Quantum Hall physics and topological states of matter - an introduction
Prof. M. Fleischhauer WS 2013

5. Sheet Tutorial: 20.12.2013, 08:15 am

Problem 15.
Consider spin full fermions in 2D with the continuous Dirac Hamiltonian,

$$\hat{\mathcal{H}} = \int d^2\boldsymbol{k}\ \underline{\hat{c}}^\dagger(\boldsymbol{k}) \sum_{j=1}^{3} d_j(\boldsymbol{k}) \underline{\underline{\sigma_j}}\ \underline{\hat{c}}(\boldsymbol{k}) \tag{1}$$

where σ_j are the Pauli matrices and $\underline{\hat{c}}(\boldsymbol{k}) = (\hat{c}_\uparrow(\boldsymbol{k}), \hat{c}_\downarrow(\boldsymbol{k}))^T$. Find the eigenenergies and eigenstates of (1). For $d_1 = k_x, d_2 = k_y, d_3 = m$ calculate the Berry gauge potential (i.e. Berry connection) and the Berry field (curvature) and verify that the Chern number of the negative energy states is half-integer.

Problem 16.
Show that no continuous gauge can be found for the Berry gauge potential of (1) in the topologically non-trivial case.

Problem 17.
Show that the tight-binding Graphene model (4.6) with $t_a = t_b = t_c$ has a C_3 symmetry. Show that this implies that a gap closing is only possible at momenta $\boldsymbol{K}$ and $\boldsymbol{K}'$ (c.f. eq. (4.3)).

Problem 18.
Consider a 1D Dirac model with

$$\hat{\mathcal{H}} = \int dx\ \underline{\hat{\Psi}}^\dagger(x)\Big(-i\partial_x\sigma_x + m(x)\sigma_z\Big)\underline{\hat{\Psi}}(x) \tag{2}$$

where $\underline{\hat{\Psi}} = \left(\hat{\Psi}_\uparrow, \hat{\Psi}_\downarrow\right)^T$. Let

$$m(x) = \begin{cases} m, & x \geq 0. \\ -m, & x < 0. \end{cases} \tag{3}$$

Show that there exists a zero energy state located at the position of the mass jump $x = 0$.