Problem 11.
Show that a spin 1/2 particle in a magnetic field is odd under TR whereas spin-orbit coupling is even under TR.

Problem 12.
Show by explicit calculation that the time reversal operator for a single spin 1/2 particle,

$$\hat{T} = e^{-i\sigma_y \pi/2} K = -Ki\sigma_y$$

(1)
gives $\hat{T}^2 = -1$. Generalize this now to two spin 1/2 particles. How does the TR operator look like in first quantization? Show that now $\hat{T}^2 = 1$.

Problem 13.
Show that the tight binding Hamiltonian of Graphene (eq.(4)-(6)) for $t_a = t_b = t_c = 1$,

$$\hat{H} = \sum_k \hat{c}_k^\dagger h(k) \hat{c}_k$$

(2)

with

$$h(k) = \begin{pmatrix} -t_a e^{-ik \cdot a_1} & -t_b e^{-ik \cdot a_2} - t_c & 0 \\ 0 & -t_a e^{ik \cdot a_1} - t_b e^{ik \cdot a_2} - t_c & 0 \\ -t_a e^{-ik \cdot a_1} & -t_b e^{-ik \cdot a_2} - t_c & 0 \end{pmatrix}$$

(3)

indeed leads to Dirac cones around $K, K'$, i.e. for

$$k = K + \kappa, \quad k = K' + \kappa$$

(4)

with $|\kappa| \ll |K|, |K'|$. Here $K = \frac{2\pi}{3} (1, 1/\sqrt{3})$ and $K' = \frac{2\pi}{3} (1, -1/\sqrt{3})$.

Problem 14.
Find the transformation properties of the Hamiltonian

$$\hat{H} = \sum_k \hat{c}_k^\dagger h(k) \hat{c}_k$$

(5)

with

$$h(k) = \sin(k_x) \sigma_x + \sin(k_y) \sigma_y + m \sigma_z$$

(6)

under TR.