
Task 21.

Verify the expression derived in the lecture for the Green function of the transverse elm. vector potential in Fourier space

$$D_C^{ij}(k) = \frac{\delta_{ij} - \frac{k_i k_j}{|\mathbf{k}|^2}}{\omega^2 - |\mathbf{k}|^2 + i\varepsilon}.$$

What is the propagator in spatial and frequency space, i.e. what is $D_C^{ij}(\omega, \mathbf{r})$?

Task 22.

Show that $S(x, x') = -i\langle 0|T\hat{\Psi}(x)\hat{\bar{\Psi}}(x')|0\rangle$ is the Green function of the Dirac equation, i.e.

$$(i\gamma^\mu \partial_\mu - m)\, S(x, x') = \delta^{(4)}(x - x'). \tag{1}$$

Task 23.

The fields $\hat{\Psi}(x)$ and $\hat{\bar{\Psi}}(x')$ do not commute if their distance is space-like, i.e. $(x - x')_\mu (x - x')^\mu = (t - t')^2 - (\mathbf{r} - \mathbf{r}')^2 < 0$. This is not a contradiction, since the fields themselves are not physical observables. The operator for the Noether current

$$j_\mu(x) = -e : \hat{\bar{\Psi}}(x)\gamma_\mu \hat{\Psi}(x) : \tag{2}$$

however, is an observable. Show that

$$[j_\mu(x), j_\nu(x')] = 0 \quad \text{if} \quad (x - x')^2 < 0! \tag{3}$$