Problem 19.
Show that electric field induced by the Chern-Simons flux attached to the electrons is given by

$$\boldsymbol{e}_{\rm CS} = 2p\frac{h}{e^2}\left(\boldsymbol{e}_z \times \boldsymbol{j}\right) \tag{1}$$

where $\boldsymbol{j}$ is the current density. For this make use of Faraday's law

$$-\frac{1}{c}\frac{\partial}{\partial t}\boldsymbol{b}(\boldsymbol{r},t) = \boldsymbol{\nabla}\times\boldsymbol{e}_{\rm CS} \tag{2}$$

where $\boldsymbol{b}(\boldsymbol{r},t) = -2p\Phi_0\rho(\boldsymbol{r},t)\boldsymbol{e}_z$ is the magnetic field associated to the $2p$ flux flux quanta per electron, and the continuity equation

$$\boldsymbol{\nabla}\cdot\boldsymbol{j} + (-e)\frac{\partial\rho}{\partial t} = 0. \tag{3}$$

Problem 20.
Show that all odd-denominator fractions can be related to an integer by repeated application of three operations

(i) composite fermionization

$$\nu \to \nu^*, \qquad (\nu^*)^{-1} = \nu^{-1} - 2p \tag{4}$$

(ii) particle-hole symmetry

$$\nu \to 1-\nu \tag{5}$$

(iii) subtraction of full Landau levels

$$\nu \to \nu - 1 \tag{6}$$

Problem 21.
Many correlation functions in the FQHE can be related to IQH states. Show for $\nu = 1$ (i.e. IQHE) wave functions

(i) $g^{(2)}(r) = 1 - \exp\left(-\frac{r^2}{2\ell^2}\right)$

(ii) $S(q) = 1 - \exp\left(-\frac{q^2}{2\ell^2}\right)$ static structure factor

(iii) interaction energy per particle, assuming Coulomb interaction $V(r) = \frac{e^2}{r}$

$$\frac{E}{N} = -\sqrt{\frac{\pi}{8}}\frac{e^2}{\ell}. \tag{7}$$