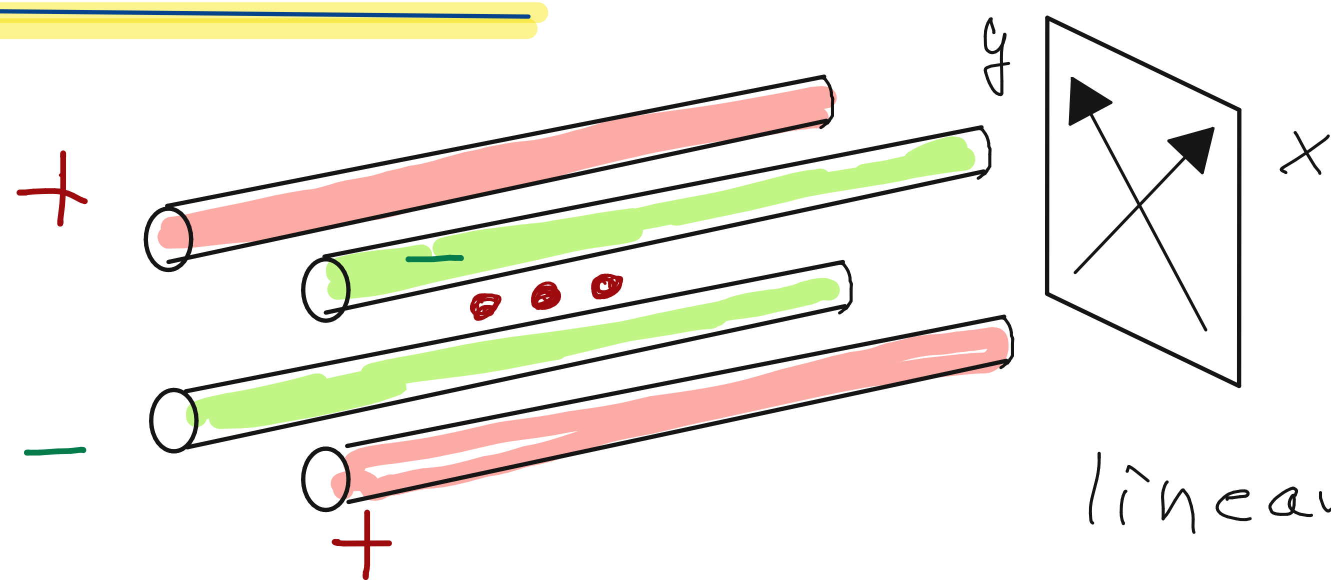
Ion trap quantum computing

idea:

J. I. Cirac & P. Zoller Phys. Rev. Lett. 74, 4091 (1995)

1. Ion traps

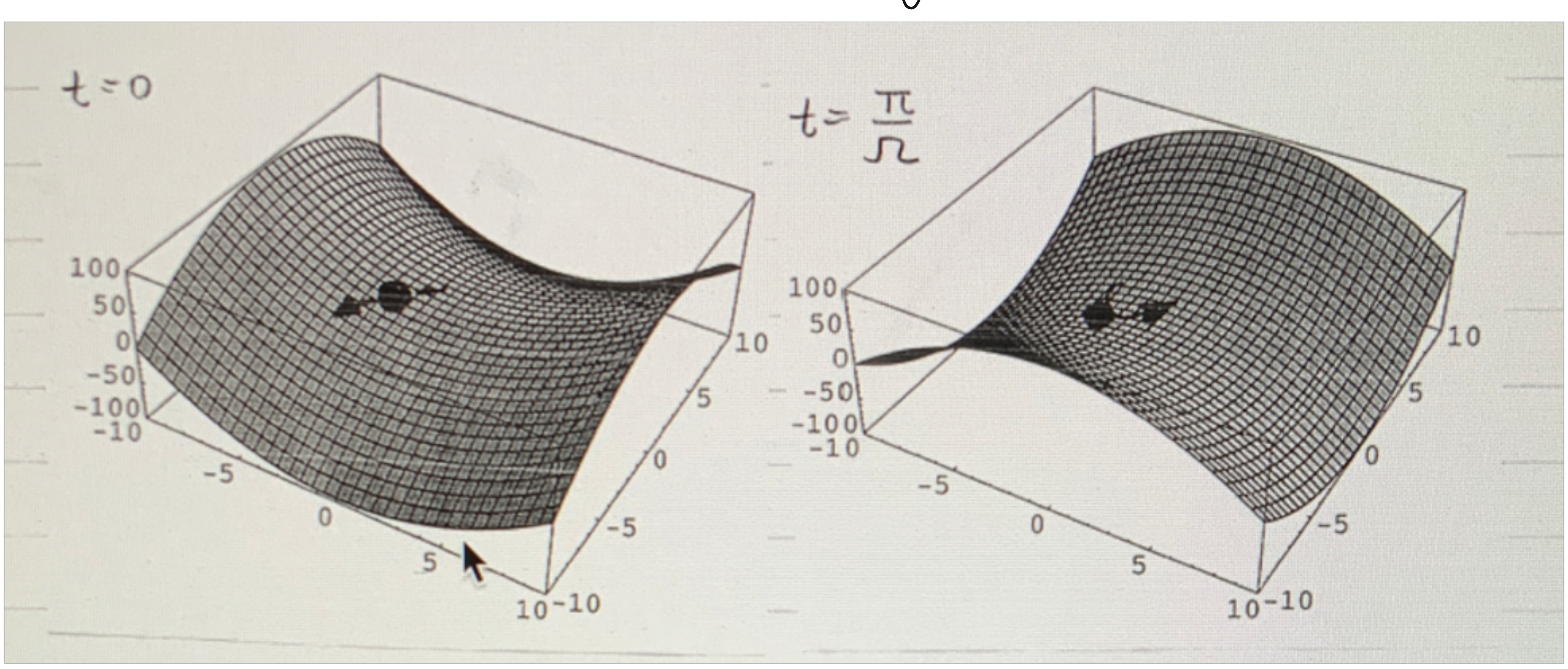


linear Paul trap

Maxwell: no extremum of $\vec{E}$ in a static source-free region

$\Rightarrow$ dynamic quadrupole field

$$\phi(x,y) = \frac{U + V\cos(\Omega t)}{2 r_0^2}\,(x^2 - y^2) \qquad (1)$$

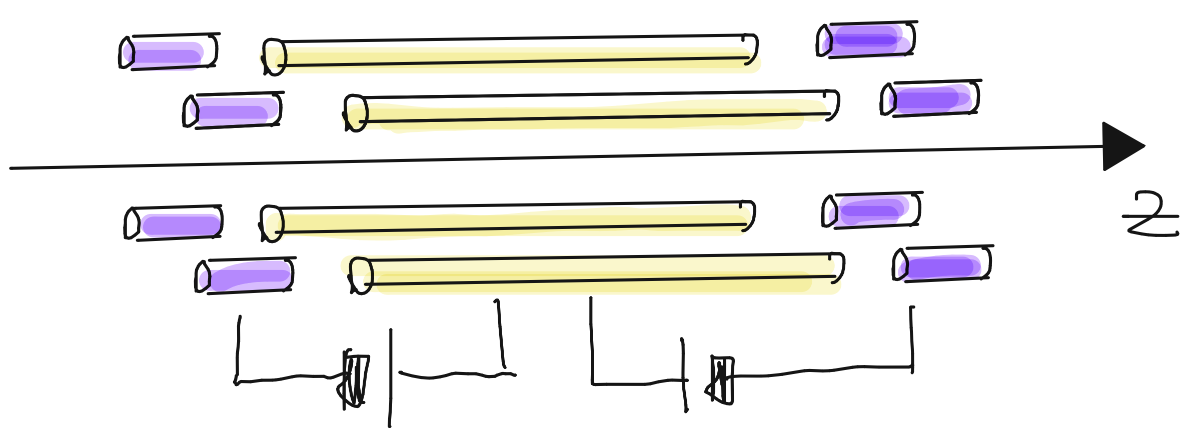


for certain parameter regimes of U, V, Ω
stable dynamics with time-averaged harmonic potential in x-y

$$(2) \qquad q\phi_{eff} = \frac{1}{2} m \omega_r^2 (x^2+y^2) \qquad \omega_r \simeq \frac{qV}{\sqrt{2}\, m\Omega r_0^2}$$

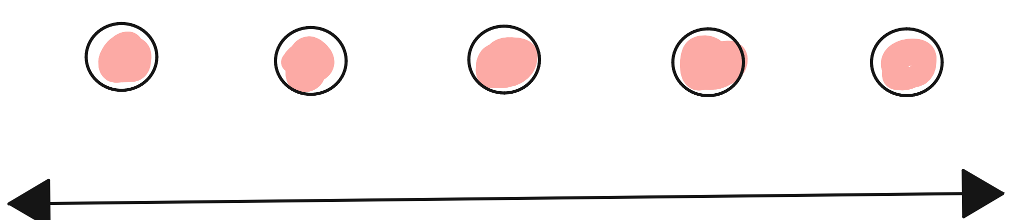
in addition: micromotion

additional electrodes at end caps yields confinement in z direction



$\omega_z < \omega_r$

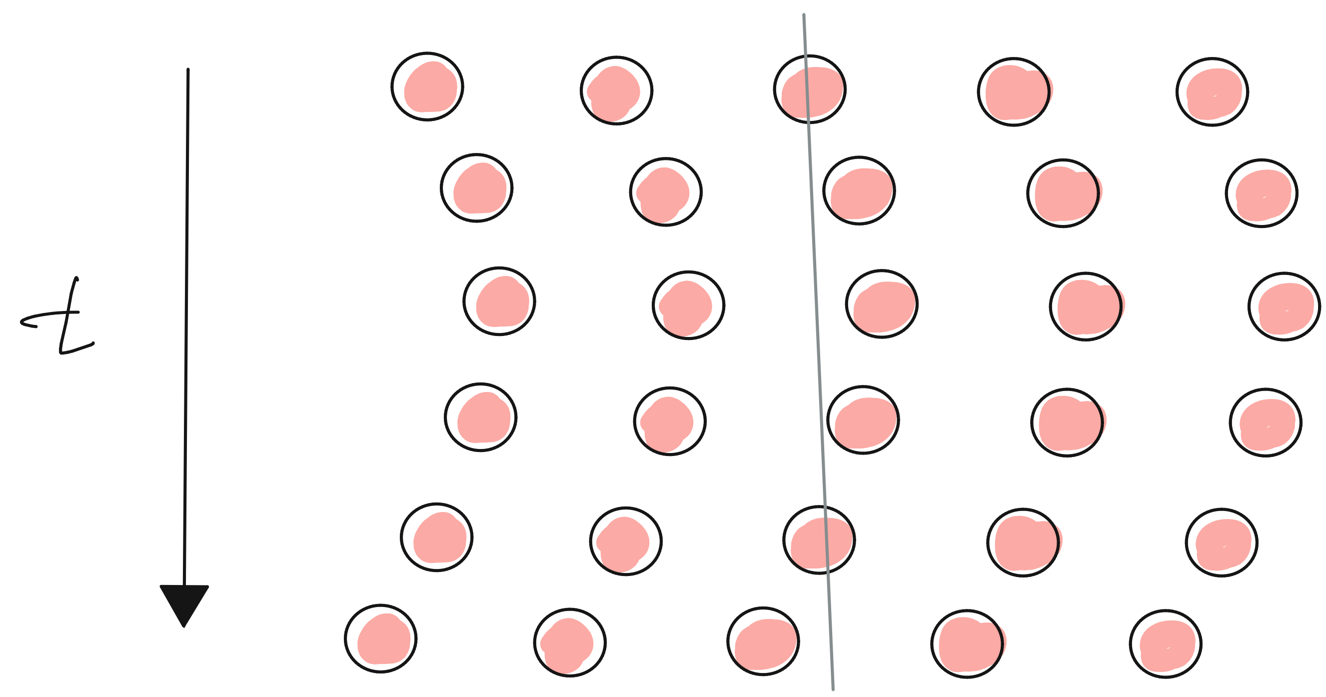
Coulomb repulsion of ions $\Rightarrow$ small linear crystal of ions (phase transition to other configurations such as zig-zag if number of ions increased or parameter changed)



elementary excitations: oscillations about equilibrium position

- <u>lowest frequency mode</u>:

  in-phase oscillation of all ions



$\nu_1 = \sqrt{3}\,\nu_0$

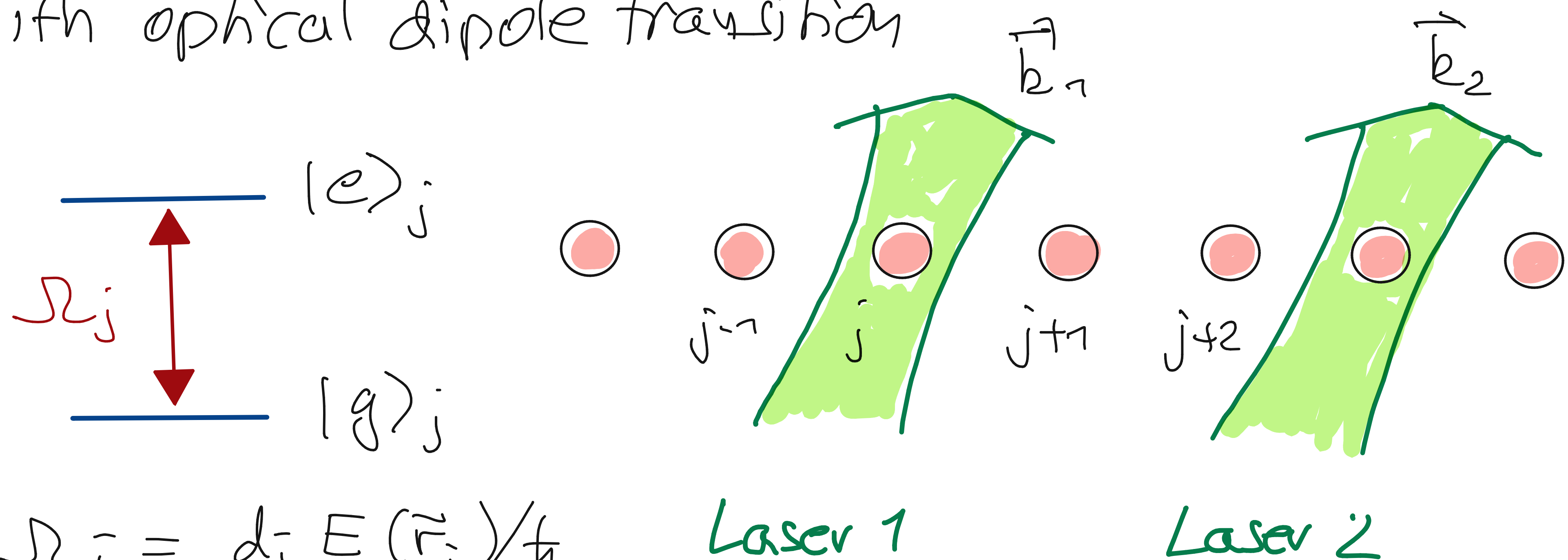
ν_0

e.g. $2\pi \times 50\,\mathrm{kHz}$

for $k_B T \ll \nu_1 - \nu_0$ only lowest mode relevant

2. qubits and Cirac Zoller gate

two internal states of ions $|g\rangle, |e\rangle$
with optical dipole transition



(3) $\Omega_j = d_j E(\vec{r}_j)/\hbar$

distance between ions $\gg \lambda$ $\rightarrow$ <u>individual addressing</u>
j – ion index

Hamiltonian

(4) $$H = H_0^{ion} + \hbar \nu_0 a^+ a + \sum_j^N \frac{\hbar \Omega_j}{2} \sin(\vec{k}_j \hat{\vec{r}}_j - \omega t)(\sigma_j^+ + \sigma_j^-)$$

harmonic oscillator of common mode "phonon"

dipole coupling of laser fields

$$\sigma_j^+ = |e\rangle_j {}_j\langle g| \qquad \sigma_j^- = |g\rangle_j {}_j\langle e|$$

for small oscillation amplitude

(5) $$\hat{\vec{r}}_j = \vec{r}_{j0} + \underbrace{\sqrt{\frac{\hbar}{2M\nu_0}}\,(\hat{a} + \hat{a}^+)}_{= \hat{z}}\,\vec{e}_z$$

$M = N \cdot m_0$

m_0: ion mass

(6) $$\vec{k}_j \hat{\vec{r}}_j = \vec{k}_j \vec{r}_{j0} + k_{jz} \hat{z}$$

(7) $$k_j \hat{z} = \eta (\hat{a} + \hat{a}^+)$$

where

(8) $$\boxed{\eta = \frac{\pi}{\lambda} \sqrt{\frac{2\hbar}{m_0 \nu_0}} \frac{1}{\sqrt{N}}}$$

Lamb-Dicke parameter

Size of η?

(9) $$\eta^2 = \frac{1}{N} \frac{\hbar}{2m_0} \left(\frac{2\pi}{\lambda}\right)^2 \frac{1}{\nu_0} = \frac{\omega_{rec}}{N \nu_0}$$

ω_{rec}: recoil frequency ($\hbar\omega_{rec}$ = energy which atom exchanges by momentum transfer when

absorbing / emitting a photon of momentum
$\hbar k = \hbar \frac{2\pi}{\lambda}$)

ω_{rec} is very small $\rightarrow$ η small

expansion of eq. (4) in lowest order of η
(Lamb - Dicke regime)

(10) $$e^{i\vec{k}_j \vec{r}_j} = \underbrace{e^{i k_j r_{j0}}}_{\hat{=}1} \left[1 + i\eta(\hat{a}^+ + \hat{a}) \right]$$

(irrelevant)

$\Rightarrow$

$$H = H_0^{ion} - \sum_j \frac{\hbar \Omega_j}{2} (\sigma_j^+ + \sigma_j^-) \sin \omega t$$

(11) $$+ \sum_j \frac{\hbar \Omega_j}{2} \eta (\hat{a} + \hat{a}^+)(\sigma_j^+ + \sigma_j^-) \cos \omega t$$

in interaction picture:

$$\sigma^+ \rightarrow \sigma^+ e^{i\omega_0 t} \qquad \hat{a} \rightarrow \hat{a} e^{-i\nu t}$$

and rotating wave approximation

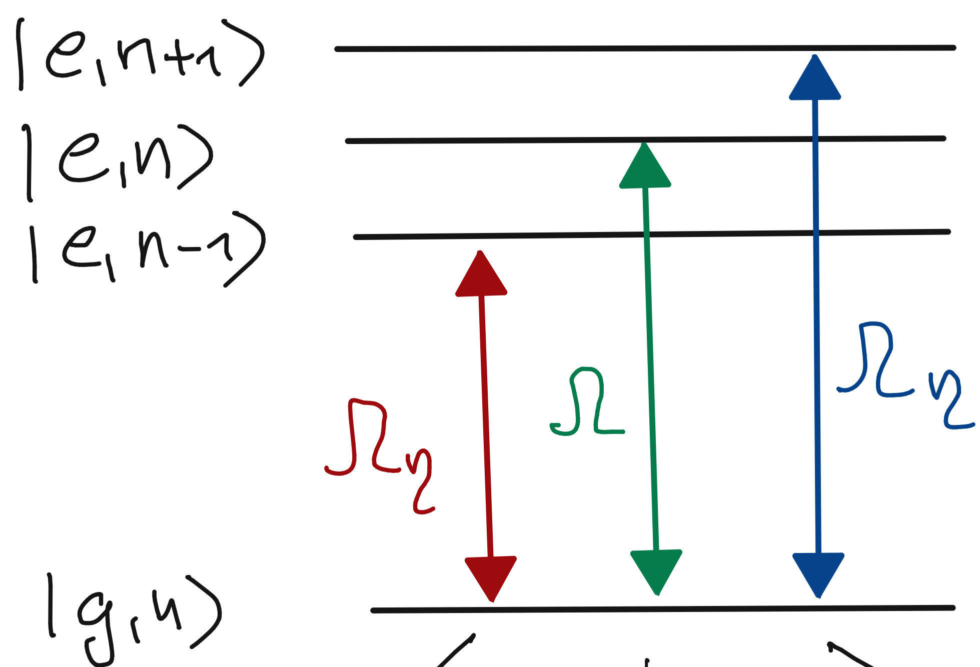
$$e^{\pm i(\omega_0 + \omega)t} \approx 0$$ averages to zero

introducing the detuning

$$\Delta = \omega - \omega_0$$

$$H = H_0^{ion} + \sum_j \frac{i\hbar\Omega_j}{2}\left(e^{-i\Delta t}\sigma_j^+ - h.a.\right)$$

$$+ \sum_j \frac{\hbar\Omega_j}{2}\,\eta\left[\left(\hat{a}^+ e^{-i(\Delta-\nu)t} + \hat{a}\, e^{i(\Delta+\nu)t}\right)\sigma_j^+ + h.a.\right] \quad (12)$$



$$\Omega_\eta = \eta\Omega \ll \Omega \quad (13)$$

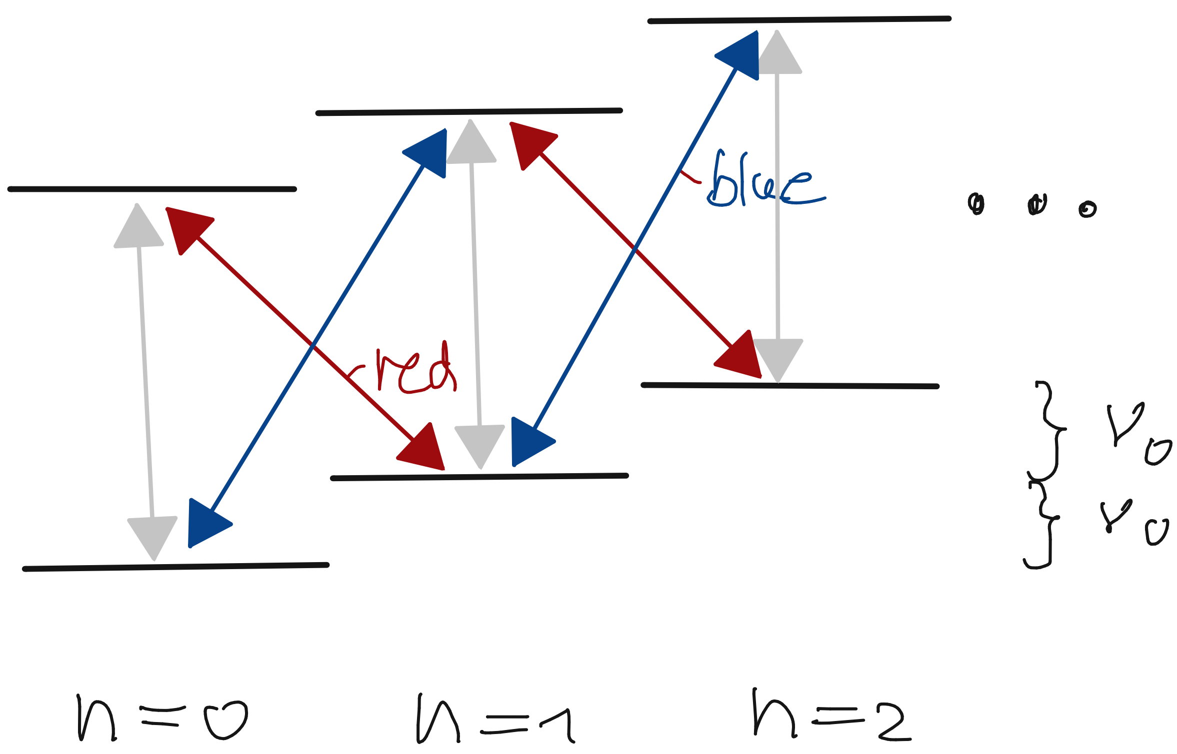
red sideband
resonant if
$\Delta + \nu = 0$
or
$\omega = \omega_0 - \nu$

blue sideband
resonant if
$\Delta - \nu = 0$
or
$\omega = \omega_0 + \nu$

central band
resonant if $\Delta = 0$
$\omega = \omega_0$

- optical excitation of an ion is accompanied by absorption (red) / emission (blue) of a phonon of collective oscillation of all ions
$\Rightarrow$ coupling to other ions (qubits)

- can also be used for cooling
- coupling on red sideband only possible if phonon mode is excited, i.e. if $n \geq 1$



If ν_0 is larger than linewidth / Rabi-frequencies the different bands can be spectroscopically resolved.

phase gate (equivalent to CNOT)

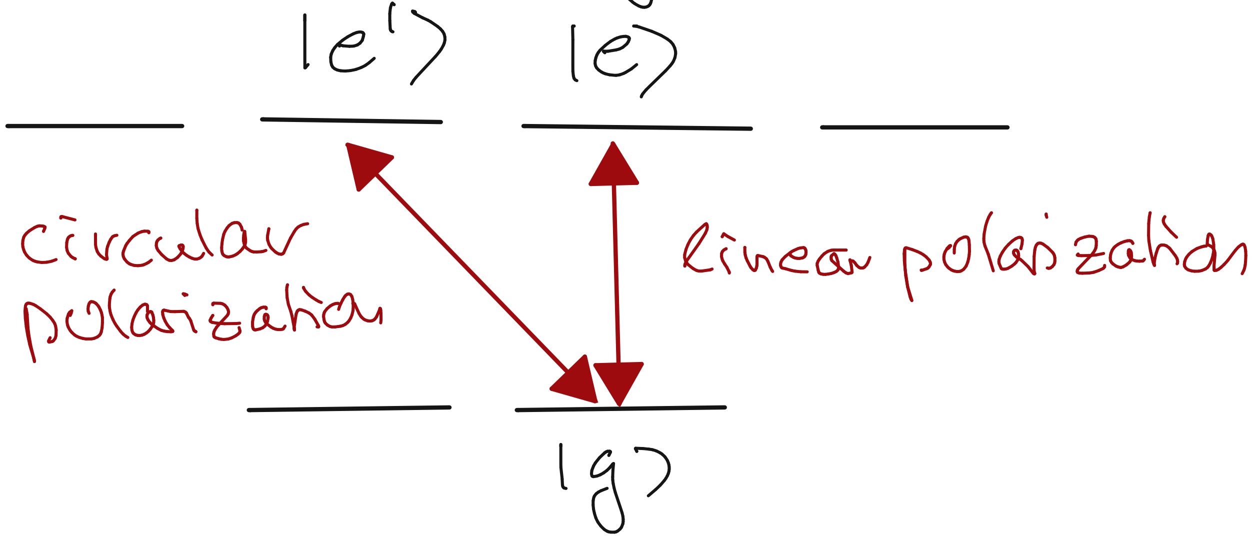
- assume phonon mode in ground state $n=0$ (cooling required)

qubits ion 1 $|g\rangle_1$, $|e\rangle_1$

ion 2 $|g\rangle_2$, $|e\rangle_2$

need additional degenerate states

e.g.:



① π laser pulse at ion 1 on red sideband

$$
\begin{aligned}
&|g\rangle_1|g\rangle_2|0\rangle & & & &|g\rangle_1|g\rangle_2|0\rangle \\
&|g\rangle_1|e\rangle_2|0\rangle & &\xrightarrow{\;n=0\,!\;} & &|g\rangle_1|e\rangle_2|0\rangle \Big\} \text{ red side band} \\
&|e\rangle_1|g\rangle_2|0\rangle & & & -i\,&|g\rangle_1|g\rangle_2|1\rangle \\
&|e\rangle_1|e\rangle_2|0\rangle & & & -i\,&|g\rangle_1|e\rangle_2|1\rangle
\end{aligned}
\qquad (13)
$$

② 2π laser pulse at ion 2 $g \leftrightarrow e'$ on red sideband

$$
\begin{aligned}
&|g\rangle_1|g\rangle_2|0\rangle & & & &|g\rangle_1|g\rangle_2|0\rangle \\
&|g\rangle_1|e\rangle_2|0\rangle & &\longrightarrow & &|g\rangle_1|e\rangle_2|0\rangle \Big\} \text{ red side-band} \\
-i\,&|g\rangle_1|g\rangle_2|1\rangle & & & +i\,&|g\rangle_1|g\rangle_2|1\rangle \\
-i\,&|g\rangle_1|e\rangle_2|1\rangle & & & -i\,&|g\rangle_1|e\rangle_2|1\rangle
\end{aligned}
\qquad (14)
$$

③ π laser pulse on ion 1 on red sideband

$$|g\rangle_1|g\rangle_2|0\rangle \qquad |g\rangle_1|g\rangle_2|0\rangle$$

$$(15) \quad |g\rangle_1|e\rangle_2|0\rangle \qquad |g\rangle_1|e\rangle_2|0\rangle$$

$$i\,|g\rangle_1|g\rangle_2|1\rangle \qquad |e\rangle_1|g\rangle_2|0\rangle$$

$$-i\,|g\rangle_1|e\rangle_2|1\rangle \qquad -|e\rangle_1|e\rangle_2|0\rangle$$

in summary

$$(16) \quad \boxed{\begin{array}{ccc} |g\rangle|g\rangle|0\rangle & & |g\rangle|g\rangle|0\rangle \\ |g\rangle|e\rangle|0\rangle & \longrightarrow & |g\rangle|e\rangle|0\rangle \\ |e\rangle|g\rangle|0\rangle & & |e\rangle|g\rangle|0\rangle \\ |e\rangle|e\rangle|0\rangle & & -|e\rangle|e\rangle|0\rangle \end{array}}$$

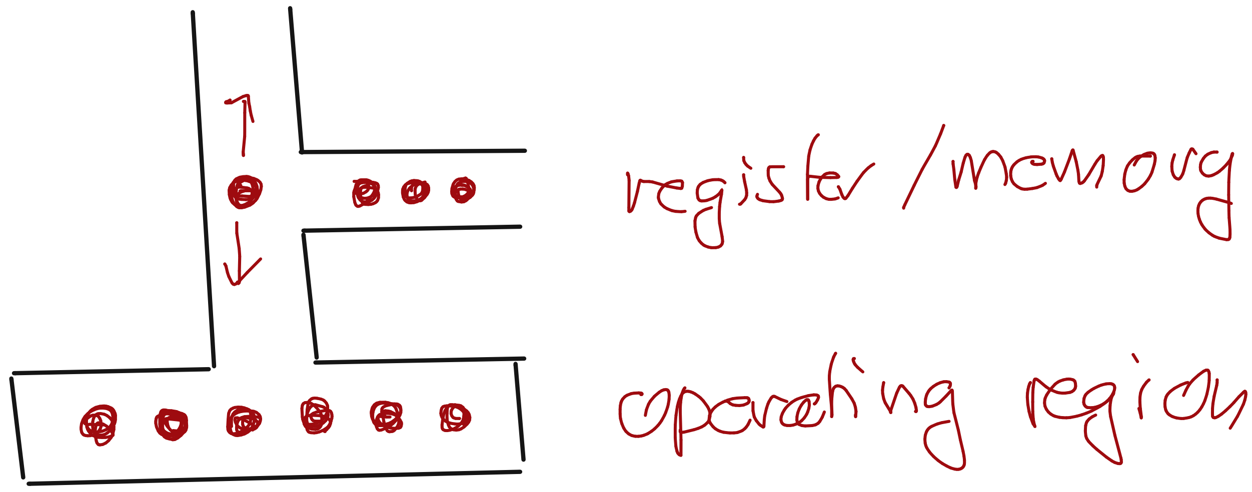
controlled phase gate

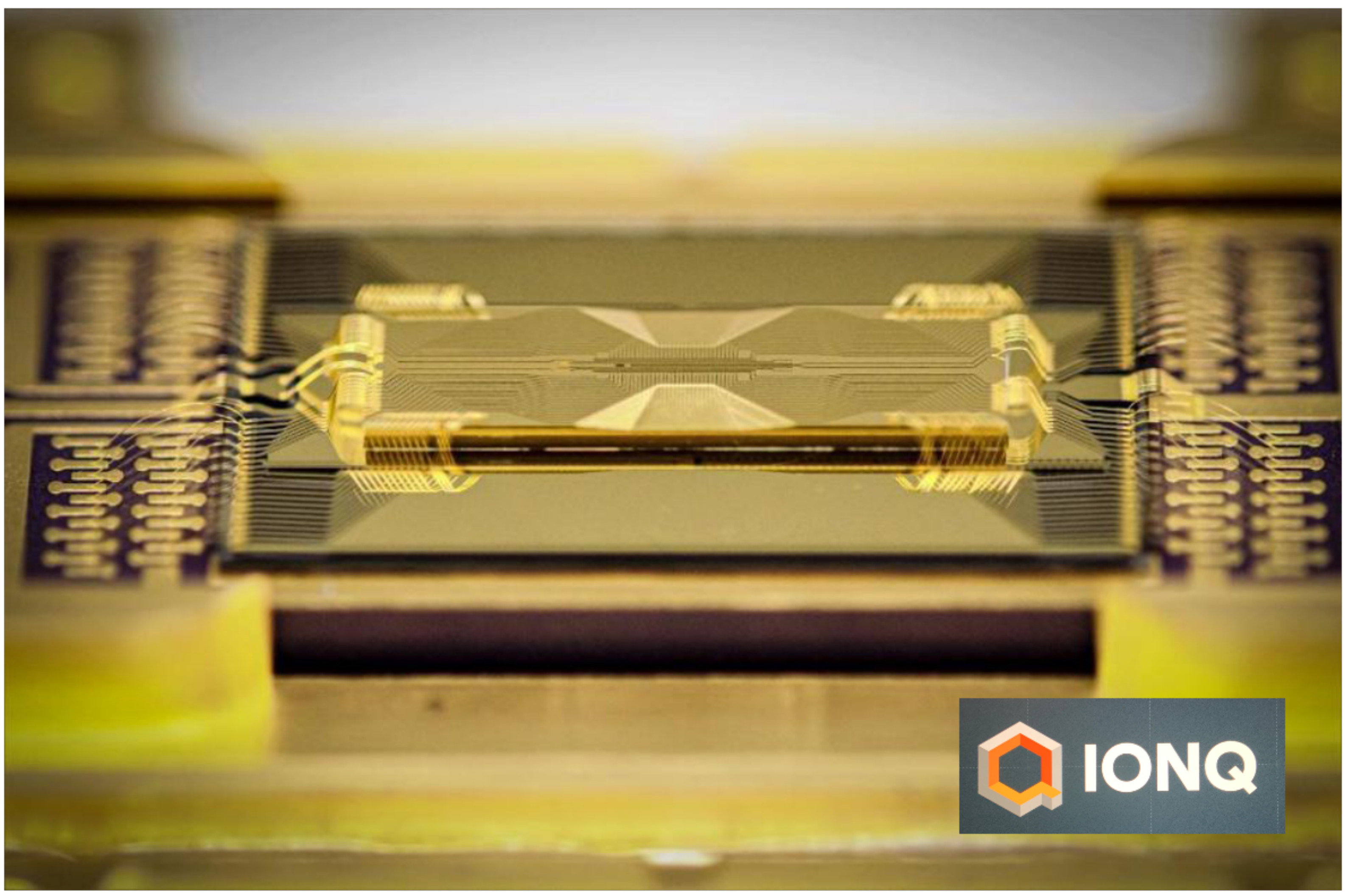
- first experimental implementation

  D. Wineland (Boulder, NIST) Nature 422, 412 (2003)

  R. Blatt (Innsbruck) Nature 421, 48 (2003)

- Scaling to large number of ions using state-preserving transport





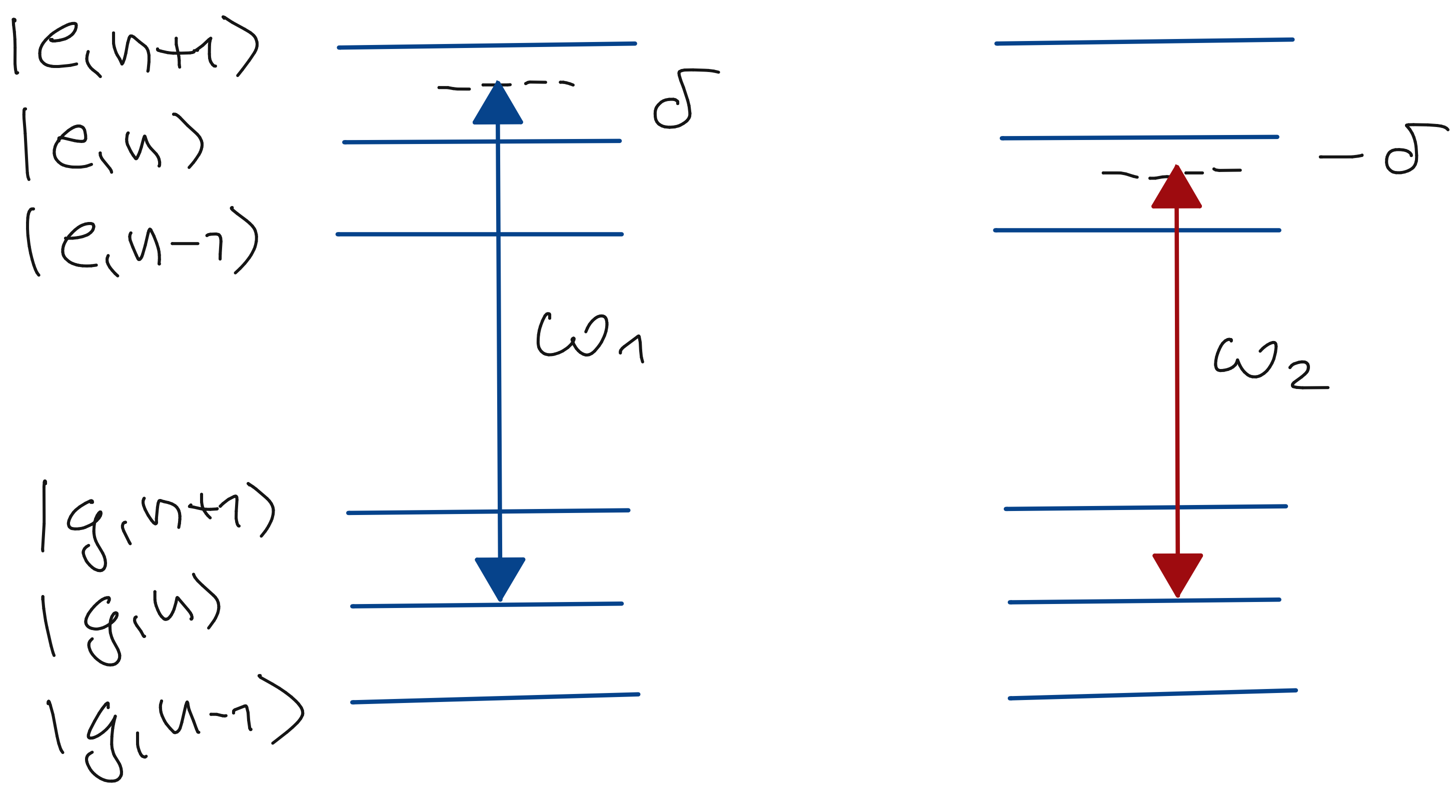
3. Mølmer-Sørensen gate

main problem of Cirac-Zoller gate:

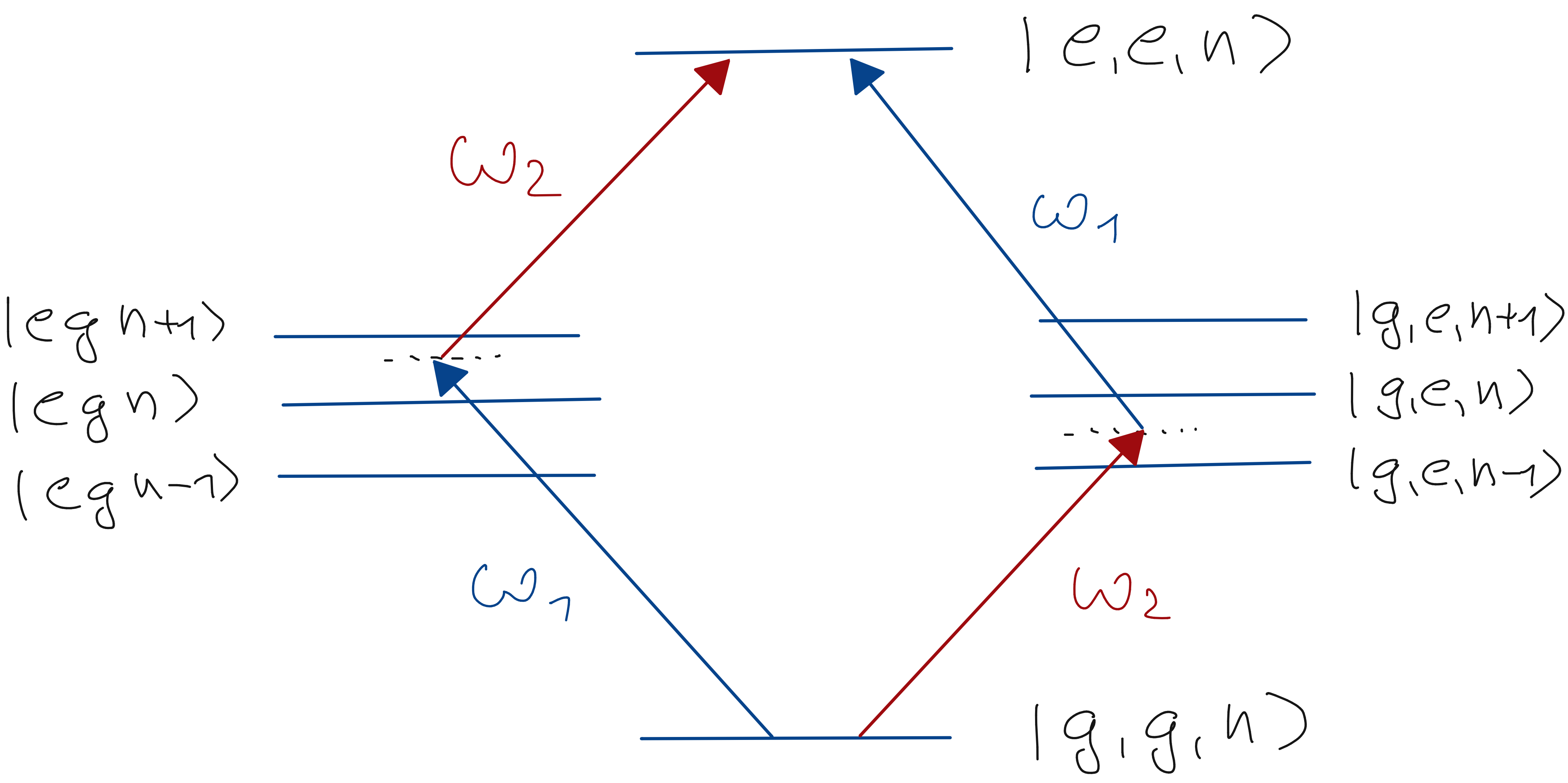
requires groundstate cooling!

⇒ gate with "hot" ions possible?

A. Sørensen, K. Mølmer Phys. Rev. Lett. 82, 1971 (1999)



two-color excitation scheme



effective 2-photon Rabi frequency $(\Omega_1 = \Omega_2 = \Omega)$

$$(17) \quad \left(\frac{\tilde{\Omega}}{2}\right)^2 \simeq \frac{1}{\hbar^2} \left| \sum_m \frac{\langle e e n | H_{int} | m \rangle \langle m | H_{int} | g g n \rangle}{E_{ggn} + \hbar \omega_i - E_m} \right|^2$$

restrict sum to $|e g n+1\rangle$ and $|g e n-1\rangle$

$\Rightarrow$

$$(18) \quad \boxed{\tilde{\Omega} = - \frac{(\eta \Omega)^2}{2(\nu - \delta)}}$$

does not depend on n !

$\delta = \omega_1 - \omega_{eg}$