Quantum Hall physics and topological states of matter - an introduction
Prof. M. Fleischhauer WS 2013

3. Sheet Tutorial: 29.11.2013, 08:15 am

Problem 8.
Consider a single particle Hamiltonian in 1D,

$$\hat{\mathcal{H}} = \sum_k \epsilon(k)\hat{c}_k^\dagger \hat{c}_k, \tag{1}$$

with

$$\frac{d\epsilon(k)}{dk} > 0. \tag{2}$$

Show that there is no elastic back-scattering from an additional potential $V(x)$. Calculate the T-matrix for $\epsilon(k) = ck$ and $V(x) = V_0\delta(x)$.

Problem 9.
Consider a lattice model for spinless particles with a TR symmetric Hamiltonian. Show that the Berry curvature is anti-symmetric in $\boldsymbol{k}$,

$$F_{ij}(-\boldsymbol{k}) = -F_{ij}(\boldsymbol{k}). \tag{3}$$

What is the consequence for the Chern number?

Problem 10.
Show that the Hall equations in $2+1$ dimensions,

$$j_i = \sigma_\mathrm{H}\epsilon_{ij}E_j, \qquad i,j = 1,2 \tag{4}$$

$$j_0 = \dot{\rho} = -\sigma_\mathrm{H}\left(\partial_1 E_2 - \partial_2 E_1\right) \tag{5}$$

are the Euler-Lagrange equations of the so-called Chern Simons Lagrangian,

$$\mathcal{L} = \frac{\sigma_\mathrm{H}}{2}\int d^2x\ A_\mu \epsilon_{\mu\nu\tau}\partial_\nu A_\tau, \tag{6}$$

where $\mu, \nu, \tau = 0, 1, 2$ are the covariant indices in $2+1$ dimensions and $\epsilon_{\mu,\nu\tau}$, $\epsilon_{i,j}$ are the corresponding Levi-cevita symbols.