PHYSIK

TECHNISCHE UNIVERSITÄT KAISERSLAUTERN

Measurement-based quantum computation

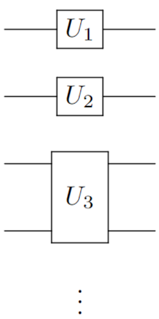
Andrei Skalkin

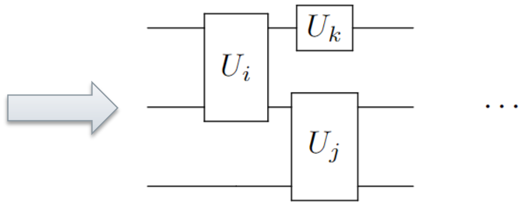
Hauptseminar
Physics Department
TU Kaiserslautern

Outline

- **Introduction**
- **MBQC principles**
- **Cluster states**
- **Experimental results**
- **Conclusion**

Circuit model

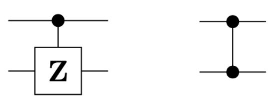




All transformations are unitary

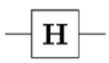
Reminder

Controlled Z (CZ)



$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

Hadamard (H)



$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$H|0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} = |+\rangle$$

[https://en.wikipedia.org/wiki/Quantum_logic_gate]

R. Raussendorf and H. Briegel proposal

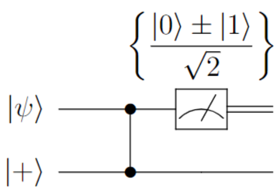
Universal quantum computing is possible by performing only single qubit measurements on a large entangled state

How possible?

[A One-Way Quantum Computer, Robert Raussendorf and Hans J. Briegel, Phys. Rev. Lett. (2001)]

PHYSIK
TECHNISCHE UNIVERSITÄT KAISERSLAUTERN

Measurements mimic unitaries



$m \in \{0,1\}$

$X^m H|\psi\rangle$

Unitary transformation

Byproduct gate, depending on measurement outcome

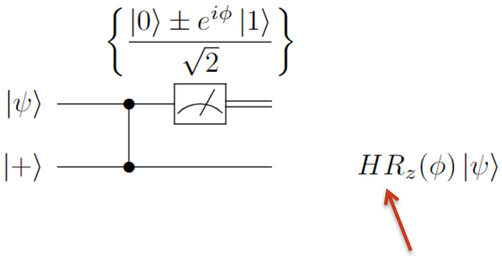
Ignore byproducts for a moment

$$|\psi\rangle_1 |+\rangle_2 = (\alpha|0\rangle_1 + \beta|1\rangle_1)\frac{|0\rangle_2 + |1\rangle_2}{\sqrt{2}} \xrightarrow{CZ_{12}}$$

$$\alpha|00\rangle_{12} + \beta|10\rangle_{12} + \alpha|01\rangle_{12} - \beta|11\rangle_{12} \xrightarrow{\frac{\langle 0|_1 \pm \langle 1|_1}{\sqrt{2}}}$$

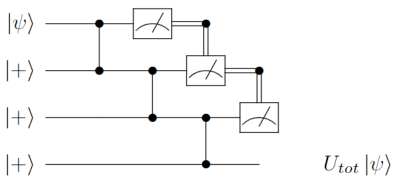
$$\alpha|0\rangle_2 \pm \beta|0\rangle_2 + \alpha|1\rangle_2 \mp \beta|1\rangle_2 = \alpha|+\rangle_2 \pm \beta|-\rangle_2 = X^m H|\psi\rangle_2$$

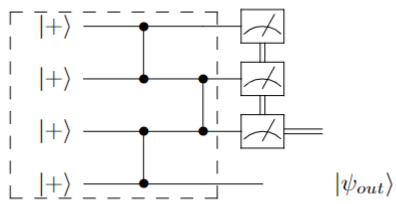
Change measurement angle



Unitary transformation, depending on measurement basis

Arbitrary rotation



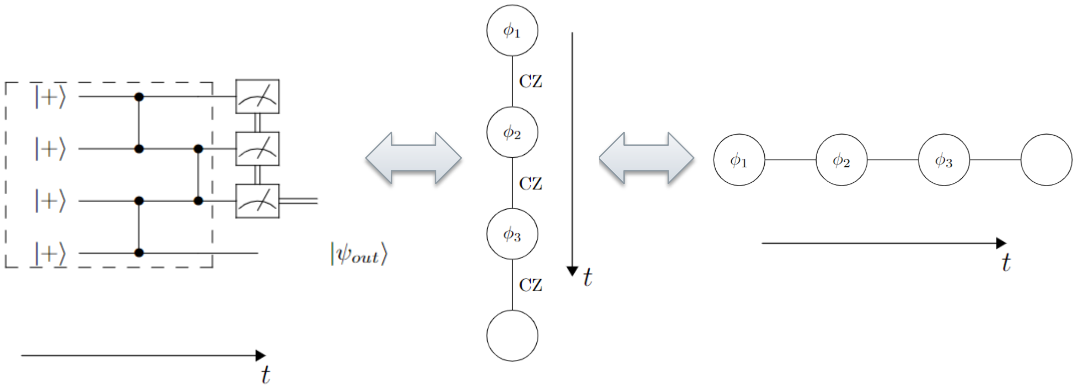


$$U_{tot} = HR_z(\phi_3)HR_z(\phi_2)HR_z(\phi_1) = HR_z(\phi_3)R_x(\phi_2)R_z(\phi_1)$$

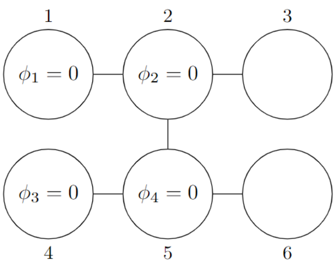
3 independent Euler angles give any Bloch sphere rotation

- Initial state does not matter
- Preparation is uniform and uses fixed unitaries

Cluster state notation



CNOT gate



- All measurements can be done simultaneously
- $U = H_6 H_3 CNOT_{3,6}$

PHYSIK

TECHNISCHE UNIVERSITÄT KAISERSLAUTERN

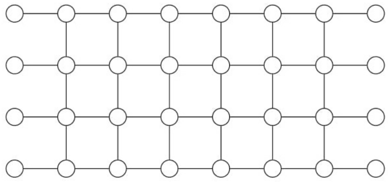
MBQC sequence

- **Prepare resource state -> Cluster state**
- **Measure qubits in a proper sequence and bases**
- **Adjust basis for the next round -> Feedforward**

PHYSIK

TECHNISCHE UNIVERSITÄT KAISERSLAUTERN

Cluster states



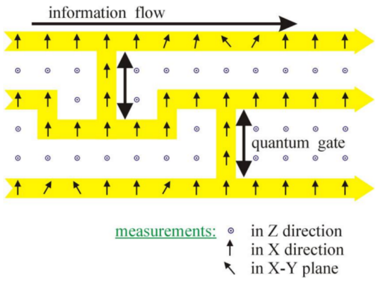
Regular structure - graph G

Prepare each qubit on vertex in plus state

Apply CZ according to the edges in G

Computation

- Z basis measurement removes a qubit from lattice
- XY plane measurements perform transformation and transport



Byproduct gates

- In general byproducts are X and Z gates
- Measurements outcomes are random

PHYSIK

TECHNISCHE UNIVERSITÄT KAISERSLAUTERN

Feedforward

Pull byproducts out:

$$
\begin{aligned}
U_R(\xi,\eta,\zeta)\,\sigma_z^s\sigma_x^{s'} &= \sigma_z^s\sigma_x^{s'}\,U_R((-1)^s\xi,(-1)^{s'}\eta,(-1)^s\zeta),\\
\mathrm{CNOT}(c,t)\,\sigma_z^{(t)\,s_t}\sigma_z^{(c)\,s_c}\sigma_x^{(t)\,s'_t}\sigma_x^{(c)\,s'_c} &= \sigma_z^{(t)\,s_t}\sigma_z^{(c)\,s_c+s_t}\sigma_x^{(t)\,s'_c+s'_t}\sigma_x^{(c)\,s'_c}\,\mathrm{CNOT}(c,t)
\end{aligned}
$$

Measurement angles should be adjusted!

PHYSIK

TECHNISCHE UNIVERSITÄT KAISERSLAUTERN

Feedforward

- Feedforward imposes time-ordering -> Measurement rounds
- Classical data - vector of 2n bits to store data about extra X and Z gates

PHYSIK

TECHNISCHE UNIVERSITÄT KAISERSLAUTERN

Universality

Universal set of gates available

Purely random measurement outcomes do not destroy the determinicity

Concatenation possible

Only specific clusters are suitable for universal computing, for example 2D rectangular

PHYSIK

TECHNISCHE UNIVERSITÄT KAISERSLAUTERN

Significance (theory)

Useful tool for analysis:

- circuits
- entanglement
- stabilizer formalizm
- correction codes
- parallelization

Significance (experiment)

Large clusters with high fidelity and generation rate required.
Generation of cluster states can be implemented more efficiently than circuit approach:

- Cold atoms in optical lattices -> Apply CZ gates in parallel
- Photons -> Probabilistic gates may lead to growth of cluster on average

Clusters of photon qubits

articles

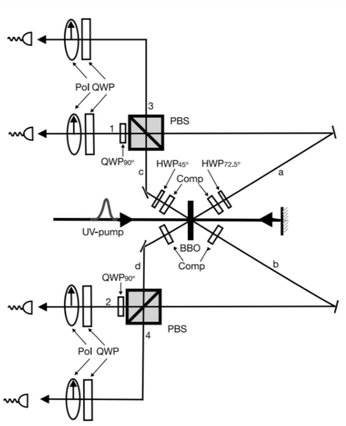
Experimental one-way quantum computing

P. Walther[1], K. J. Resch[1], T. Rudolph[2], E. Schenck[1*], H. Weinfurter[3,4], V. Vedral[1,5,6], M. Aspelmeyer[1] & A. Zeilinger[1,7]

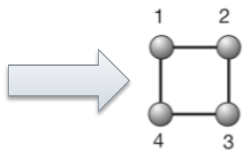
[1]*Institute of Experimental Physics, University of Vienna, Boltzmanngasse 5, 1090 Vienna, Austria*
[2]*QOLS, Blackett Laboratory, Imperial College London, London SW7 2BW, UK*
[3]*Department of Physics, Ludwig Maximilians University, D-80799 Munich, Germany*
[4]*Max Planck Institute for Quantum Optics, D-85741 Garching, Germany*
[5]*The Erwin Schrödinger Institute for Mathematical Physics, Boltzmanngasse 9, 1090 Vienna, Austria*
[6]*The School of Physics and Astronomy, University of Leeds, Leeds LS2 9JT, UK*
[7]*IQOQI, Institute for Quantum Optics and Quantum Information, Austrian Academy of Sciences, Boltzmanngasse 3, 1090 Vienna, Austria*

* Permanent address: Ecole normale supérieure, 45, rue d'Ulm, 75005 Paris, France

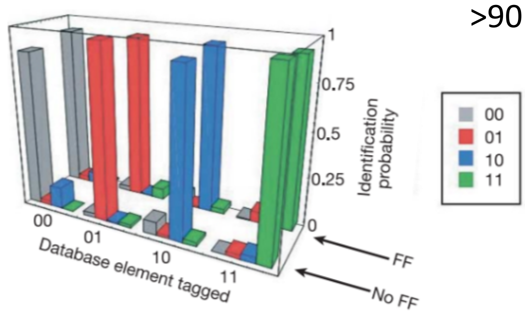
Setup



$$F_{\square} = (0.63 \pm 0.02)$$



Grover’s algorithm



>90 % identification success

Summary

- New approach to quantum computing
  - Resource does not depend on the algorithm
  - Insight on the role of entanglement
- Applicable to all physical platforms, but large cluster states are still a challenge

Thank you!

Bonus: CNOT between distant qubits