Quantum Hall physics and topological states of matter - an introduction
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1. Sheet Tutorial: 08.11.2013, 08:15 am

Problem 1.
Making use of the complerteness relation of the eigenfunctions in the symmetric gauge,

$$\sum_{m,n} \psi_{n,m}(\boldsymbol{r}')\psi^*_{n,m}(\boldsymbol{r}) = \delta^{(2)}(\boldsymbol{r}-\boldsymbol{r}') \tag{1}$$

where n and m are the eigenvalues corresponding to the Landau level oscillator $(\hat{a}^\dagger\hat{a})$ and the angular momentum $(\hat{b}^\dagger\hat{b})$, show the LLL projection of the δ-function is

$$\delta^{(2)}_p(\boldsymbol{r}-\boldsymbol{r}') = \frac{1}{2\pi}\exp\left[\frac{1}{2}zz' - \frac{1}{4}\left(|z|^2+|z'|^2\right)\right]. \tag{2}$$

Problem 2.
Show that the Hamiltonian (1.73)

$$\hat{\mathcal{H}} = -t\sum_{i,j}\left(e^{-2\pi i\alpha j}\hat{a}^\dagger_{i+1,j}\hat{a}_{i,j} + \text{hc}\right) - t\sum_{i,j}\left(\hat{a}^\dagger_{i,j+1}\hat{a}_{i,j} + \text{hc}\right) + 4t\sum_{i,j}\hat{a}^\dagger_{i,j}\hat{a}_{i,j} \tag{3}$$

in the limit $\Delta x \to 0$ reproduces

$$\hat{\mathcal{H}} = \int d^2\boldsymbol{r}\;\hat{\psi}^\dagger(\boldsymbol{r})\frac{1}{2m_0}\left(\frac{\hbar}{i}\boldsymbol{\nabla} + \frac{e}{c}\boldsymbol{A}(\boldsymbol{r})\right)^2\psi(\boldsymbol{r}) \tag{4}$$

in the Landau gauge. Calculate the spectrum of (3) for the case $\alpha = 0$, i.e. without magnetic field.

Problem 3.
Consider a quantum Hall system with parabolic confinement $(\boldsymbol{A}(\boldsymbol{r}) = \frac{B}{2}(-y,x,0))$,

$$\hat{\mathcal{H}} = \frac{1}{2m_0}\left(\boldsymbol{p} + \frac{e}{c}\right)^2 + \frac{1}{2}m_0\omega_0^2\left(x^2+y^2\right). \tag{5}$$

Show that (5) can be written as

$$\hat{\mathcal{H}} = \frac{\hbar\Omega}{2}\left(-\boldsymbol{\nabla}^2 + \frac{r^2}{4} + \hat{L}_z\right) - \frac{\Omega-\omega_\mathrm{c}}{2}\hat{L}_z, \tag{6}$$

where $\Omega^2 = \omega_\mathrm{c}^2 + 4\omega_0^2$. ω_c is the cyclotron frequency and $\hat{L}_z$ the angular momentum operator in z-direction. What happens in the absence of the parabolic confinement $\omega_0 \to 0$? From this consideration determine the spectrum and the eigenfunctions of (5).