Problem 11.
Show that a spin 1/2 particle in a magnetic field is odd under TR whereas spin-orbit coupling is even under TR.
Problem 12.
Show by explicit calculation that the time reversal operator for a single spin 1/2 particle,

$$\hat{T} = e^{-i\sigma^y \pi/2} K = -K i\sigma^y \tag{1}$$

gives $\hat{T}^2 = -1$. Generalize this now to two spin 1/2 particles. How does the TR operator look like in first quantization? Show that now $\hat{T}^2 = 1$.
Problem 13.
Show that the tight binding Hamiltonian of Graphene (eq.(4)-(6)) for $t_a = t_b = t_c = 1$,

$$\hat{\mathcal{H}} = \sum_{\boldsymbol{k}} \hat{c}^\dagger_{\boldsymbol{k}} h(\boldsymbol{k}) \hat{c}_{\boldsymbol{k}} \tag{2}$$

with

$$h(\boldsymbol{k}) = \begin{pmatrix} 0 & -t_a e^{i\boldsymbol{k}\cdot\boldsymbol{a}_1} - t_b e^{i\boldsymbol{k}\cdot\boldsymbol{a}_2} - t_c \\ -t_a e^{-i\boldsymbol{k}\cdot\boldsymbol{a}_1} - t_b e^{-i\boldsymbol{k}\cdot\boldsymbol{a}_2} - t_c & 0 \end{pmatrix} \tag{3}$$

indeed leads to Dirac cones around $\boldsymbol{K}, \boldsymbol{K}'$, i.e. for

$$\boldsymbol{k} = \boldsymbol{K} + \boldsymbol{\kappa}, \qquad \boldsymbol{k} = \boldsymbol{K}' + \boldsymbol{\kappa} \tag{4}$$

with $|\boldsymbol{\kappa}| \ll |\boldsymbol{K}|, |\boldsymbol{K}'|$. Here $\boldsymbol{K} = \frac{2\pi}{3}\left(1, 1/\sqrt{3}\right)$ and $\boldsymbol{K}' = \frac{2\pi}{3}\left(1, -1/\sqrt{3}\right)$.
Problem 14.
Find the transformation properties of the Hamiltonian

$$\hat{\mathcal{H}} = \sum_{\boldsymbol{k}} \hat{c}^\dagger_{\boldsymbol{k}} h(\boldsymbol{k}) \hat{c}_{\boldsymbol{k}} \tag{5}$$

with

$$h(k) = \sin(k_x)\sigma_x + \sin(k_y)\sigma_y + m\sigma_z \tag{6}$$

under TR.