SEMINAR II

TECHNISCHE UNIVERSITÄT KAISERSLAUTERN

Bell Inequalities

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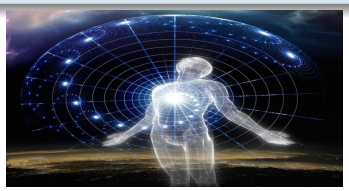
History

- New theory: Quantum Mechanics
- No interpretation of theory
- Einstein was a vocal enemy of Quantum Mechanics
- 1935 Einstein, Podolsky and Rosen "Can Quantum-Mechanical Descriptions of Physical Reality Be Considered Complete?"

$$i\hbar\frac{\partial}{\partial t}\left|\psi(t)\right\rangle = \hat{H}\left|\psi(t)\right\rangle$$



[www.thesacredscience.com/the-science-behind-shamanism-non-locality/]

Motivation

- 1964 Bell responds to EPR: "On the Einstein-Podolsky-Rosen-Paradox"
  $\Rightarrow$ Laid groundwork for all presentations in this seminar

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. Einstein, B. Podolsky and N. Rosen, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

Motivation

- 1964 Bell responds to EPR: "On the Einstein-Podolsky-Rosen-Paradox"
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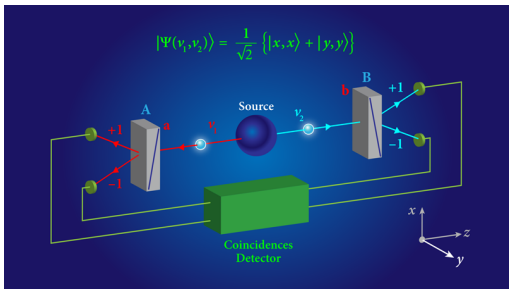
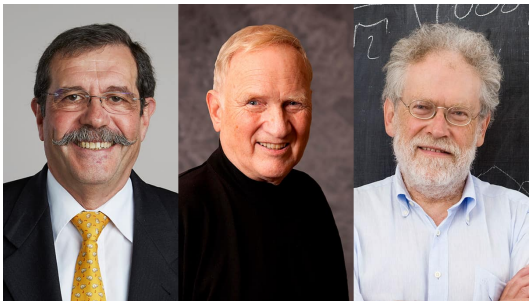
- Experiments followed by Clauser and Aspect



[Alain Aspect Physics Viewpoint]

Motivation

- 1964 Bell responds to EPR: "On the Einstein-Podolsky-Rosen-Paradox"
  $\Rightarrow$ Laid groundwork for all presentations in this seminar
- Experiments followed by Clauser and Aspect
  $\Rightarrow$ Nobel Price of 2022 after all Loopholes were closed in 2015



[Spie.org 2022 Nobel price in physics]

Outline

Physics in the late 19th century

- "in this field [physics], almost everything is already discovered, and all that remains is to fill a few unimportant holes." by Phillip Jolly in 1878
- Quantum Mechanics was discovered

  $\rightarrow$ view changed drastically

Significant contributions

- Planck 1900: Black Body Radiation
- Einstein 1905: Photo Electrical Effect

Two fronts

- Initially no conventional interpretation of Quantum Mechanics
  $\rightarrow$ a lot of interpretations existed
- 1927 Nils Bohr, Werner Heisenberg; In Agreement that Quantum Mechanics is complete

Other Opinion

- Einstein disagrees fundamentally
- Quantum Mechanics can't be complete

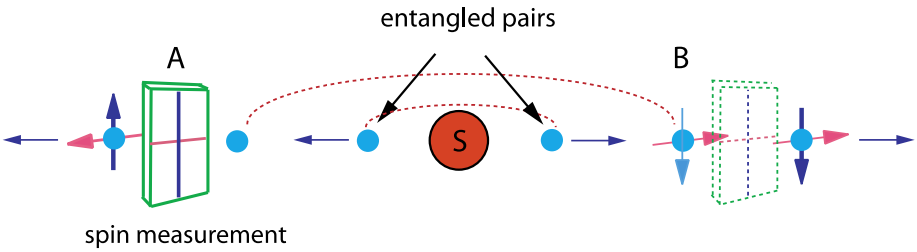
EPR-Paradox

- Einstein, Podolsky and Rosen "Can Quantum-Mechanical Descriptions of Physical Reality Be Considered Complete?"

- non commuting variables: Independent from each other

$$[x_1, p_1] = -i\hbar$$
$$[p_1 - p_2, x_1 + x_2] = [p_1, x_1] + [x_2, p_2] = 0$$

EPR-Paradox



[Scientific Background on the Nobel Price in physics 2022]

EPR conclude there must be hidden variables that make Quantum Mechanics complete

- version Bohm (1952): Spins

$$[S_x, S_z] = -i\hbar S_y$$

Hidden variables

Assume hidden variables: expectation value is

$$P(\vec{a},\vec{b}) = \int \mathrm{d}\lambda\, \rho(\lambda)\, A(\vec{a},\lambda) B(\vec{b},\lambda)$$

$\vec{a},\vec{b}$ angle of measurement, P between ± 1 represent correlation and $A, B \in -1, 1$, ρ probability density

$$P(\vec{a},\vec{b}) = \int \mathrm{d}\lambda\, \rho(\lambda)\, A(\vec{a},\lambda) B(\vec{b},\lambda) = P(\vec{a},\vec{b}) = -\int \mathrm{d}\lambda\, \rho(\lambda)\, A(\vec{a},\lambda) A(\vec{b},\lambda)$$

since $A(\vec{a},\lambda) = -B(\vec{b},\lambda)$ in case of $\vec{a}$ and $\vec{b}$ parallel so $\vec{a} = \vec{b}$

consider

$$P(\vec{a},\vec{c}) = -\int \mathrm{d}\lambda\, \rho(\lambda)\, A(\vec{a},\lambda)A(\vec{c},\lambda)$$

subtract both

$$P(\vec{a},\vec{b}) - P(\vec{a},\vec{c}) = -\int \mathrm{d}\lambda\rho(\lambda)\left[A(\vec{a},\lambda)A(\vec{b},\lambda) - A(\vec{a},\lambda)A(\vec{c},\lambda)\right]$$

$$P(\vec{a},\vec{b}) - P(\vec{a},\vec{c}) = -\int \mathrm{d}\lambda\rho(\lambda)A(\vec{a},\lambda)A(\vec{b},\lambda)\left[1 - A(\vec{b},\lambda)A(\vec{c},\lambda)\right]$$

since $|A(\vec{b},\lambda)|^2 = +1$

Introduce Inequality by $A(\vec{a},\lambda)A(\vec{b},\lambda) \leq 1$

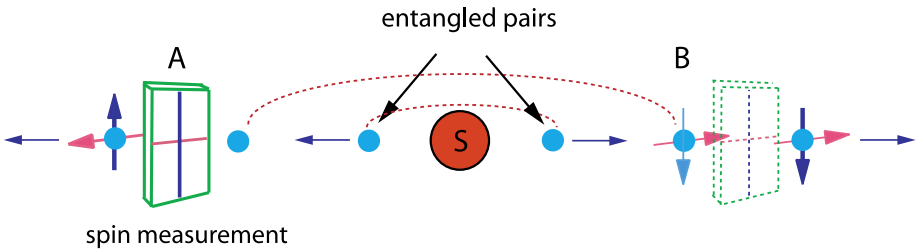
$$P(\vec{a},\vec{b}) - P(\vec{a},\vec{c}) \leq -\int \mathrm{d}\lambda \rho(\lambda)\left[1 - A(\vec{b},\lambda)A(\vec{c},\lambda)\right]$$

$$|P(\vec{a},\vec{b}) - P(\vec{a},\vec{c})| \leq 1 + P(\vec{b},\vec{c})$$

violated for $\vec{a} \perp \vec{b}$ and $\vec{c}$ at 45°

$$\frac{1}{\sqrt{2}} \leq 1 - \frac{1}{\sqrt{2}}$$
$$1 \leq 0.4$$

Bell's Gedankenexperiment and its problems



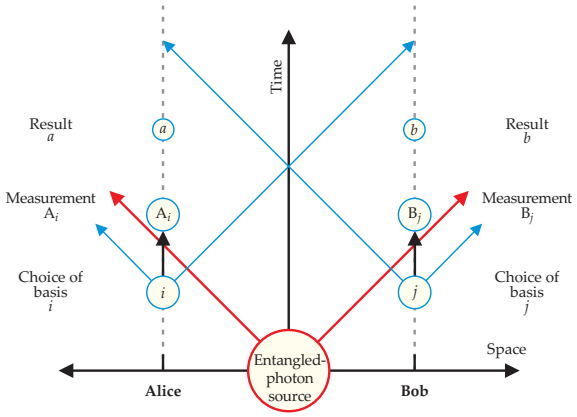
[Scientific Background on the Nobel Price in physics 2022]

- **Near perfect anti-correlation needed**

$\Rightarrow$ **Bell Inequalities are less suitable for experiments**

- **Loopholes are present**

Locality Loophole



[Physics Today 69, 1, 14 (2016); doi: 10.1063/PT.3.3039]

How to prevent the Loophole

choice of basis after entangled particle send to Alice and Bob
$\Rightarrow$ Photons better suited than Spins

Detection Loophole

Low efficiency creates sample bias

How to prevent the Loophole

High detection efficiency is needed
⇒ Spins are better suited: easier to detect

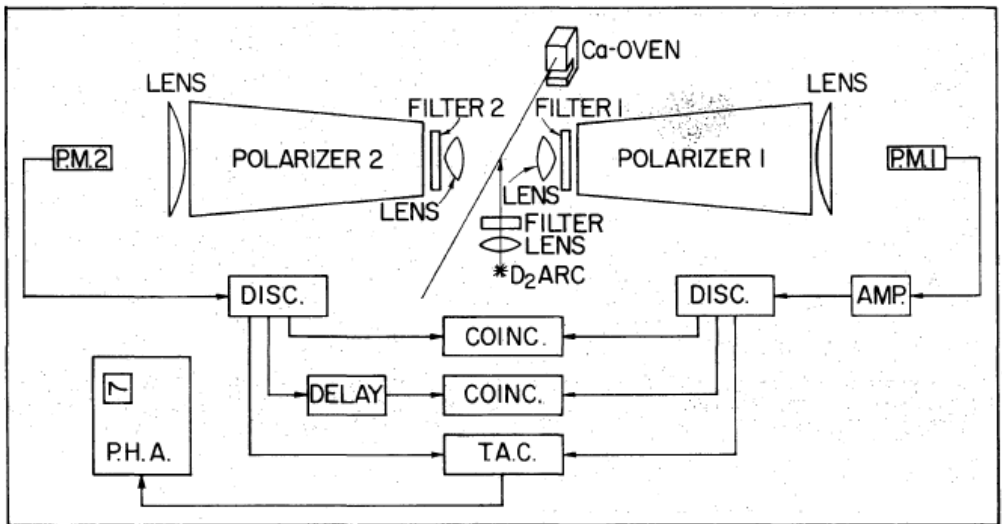
Different variants of the Bell inequality

Bell inequalities are less suited, thus new versions were made

$2 \geq |E(a,b) - E(a,b') + E(a',b') + E(a',b)|$
with
$E(a,b) = \frac{N_{++} - N_{+-} - N_{-+} + N_{--}}{N_{++} + N_{+-} + N_{-+} + N_{--}}$
for $a = 0, a' = 45, b = 22.5, b' = 67.5$

$-1 \geq p_{\mathrm{i,j}} + (a,b) + p_{\mathrm{i,j}}(a,b') + p_{\mathrm{i,j}}(a',b') - p_{\mathrm{i,j}}(a') - p_{\mathrm{i,j}}(b) \geq 0$
p are the probability of coincidences and i and j are the results $+$ and $-$

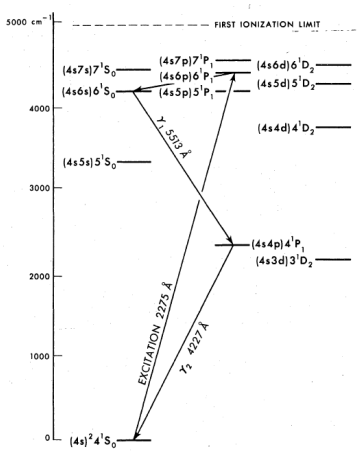
Experiment of Clauser (1972)



[Scientific Background on the Nobel Price in physics 2022]

Creating entangled Photons

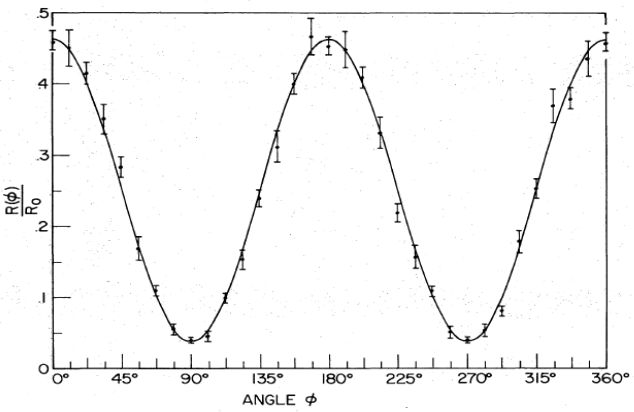
$$|\psi_\pm\rangle = \frac{1}{\sqrt{2}}\left[|{+}{+}\rangle \pm |{-}{-}\rangle\right]$$



[Scientific Background on the Nobel Price in physics 2022]

Created a Bell state

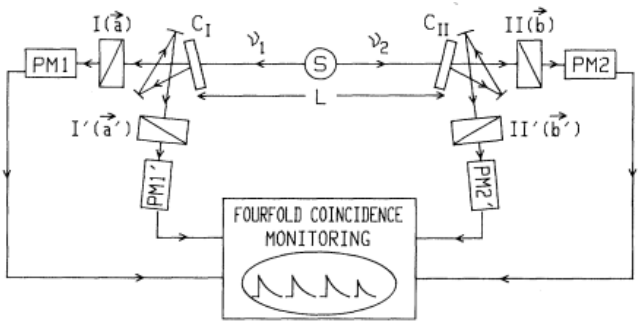
Results of Clauser (1972)



[Scientific Background on the Nobel Price in physics 2022]

- With the new equation, Bell Tests are possible
- Loopholes make the results untrustworthy

Experiment of Aspect (1982)



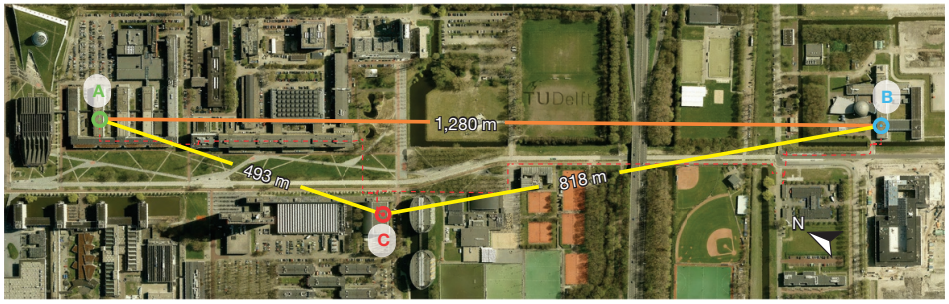
[Scientific Background on the Nobel Price in physics 2022]

Results of Aspect (1982)

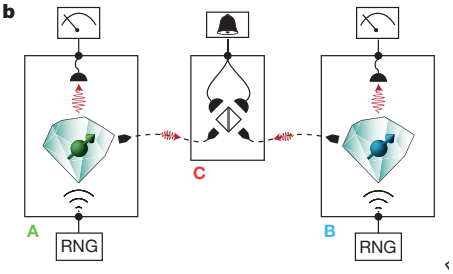
- The inequality used required $-1 \leq S \leq 0$
  The measurements gave $S = 0.101 \pm 0.020$
- It gained a lot of attention of the scientific community and started a lot more experiments

Locality Loophole was closed!*

Loophole free experiment (2015)



[nature15759 682..686 https://doi.org/10.1038/nature15759]



[nature15759 682..686 https://doi.org/10.1038/nature15759]

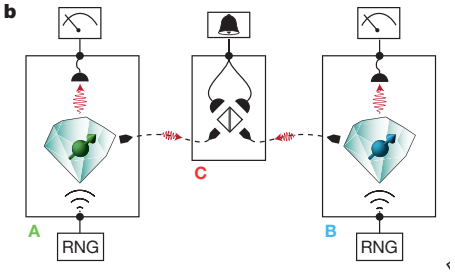
- Large experimental set-up
- clever combination of Photons and Spin to close the Loophole

How to entangle NV (nitrogen-vacancy) centrums

Consider

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}\left[|+_1\uparrow_1\rangle + |-_1\downarrow_1\rangle\right]$$

$$|\psi_2\rangle = \frac{1}{\sqrt{2}}\left[|+_2\uparrow_2\rangle + |-_2\downarrow_2\rangle\right]$$



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multiplication of both gives

$$\frac{1}{2}\left[|+_1+_2\rangle\,|\uparrow_1\uparrow_2\rangle + |+_1-_2\rangle\,|\uparrow_1\downarrow_2\rangle + |-_1+_2\rangle\,|\downarrow_1\uparrow_2\rangle + |-_1-_2\rangle\,|\downarrow_1\downarrow_2\rangle\right]$$

$$\frac{1}{\sqrt{2}}\left[|+_1-_2\rangle\,|\uparrow_1\downarrow_2\rangle + |-_1+_2\rangle\,|\downarrow_1\uparrow_2\rangle\right]$$

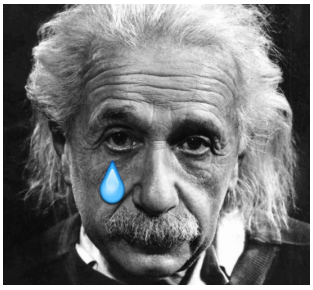
Results of the Loophole free experiment (2015)

- Violation of Bell Inequalities
- Detection efficiency of 75% of reached
- The final goal has been achieved after almost 100 years

All Loopholes are closed

Summary

- No interpretation for Quantum Mechanics
- Bell proofed theoretically hidden variable can't explain EPR-Paradox



[https://www.wellsreserve.org/blog/fools-russian]

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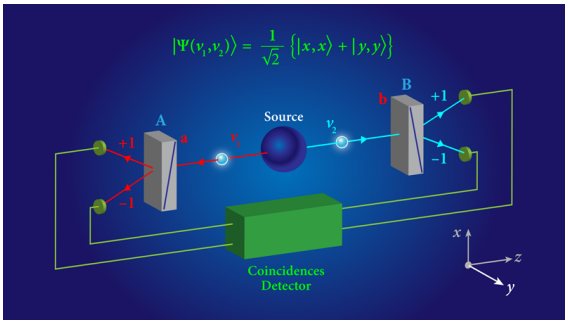

[Alain Aspect Physics Viewpoint]

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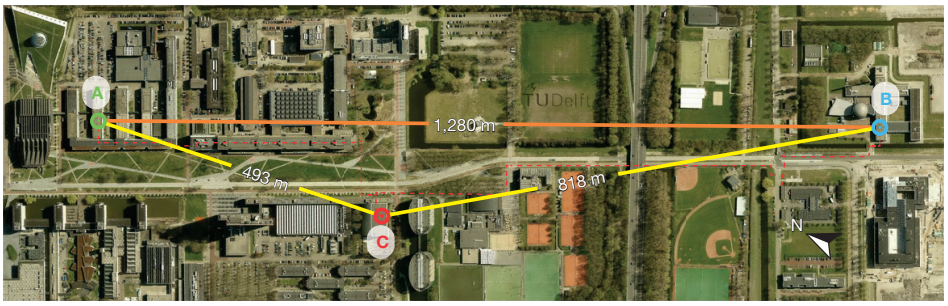
- 2015 all Loopholes were closed



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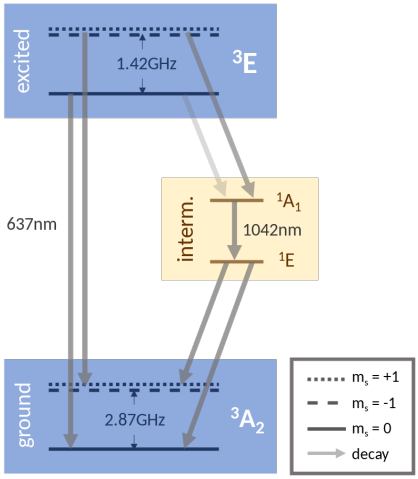
Outlook

- Now everything can begin, the basics and the beginning are established
  Every future presentation is only possible because of the work of Bell, Clauser and Aspect
- Next presentation will be about Quantum Teleportation
- Quantum information/cryptography

Thank you!

NV-Centrum energy levels



[Nitrogen-vacancy center Wikipedia]