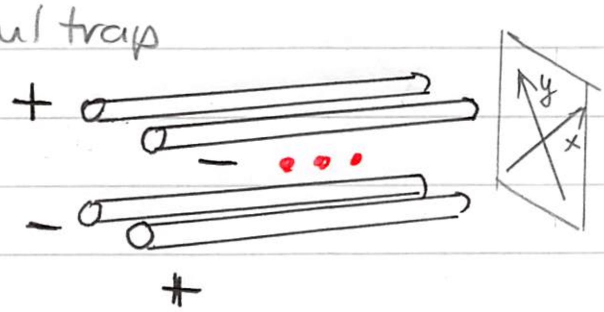
(vi) the ability to interconvert "stationary" into "flying" qubits and vice versa
(vii) the ability to faithfully transmitt flying qubits between different locations

We will discuss here only a few important examples.

7.1 ion traps

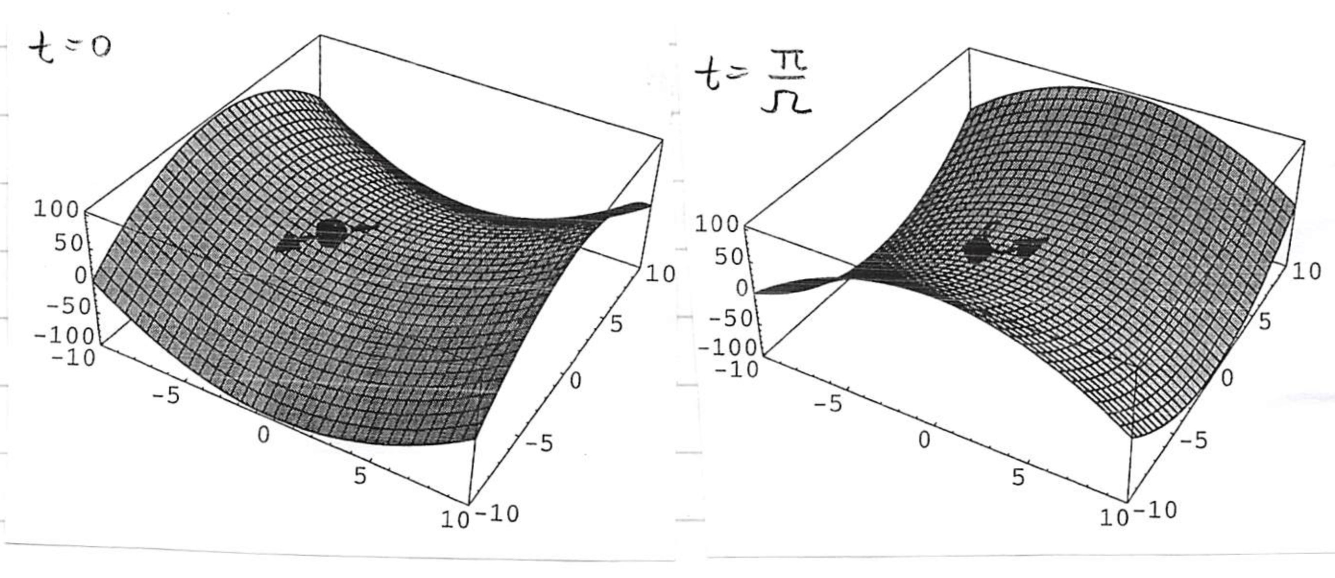
One of the first and currently the most precise (highest fidelities) implementations make use of the strong Coulomb interaction between trapped ions. Scalability is however an issue, J.I. Cirac and P. Zoller (Phys. Rev. Lett. 74, 4091 (1995))

linear Paul trap



quadrupole field (dynamic! static field cannot produce extremum in center)

$$\Phi = \frac{U + V\cos(\Omega t)}{2 r_0^2}\,(x^2 - y^2) \qquad (1)$$

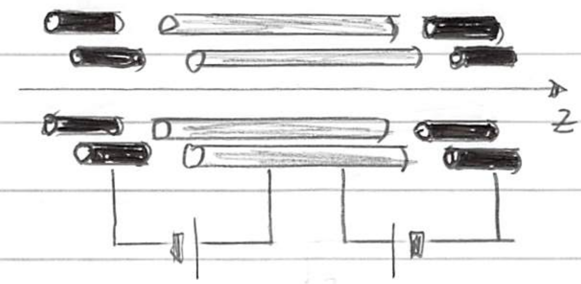


oscillating or rotating saddle point

for certain parameter regime of U, V, Ω
stable dynamics with time-averaged harmonic potential

(2) $$q\Phi_{eff} = \frac{1}{2} m \omega_r^2 (x^2+y^2) \qquad \omega_r \simeq \frac{qV}{\sqrt{2}\, m \Omega r_0^2}$$

in addition: micromotion (on short timescales).
Additional electrodes at end caps yield confinement in z-direction



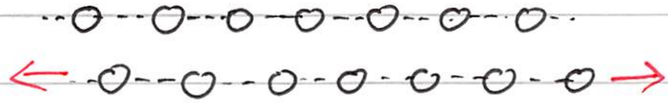
potential in z-direction
$\omega_z < \omega_r$

Coulomb repulsion of ions $\Rightarrow$ small linear crystals of ions (phase transition to other configurations if # of ions increased or parameter modified)

e.g. 0,2mm

elementary excitations: oscillations about equilibrium position.

lowest mode: in-phase oscillation of all ions



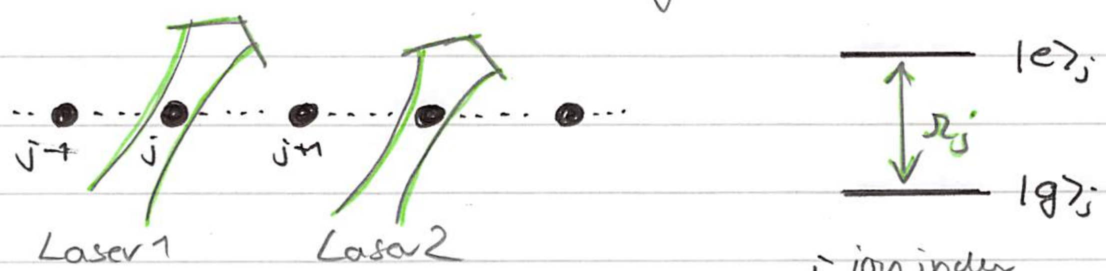
ν_0

e.g. $\nu_0 = 2\pi \times 50\,\mathrm{kHz}$
$\nu_1 = \sqrt{3}\,\nu_0$

for $k_B T \ll \nu_1 - \nu_0 \Rightarrow$ only lowest oscillation mode relevant

<u>qubit:</u> two internal states $|g\rangle, |e\rangle$
with optical transition $|g\rangle \leftrightarrow |e\rangle$

distance of ions $\gg$ optical wavelength $\Rightarrow$ <u>individual addressing</u>



j ion index

Ω_j Rabi-frequency of optical drive

$\Omega_j = dE_j/\hbar$

Hamiltonian

$$(3) \qquad H = H_0^{ions} + \hbar\nu_0 a^+ a + \sum_{j=1}^{N} \frac{\hbar\Omega_j}{2} \sin(\vec{k}_j \vec{r}_j - \omega t)(\sigma_j^+ + \sigma_j^-)$$

harmonic oscillator of common mode

$$\sigma_j^+ = |e\rangle_j {}_j\langle g| \qquad \sigma_j^- = |g_j\rangle_j\langle e|$$

For small oscillation amplitudes $\quad m = N m_0 \quad$ m_0 mass of one ion

$$(4) \qquad \hat{\vec{r}}_j = \hat{\vec{r}}_{j0} + \underbrace{\sqrt{\frac{\hbar}{2m\nu_0}}\,(\hat{a} + \hat{a}^+)}_{=\hat{z}}\,\vec{e}_z$$

$$(5) \qquad \vec{k}_j \vec{r}_j = \vec{k}_j \vec{r}_{j0} + k_{jz}\hat{z}$$

$$(6) \qquad k_{jz}\hat{z} = \eta(\hat{a}^+ + \hat{a})$$

$$(7) \qquad \eta = \frac{\pi}{\lambda}\sqrt{\frac{2\hbar}{m_0\nu}}\frac{1}{\sqrt{N}} \qquad \text{\underline{Lamb-Dicke parameter}}$$

(8) $$\eta^2 = \frac{1}{N}\frac{\hbar}{2m_0}\left(\frac{2\pi}{\lambda}\right)^2\frac{1}{\nu_0} = \frac{\omega_{rec}}{N\nu_0}$$

ω_{rec} = recoil frequency (recoil energy = kinetic energy which an atom exchanges by momentum transfer when absorbing / emitting a photon of momentum $\hbar k$)
ω_{rec} is very small $\sim$ $\underline{\eta \text{ small}}$

expansion of eq. (3) (<u>Lamb-Dicke regime</u>)

(9) $$e^{i\vec{k}_j \vec{r}_j} \simeq 1 + i\eta(\hat{a}^+ + \hat{a})$$

then

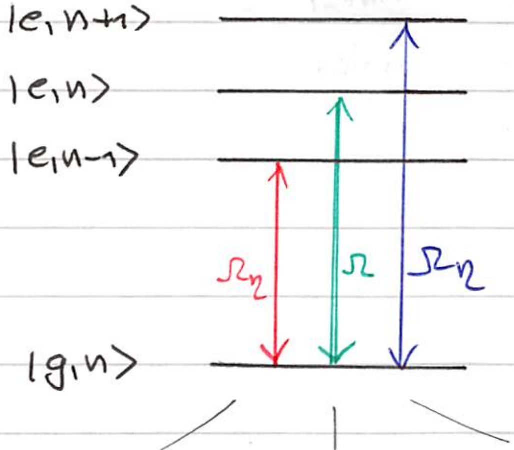
$$H = H_0^{ion} - \sum_{j=1}^{N} \frac{\hbar\Omega_j}{2}(\sigma_j^+ + \sigma_j^-)\sin\omega t$$
(10)
$$+ \sum_{j=1}^{N} \frac{\hbar\Omega_j}{2}\eta(\hat{a}+\hat{a}^+)(\sigma_j^+ + \sigma_j^-)\cos\omega t$$

In the interaction picture $\sigma^+ \to \sigma^+ e^{i\omega_0 t}$, $\hat{a} \to \hat{a}e^{-i\nu t}$
and <u>rotating wave approximation</u> (RWA)

(11) $$e^{\pm i(\omega_0+\omega)t} \approx 0$$ averages to zero

$$H = H_0^{ion} + \sum_{j=1}^{N} i\frac{\hbar\Omega_j}{2}\left(e^{-i\Delta t}\sigma_j^+ - h.a.\right)$$
(12)
$$+ \sum_{j=1}^{N} \frac{\hbar\Omega_j}{2}\eta\left[\left(\hat{a}^+ e^{-i(\Delta-\nu)t} + \hat{a}e^{i(\Delta+\nu)t}\right)\sigma_j^+ + h.a.\right]$$

where $\Delta = \omega - \omega_0$. Thus an individual ion has the following transitions



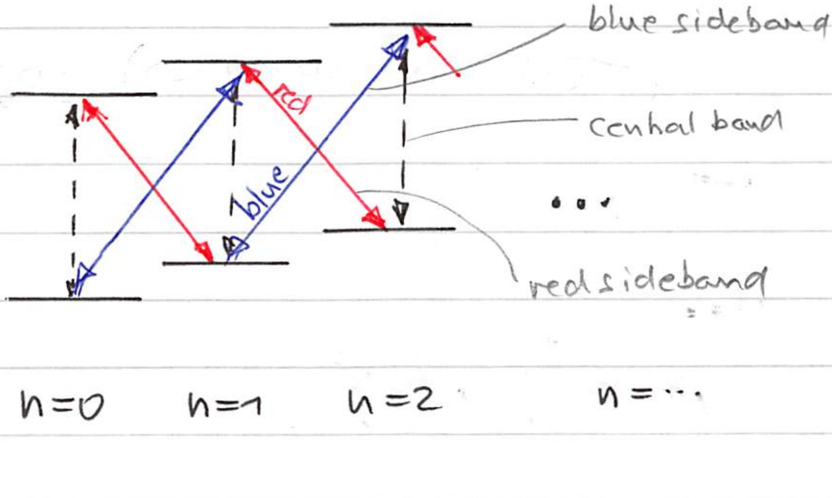
$$\Omega_\eta = \eta\Omega \ll \Omega$$

red sideband
resonant if $\Delta + \nu = 0$
$\omega = \omega_0 - \nu$

blue sideband
resonant if $\Delta - \nu = 0$
$\omega = \omega_0 + \nu$

resonant if $\Delta = 0$
$\omega = \omega_0$

optical excitation of an ion on a sideband is associated with absorption / emission of a phonon of the collective oscillations of all ions $\Rightarrow$ coupling to other ions (qubits). Coupling on red sideband only possible if phonon mode is excited, i.e. if $n \geq 1$



If ν is larger than the decay rates and Rabi-frequencies the different bands can be spectroscopically resolved.

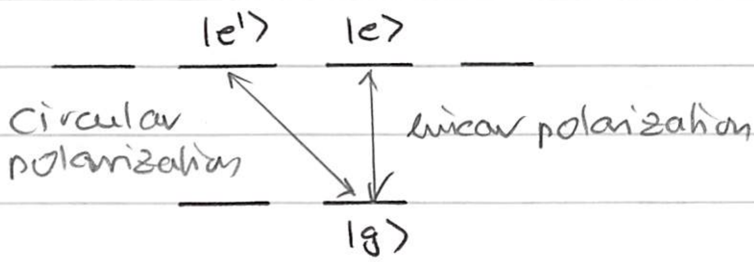
phase gate (equivalent to CNOT)

phonon mode in ground state, i.e. $n=0$
(cooling required!)

qubits: ion 1 $\{|g\rangle_1, |e\rangle_1\}$
ion 2 $\{|g\rangle_2, |e\rangle_2\}$

need degenerate excited states (auxiliary level)

e.g.



(i) π laser pulse at ion 1 on red sideband

$$\begin{aligned}
|g_1\rangle|g_2\rangle|0\rangle \quad & \qquad |g_1\rangle|g_2\rangle|0\rangle \\
|g_1\rangle|e_2\rangle|0\rangle \quad & \longrightarrow \quad |g_1\rangle|e_2\rangle|0\rangle \\
|e\rangle_1|g_2\rangle|0\rangle \quad & \qquad -i\,|g_1\rangle|g_2\rangle|1\rangle \\
|e\rangle_2|e\rangle_2|0\rangle \quad & \qquad -i\,|g_1\rangle|e_2\rangle|1\rangle
\end{aligned}$$

} red sideband and $n=0$

(ii) 2π laser pulse at ion 2 ($g \leftrightarrow e'$) on red sideband

$$\begin{aligned}
|g_1\rangle|g_2\rangle|0\rangle \quad & \qquad |g_1\rangle|g_2\rangle|0\rangle \\
|g_1\rangle|e_2\rangle|0\rangle \quad & \longrightarrow \quad |g_1\rangle|e_2\rangle|0\rangle \\
-i\,|g_1\rangle|g_2\rangle|1\rangle \quad & \qquad +i\,|g_1\rangle|g_2\rangle|1\rangle \\
-i\,|g_1\rangle|e_2\rangle|1\rangle \quad & \qquad -i\,|g_1\rangle|e_2\rangle|1\rangle
\end{aligned}$$

} red sideband

(iii) π laserpulse at ion 1 on red sideband

$$
\begin{aligned}
|g_1\rangle|g_2\rangle|0\rangle & & |g_1\rangle|g_2\rangle|0\rangle \\
|g_1\rangle|e_2\rangle|0\rangle & \longrightarrow & |g_1\rangle|e_2\rangle|0\rangle \\
i\,|g_1\rangle|g_2\rangle|1\rangle & & |e_1\rangle|g_2\rangle|0\rangle \\
-i\,|g_1\rangle|e_2\rangle|1\rangle & & -|e_1\rangle|e_2\rangle|0\rangle
\end{aligned}
$$

red sideband

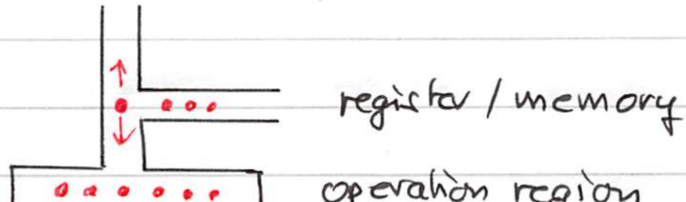
alltogether Phase-gate

$$
\boxed{
\begin{aligned}
|g\rangle|g\rangle & \longrightarrow |g\rangle|g\rangle \\
|g\rangle|e\rangle & \longrightarrow |g\rangle|e\rangle \\
|e\rangle|g\rangle & \longrightarrow |e\rangle|g\rangle \\
|e\rangle|e\rangle & \longrightarrow -|e\rangle|e\rangle
\end{aligned}
}
$$

- first experimental implementations
  D. Wineland (Boulder, NIST) Nature 422, 412 (2003)
  R. Blatt (Innsbruck) Nature 421, 48 (2003)

- problem of Cirac-Zoller gate: cooling of ions to vibrational ground state ($n=0$). proposal without need of ground-state cooling: A. Sørensen, K. Mølmer Phys. Rev. Lett 82, 1971 (1999)

- scaling to large system using (state-preserving) transport of ions



7.2 superconducting qubits

Scalable quantum computing most likely means to go to the solid state. A promising avenue are superconducting qubits and Josephson elements.

- Solid-state devices are typically <u>dissipative</u> (Ohmic losses)

  $\Longrightarrow$ superconducting circuits

- transition frequencies in superconducting qubits (SCQ) are in the range of 5-20 GHz; to avoid thermal <u>fluctuations</u> and <u>noise</u>

  $\Longrightarrow$ low temperatures

- to create systems with two relevant states (qubits) we need nonlinear elements

  $\Longrightarrow$ Josephson junctions

two types of qubits

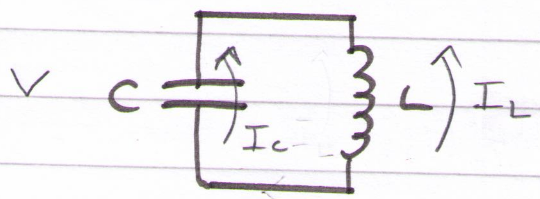
(i) charge qubit
(ii) flux qubit

literature:

- Y. Makhlin, G. Schön, A. Shnirman Rev. Mod. Phys. 73, 357 (2001)
- G. Wendin, V.S. Shumeiko, arxiv 0508729
- M. Devoret, A. Wallraff, J. Martinis, arxiv 0411174

(A) quantum LC oscillator

consider quantum description of an LC circuit



V voltage at capacitor
I current

Q charge on capacitor
Φ magn. flux through inductor

elements of circuit

(13) $I_C = \frac{d}{dt}Q = C\frac{dV_C}{dt}$ $\qquad I_L = \frac{\Phi}{L}$

(14) $V_L = \frac{d}{dt}\Phi$

kirchhoffs laws: $\quad V_L = V_C \qquad I_C + I_L = 0$

(15) $$\boxed{\frac{d^2}{dt^2}\Phi + \frac{\Phi}{LC} = 0}$$

harmonic oscillator

(16) $$\omega_{LC} = \frac{1}{\sqrt{LC}}$$

energy of capacitor and of inductor

(17) $H_C = \frac{1}{2C} Q^2$ $\qquad$ $H_L = \frac{1}{2L} \Phi^2$

Hamiltonian

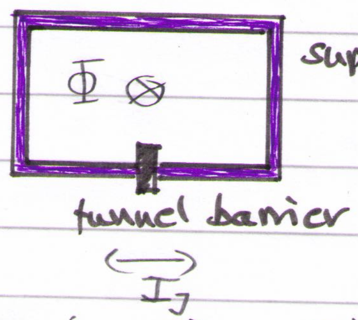
(18) $$\boxed{H = \frac{1}{2C}\hat{Q}^2 + \frac{1}{2L}\hat{\Phi}^2} \qquad \boxed{E_n = \hbar\omega_{LC}(n+\tfrac{1}{2})}$$

canonical coordinate: Φ $\qquad$ momentum: Q

(19) $$[\hat{\Phi}, \hat{Q}] = i\hbar$$

(B) Josephson element

harmonic oscillator has aequidistant spectrum. In order to obtain a qubit with two relevant states we need a nonlinear element $\Rightarrow$ Josephson tunnel junction



Φ flux through ring
I_J current through barrier

Analogously to a Bose-Einstein condensate the cooper pairs of a superconductor are described by a complex wave function

(20) $$\Psi = |\Psi| e^{i\varphi}$$

If a magnetic flux Φ penetrates the loop the cooper pairs pick up a phase proportional to the flux when going around the loop

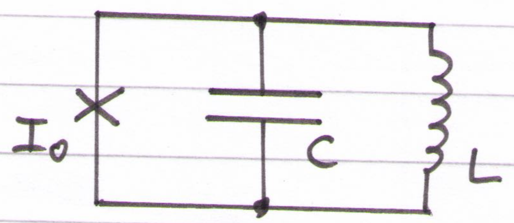
(21) $$\theta = \Delta\varphi = \frac{2e}{\hbar}\Phi = 2\pi\frac{\Phi}{\Phi_0}$$

$\Phi_0 = h/2e$ flux quantum

The tunnel current increases with increasing flux but must be periodic in θ mod 2π

(22) $$\boxed{I_J = I_0 \sin\theta}$$ I_0 critical Josephson current

The loop acts as a primitive coil and has a capacity. The equivalent circuit diagram is thus



If there is in addition to the flux Φ created by the oscillating current an external magnetic flux Φ_e (used for tuning purposes)

(23) $$I_L = \frac{(\Phi - \Phi_e)}{L} = \frac{\hbar}{2eL}(\theta - \theta_e)$$

Kirchhoff yields then $I_C + I_L + I_J = 0$

(24) $$C\frac{d^2}{dt^2}\Phi + I_0 \sin(2\pi\,\Phi/\Phi_0) + \frac{\Phi - \Phi_e}{L} = 0$$

or equivalently

(25) $$\frac{\hbar}{2e} C \ddot{\theta} + I_0 \sin\theta + \frac{\hbar}{2eL}(\theta - \theta_e) = 0$$

It is easy to see that this is the Euler-Lagrange equation

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\theta}} - \frac{\partial \mathcal{L}}{\partial \theta} = 0$$

for

(26) $$\boxed{\mathcal{L} = \frac{\hbar^2}{4E_C}\dot{\theta}^2 - E_J(1-\cos\theta) - E_L\frac{(\theta-\theta_e)^2}{2}}$$

where

(27) $$\boxed{E_C = \frac{(2e)^2}{2C}}$$ charging energy

= energy of capacitor C with charge difference of one Cooperpair 2e

and

(28) $$\boxed{E_J = \frac{\hbar}{2e} I_0}$$ Josephson energy

and

(29) $$\boxed{E_L = \frac{\Phi_0^2}{4\pi^2 L}}$$ induction energy (one flux quantum)

Hamiltonian $H = p\dot{\theta} - \mathcal{L}$

with

(30) $$p = \frac{\partial \mathcal{L}}{\partial \dot{\theta}} = \left(\frac{\hbar}{2e}\right)^2 C\dot{\theta}$$

Since $\dot{\theta} = \frac{2e}{\hbar}\dot{\Phi}$ (see eq. (21)) and $\dot{\Phi} = V_L$ one finds

(31) $$p = \frac{\hbar}{2e} CV = \hbar \frac{Q}{2e} = \hbar n$$

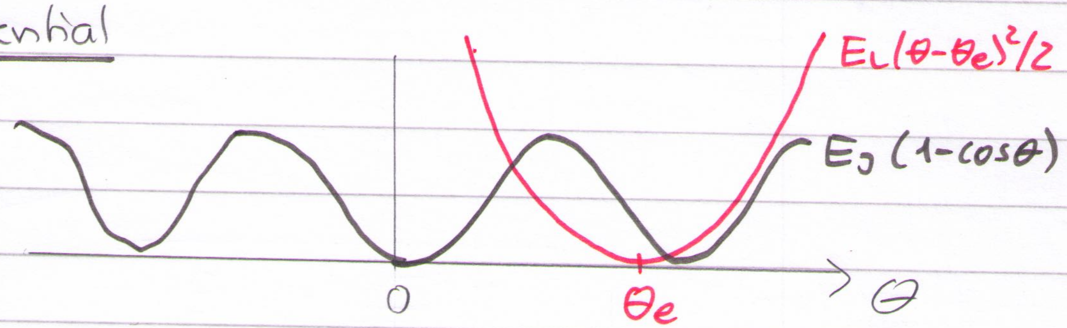
where n is the number of Cooper pairs

(32) $$\boxed{H = E_c \hat{n}^2 + E_J(1-\cos\hat{\theta}) + E_L \frac{(\hat{\theta}-\theta_e)^2}{2}}$$

fundamental commutator

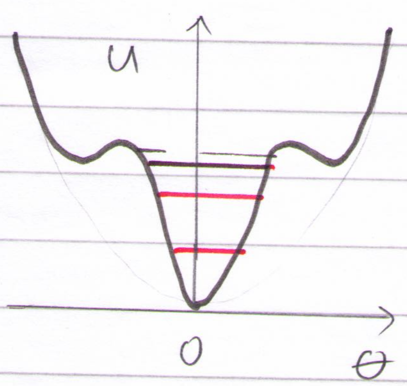
(33) $$\boxed{[\hat{\theta}, \hat{n}] = i}$$

potential



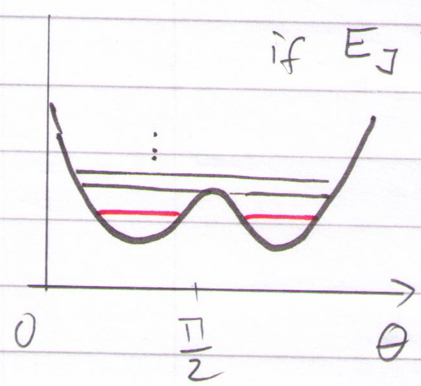
$U = E_J(1-\cos\theta) + E_L(\theta-\theta_e)^2/2$

$\boxed{\theta_e = 0}$



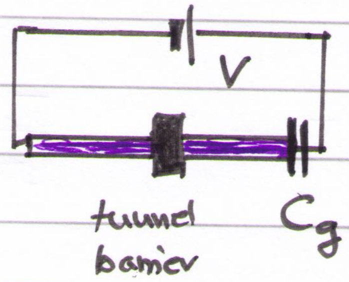
$\boxed{\theta_e = \pi/2}$

if $E_J > E_L$



(c) single cooper-pair box (SCB): charge qubit

- qubit



application of external voltage V via gate capacitance allows to change number of cooper pairs; $E_L \approx 0$

(34) $$\boxed{H = E_C(\hat{n} - n_g)^2 - E_J \cos\hat{\theta}}$$

now $$E_C = \frac{(2e)^2}{2(C_g + C_J)} \qquad \underbrace{2e\, n_g}_{\text{charge}} = C_g V$$

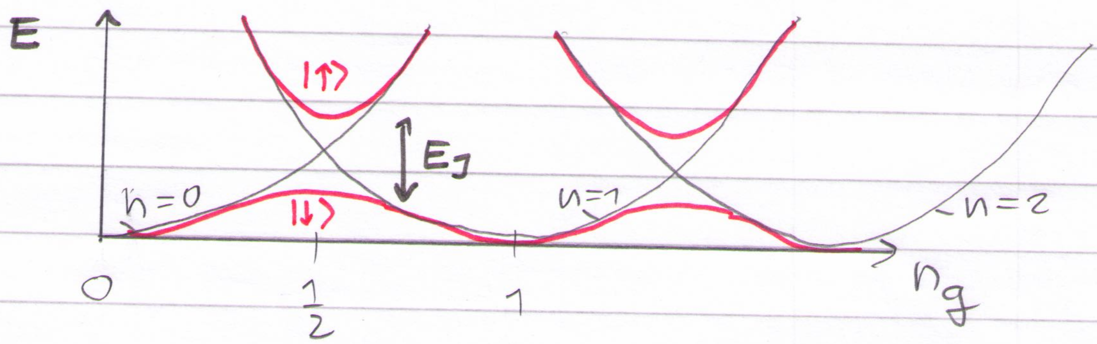
$\boxed{E_C \gg E_J}$

introduce basis states of number of cooper pairs $\hat{n}$

(35) $$H = \sum_n E_C (n - n_g)^2 |n\rangle\langle n| - \frac{1}{2} E_J \left(|n\rangle\langle n+1| + |n+1\rangle\langle n| \right)$$

as $\langle n|\cos\hat{\theta}|n'\rangle = \delta_{n,n'\pm 1}$

Energy sequence of parabolas as function of n_g plus small perturbation $\sim E_J$



for $n_g \simeq \frac{1}{2}$ effective two-level system

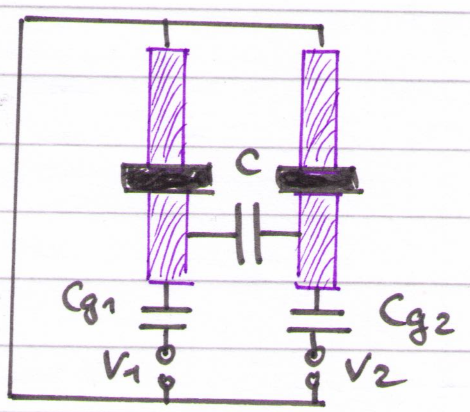
(36) $$\boxed{H = -\frac{1}{2} B_z \sigma_z - \frac{1}{2} B_x \sigma_x}$$

(37) $$B_z = E_C (1 - 2n_g) \qquad B_x = E_J$$

varying n_g through the applied voltage allows to perform single-bit operations

- quantum gates

parallel tunnel junctions with capacitive coupling

(38) 

$$E_C' = \frac{(2e)^2}{2C}$$

charging energy of coupling capacitor

(39) $$H_{int} = \sum_{n_1, n_2} E_C' (n_1 - n_2)^2 |n_1, n_2\rangle\langle n_1, n_2|$$

for $n_{g1} \approx n_{g2} \approx \frac{1}{2}$ $\quad |n_1, n_2\rangle \in \{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$

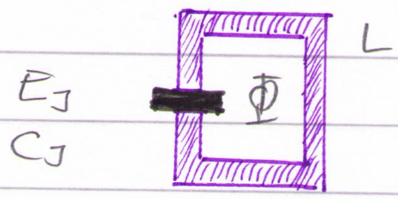
$$H_{int} = E_C' \left(|01\rangle\langle 01| + |10\rangle\langle 10| \right)$$

$$= E_C' \left((1+\sigma_z^1)(1-\sigma_z^2) + (1-\sigma_z^1)(1+\sigma_z^2) \right)$$

(40) $$\boxed{H_{int} = E_C' \left(2 - \sigma_z^1 \sigma_z^2 \right)}$$ Ising interaction

(D) Josephson flux qubit

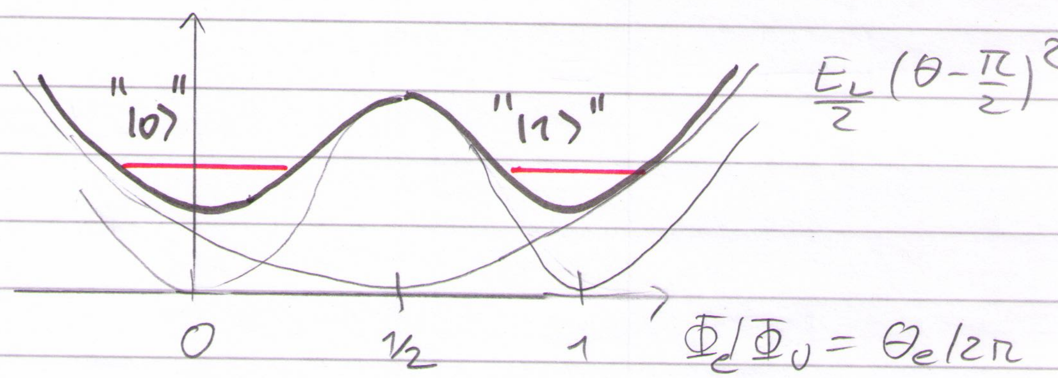
$\boxed{E_J \gg E_C}$



now E_L cannot be ignored

(41) $\hat{=}$ (32) $$H = E_C \hat{n}^2 + E_J (1 - \cos\hat{\theta}) + E_L \frac{(\hat{\theta} - \theta_e)^2}{2}$$

Choose $\theta_e \simeq \pi/2$ i.e. $\Phi_e \simeq \Phi_0/2$



single-bit hamiltonian

(42) $$H = -\frac{1}{2} B_z \sigma_z - \frac{1}{2} B_x \sigma_x$$

here

(43) $$B_z = 4\pi \sqrt{6(\beta_L - 1)}\, E_J \left(\frac{\Phi_e}{\Phi_0} - 1\right)$$

$$\beta_L = E_J / (\Phi_0^2 / 4\pi^2 L)$$

Two qubit gates can be realized via an inductive coupling of

flux qubit 1   flux qubit 2

= the end =