Quantum Hall physics and topological states of matter - an introduction
Prof. M. Fleischhauer **WS 2013**

2. Sheet Tutorial: 15.11.2013, 08:15 am

Problem 4. Two level system
In the class the Berry curvature of two level system with

$$\hat{\mathcal{H}} = \boldsymbol{R} \cdot \hat{\boldsymbol{\sigma}} \tag{1}$$

was calculated via the wave functions. Repeat the same calculation using the Hamiltonian approach (2.45).

Problem 5. Spin in a magnetic field
Consider a spin-S particle in a homogeneous magnetic field,

$$\hat{\mathcal{H}} = \boldsymbol{B} \cdot \boldsymbol{S}. \tag{2}$$

Calculate the Berry curvature (Berry field) $\boldsymbol{V}_n$ and the Berry phase when the magnetic field is slowly rotated around a loop $\mathcal{L}$. What is the difference between half-integer (fermions) and integer spin particles (bosons)?

Problem 6.
Consider a 2D electron gas in a constant magnetic field perpendicular to the 2D plane. Consider a bulk system, i.e. with no boundaries. Can you find a continuous smooth gauge (Berry connection)?

Problem 7. Anisotropic Harper model
Consider a square lattice with anisotropic hoppings in a homogeneous magnetic field,

$$\hat{\mathcal{H}} = -t_x \sum_{m,n} \left(\hat{c}^\dagger_{m+1,n} \hat{c}_{m,n} + \text{h.c.} \right) - t_y \sum_{m,n} \left(\hat{c}^\dagger_{m,n+1} \hat{c}_{m,n} e^{i2\pi mp/q} + \text{h.c.} \right) \tag{3}$$

(a) What is the Harper equation for this system?

(b) Consider the limit $t_y \to 0$ and solve for the spectrum. Find general conditions for level crossings.