Quantum field theory

WS 2014 / 2015

Literature: Franz Gross "Relativistic Quantum Mechanics & Field Theory" Wiley 2004 (© 1993)

F. Mandl, G. Shaw "Quantum Field Theory" Wiley 2006 (© 1984)

0. Introduction

- Why a quantum theory of fields?

(i) inconsistency in quantization

classical theory $\dot{\vec{p}} = -q\vec{E}$

quantum mechanics $\vec{p} \to \hat{\vec{p}} \Rightarrow \vec{E}$?

(ii) $\nexists$ relativistic single-particle quantum mechanics
Dirac equation cannot be interpreted as single-particle quantum mechanics; there is no conservation of positive definite probability

▶ <u>what is a quantum field?</u> (canonical quantization)

classical field $\mathbb{R}^3 \otimes \mathbb{R} \rightarrow \mathbb{R}, \mathbb{C}, \mathbb{R}^n, \mathbb{C}^n$

$\vec{r} \quad t \qquad \phi(\vec{r}, t)$

quantum field $\mathbb{R}^3 \otimes \mathbb{R} \rightarrow \mathcal{L}(\mathcal{H})$

In q-mechanics we introduced an operator for space. In QFT space will be a <u>continuous index</u>

$$\underset{\text{discrete index}}{\hat{q}_j} \longrightarrow \underset{\text{continuous index}}{\hat{O}(x)}$$

<u>remark:</u> the transition to continuum will cause some problems lateron

▶ <u>how will a quantum theory of fields look like?</u>

QFT is a quantum theory, so all postulates of quantum theory should hold

- system described by statevector
  $|\psi\rangle \in \mathcal{H}$ (or generalization of $\mathcal{H}$)

- observables are self-adjoint operators
  $\hat{O} : \mathcal{H} \rightarrow \mathcal{H} \qquad \hat{O}^+ = \hat{O}$

- measurement leads to projection of state to eigenspace of observable; probability density of measurement outcome given by projection to eigenspace of observable with eigenvalue λ (range of eigenvalues)

$$p_\lambda = \langle \psi | \hat{P}_\lambda | \psi \rangle \qquad \hat{P}_\lambda \text{ - projector}$$

$$|\psi\rangle\langle\psi| \longrightarrow \frac{\hat{P}_\lambda |\psi\rangle\langle\psi| \hat{P}_\lambda}{\langle\psi|\hat{P}_\lambda|\psi\rangle}$$

- time evolution: unitary operator

$$\hat{U}(t, t_0) = e^{-i\hat{H}(t-t_0)}$$

- observables can be reduced to fundamental set with fundamental commutation relations

How to quantize a field theory?

$\rightarrow$ analogy between classical and quantum mechanics

classical mechanics	quantum mechanics
$L(q_j, \dot{q}_j, t)$	
$H = \sum_j \dot{q}_j \frac{\partial L}{\partial \dot{q}_j} - L$	$\hat{H}$
$q_j \quad p_j = \frac{\partial L}{\partial \dot{q}_j}$	$\hat{q}_j \ , \ \hat{p}_j$

$$\{q_j, q_k\} = \{p_j, p_k\} = 0 \qquad [\hat{q}_j, \hat{q}_k] = [\hat{p}_j, \hat{p}_k] = 0$$

$$\{q_j, p_k\} = \delta_{jk} \qquad [\hat{q}_j, \hat{p}_k] = i\hbar\,\delta_{jk}$$

$$\dot{q}_j = \{q_j, H\} \qquad \frac{d}{dt}\hat{q}_j = -\frac{i}{\hbar}[\hat{q}_j, \hat{H}]$$

$$\dot{p}_j = \{p_j, H\} \qquad \frac{d}{dt}\hat{p}_j = -\frac{i}{\hbar}[\hat{p}_j, \hat{H}]$$

or $f = f(q_j, p_j, t)$

$$\frac{d}{dt} f = \{f, H\} + \frac{\partial f}{\partial t} \qquad \frac{d}{dt}\hat{f} = -\frac{i}{\hbar}[\hat{f}, H] + \frac{\partial \hat{f}}{\partial t}$$

$\Rightarrow$ need formulation of classical field theory in terms of Lagrangians / Hamiltonians

▶ specialities and remarks:

- fundamental field theories are relativistic
  $\Rightarrow$ Lorentz invariance will play important role

- Hamilton formalism singles out time and is not explicitly Lorentz invariant ☹
  better suited: path integral quantization is explicit Lorentz invariant but less convenient to interpret (will not be done in this lecture)

- mathematically consistent formulation of QFT has subtleties: often quantities divergent! Consistent removal of infinities is a difficult and not always possible task $\Rightarrow$ renormalization

- an important role will be played by gauge field theories; all observed interactions in nature can be described by gauge-field theories

▶ new physics from quantum field theory?

yes!

- while energy is (in general) conserved, particle number is not

  → creation and annihilation of particle (pairs) possible

  → spontaneous decay of heavy particles in several lighter particles

- in the non-relativistic, low-energy limit quantum many-particle theory is recovered

- **is QFT the ultimate wisdom?**

no! unsolved:
- quantization of gravity
- explanation of mass of fundamental particles

- **structure of lecture**

(I) classical field theory

Lagrange-Hamilton for fields; symmetries and conservation laws; Schrödinger- and Dirac quantum mechanics as classical field theory; gauge fields

(II) canonical quantization of fields

(III) some simple consequences of field quantization (non-interacting fields)

Casimir force; spontaneous decay; Lamb-shift (Bethe formula);

(IV) QED with interactions: diagrammatic perturbation theory

(V) Renormalization

(VI) Outlook to QCD and Standard model; Higgs mechanism