Problem 4. Two level system
In the class the Berry curvature of two level system with
\[ \hat{H} = R \cdot \hat{\sigma} \]  
was calculated via the wave functions. Repeat the same calculation using the Hamiltonian approach (2.45).

Problem 5. Spin in a magnetic field
Consider a spin-$S$ particle in a homogeneous magnetic field,
\[ \hat{H} = B \cdot S. \]  
Calculate the Berry curvature (Berry field) $V_n$ and the Berry phase when the magnetic field is slowly rotated around a loop $L$. What is the difference between half-integer (fermions) and integer spin particles (bosons)?

Problem 6.
Consider a 2D electron gas in a constant magnetic field perpendicular to the 2D plane. Consider a bulk system, i.e. with no boundaries. Can you find a continuous smooth gauge (Berry connection)?

Problem 7. Anisotropic Harper model
Consider a square lattice with anisotropic hoppings in a homogeneous magnetic field,
\[ \hat{H} = -t_x \sum_{m,n} \left( \hat{c}_{m+1,n}^\dagger \hat{c}_{m,n} + \text{h.c.} \right) - t_y \sum_{m,n} \left( \hat{c}_{m,n+1}^\dagger \hat{c}_{m,n} e^{i2\pi mp/q} + \text{h.c.} \right) \]  

(a) What is the Harper equation for this system?
(b) Consider the limit $t_y \to 0$ and solve for the spectrum. Find general conditions for level crossings.