OSCAR iWeek
List of recommendable literature

Original papers. The two classic articles. Schwinger describes the method rather sharply in terms of operators. Keldysh introduces the rotation to the more physical "RAK" representation which is named after him.

- Schwinger, J. (1961). Brownian motion of a quantum oscillator. Journal of Mathematical Physics, 2(3), 407–432.
  https://doi.org/10.1063/1.1703727.
- Keldysh, L. V. (1964). Diagram technique for nonequilibrium processes. JETP, 20(5), 1080.
  https://doi.org/10.1007/BF02724324.

Reviews. A list of reviews that are good starting points for getting familiar with the technique. The first article is about the fluctuation-dissipation theorem in equilibrium, which can give a good motivation for the more general non-equilibrium problem.

The second one is probably the best place to start: it derives the Keldysh path integral from a master equation and introduces the one-particle irreducible effective action. Also treats the functional renormalization group.

The two Berges reviews are highly recommendable, although perhaps more advanced, as they are base on the *two-particle* irreducible effective action (2PIEA). They have nice discussions of initial conditions and advanced nonperturbative approximation techniques.

The last one is an older piece that is possibly good for comparison and further reading, but mostly here for completeness.

- Kubo, R., 1966. The fluctuation-dissipation theorem. Reports on Progress in Physics, 29(1), 255.
  https://doi.org/10.1088/0034-4885/29/1/306
- Sieberer, L. M., Buchhold, M., and Diehl, S. (2015). Keldysh Field Theory for Driven Open Quantum Systems. Reports on Progress in Physics, 79(9), 96001.
  https://arxiv.org/abs/1512.00637
- Berges, J. (2004). Introduction to Nonequilibrium Quantum Field Theory.
  https://arxiv.org/abs/hep-ph/0409233.

- Berges, J. (2015). Nonequilibrium Quantum Fields: From Cold Atoms to Cosmology. http://arxiv.org/abs/1503.02907.
- Chou, K., Su, Z., Hao, B., and Yu, L. (1985). Equilibrium and Nonequilibrium Formalisms Made Unified. Phys. Rep., 118, 1. https://doi.org/10.1016/0370-1573(85)90136-X.

Books. Kamenev's book is sort of the standard for the path-integral based approach. He gives another derivation of the path integral from a discrete-time expansion in terms of coherent states. Among the applications are interacting Bose gases. He uses the effective-action concepts without ever stating it clearly, which is a drawback. Has a nice chapter on classical stochastics.

Altland's book has a lot of material on field theory methods both for classical and quantum non-equilibrium problems, presented in two chapters. The derivation of the Keldysh path integral is practically identical to Kamenev, but there is more material that may provide a complementary perspective.

Calzetta's book is pretty cryptic when it comes to notation, but it has a very good chapter on Bose gases with a discussion of existing approaches (Φ-derivable, conserving and gapless approximations from 2PIEA).

Rammer's is based solely on operators, for those who prefer that. Very abstract, but good for looking up identities and reading about general diagrammatics concepts.

Stefanucci's book is also based on operators and derives everything in terms of the so-called "Langereth rules". Used to use it when starting out with non-equilibrium systems, but abandoned it at some point for lack of intuitive transparency.

- Kamenev, A. (2011). Field Theory of Non-Equilibrium Systems. Cambridge University Press.
- Altland, A., & Simons, B. D. (2010). Condensed matter field theory. Cambridge University Press.
- Calzetta, E. A., and Hu, B. L. B. (2008). Nonequilibrium quantum field theory. Cambridge University Press.
- Rammer, J. (2007). Quantum field theory of non-equilibrium states. Cambridge University Press.
- Stefanucci, G., and Van Leeuwen, R. (2013). Nonequilibrium many-body theory of quantum systems: a modern introduction. Cambridge University Press.

Articles about cold Bose gases. The first article is a good resource for an example of how the 2PIEA technique can actually be applied. Also discusses the famous Hartree-Fock-Bogoliubov equations and their derivation.

The second article is interesting because it compares classical and quantum dynamics from the perspective of the Schwinger-Keldysh path integral. There one can clearly see how the vertex with the cubic response field gives rise to the nonclassical dynamics.

- Gasenzer, T., Berges, J., Schmidt, M. G., and Seco, M. (2005). Nonperturbative dynamical many-body theory of a Bose-Einstein condensate. Physical Review A, 72(6), 1–20. https://arxiv.org/abs/cond-mat/0507480.

- Berges, J., and Gasenzer, T. (2007). Quantum versus classical statistical dynamics of an ultracold Bose gas. Physical Review A, 76(3).
  https://arxiv.org/abs/cond-mat/0703163.

Relation between master equation and Keldysh functional integral. This article contains a relatively detailed derivation of the "Markovian dissipative action" that is the functional equivalent of the Lindblad master equation in the usual Born-Markov approximation. This is well suited for a comparison to Schwinger's original treatment.

- Sieberer, L. M., Huber, S. D., Altman, E., & Diehl, S. (2014). Nonequilibrium functional renormalization for driven-dissipative Bose-Einstein condensation. Physical Review B, 89(13), 1–34.
  https://arxiv.org/abs/1309.7027.

Using Keldysh to describe a "sudden modulation". This reference is the only article that I know of where the method is applied in the original style given by Schwinger, hence not uninteresting.

- Busch, X., Carusotto, I., and Parentani, R. (2014). Spectrum and entanglement of phonons in quantum fluids of light. Physical Review A, 89(4), 1–16.
  https://arxiv.org/abs/1311.3507.

Classical non-equilibrium systems. The classic resources for the Martin-Siggia-Rose-Jansen-De Dominicis (MSRJD) functional integral. The "response field" also occurring in Keldysh is introduced here for classical systems, which gives e.g. an alternative approach to the Onsager-Machlup path integral for Brownian motion.

- Martin, P. C., Siggia, E. D., & Rose, H. A. (1973). Statistical dynamics of classical systems. Physical Review A, 8(1), 423.
  https://doi.org/10.1103/PhysRevA.8.423.
- Janssen, H. K. (1976). On a Lagrangean for classical field dynamics and renormalization group calculations of dynamical critical properties. Zeitschrift für Physik B Condensed Matter, 23(4), 377-380.
  https://doi.org/10.1007/BF01316547.
- De Dominicis, C., & Peliti, L. (1978). Field-theory renormalization and critical dynamics above T c: Helium, antiferromagnets, and liquid-gas systems. Physical Review B, 18(1), 353.
  https://doi.org/10.1103/PhysRevB.18.353.

Another instructive derivation of the classical path integral. This article gives a more rigorous approach to the path integral for general stochastic processes from a probability-theory point of view. One should pay attention, however, to the different time discretization chosen here.

- Hänggi, P. (1989). Path integral solutions for non-Markovian processes. Zeitschrift Für Physik B Condensed Matter, 75(2), 275–281.
  https://doi.org/10.1007/BF01308011.