Irreversible Photon Transfer in an Ensemble of Λ-Type Atoms and a Photon Diode

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(Received 3 July 2009; published 15 October 2009)

We show that a pair of quantized modes interacting with a spectrally broadened ensemble of Λ-type atoms is analogous to an ensemble of two-level systems coupled to a bosonic reservoir. This enables an irreversible photon transfer between photon modes. The reservoir can be engineered which allows the observation of effects such as the Zeno and anti-Zeno effect, the destructive interference of decay channels, and the decay in a squeezed vacuum. We also consider a photon diode, i.e., a device which directs a single photon from any one of two input ports to a common output port.

DOI: 10.1103/PhysRevLett.103.163603 PACS numbers: 42.50.Dv, 03.65.Xp, 03.65.Yz, 42.25.Kb

In the present Letter we propose a photonic analogue of the irreversible decay of an ensemble of two-level systems coupled to a bosonic reservoir [1]. In particular, we consider a pair of quantized cavity modes interacting with a spectrally broadened ensemble of Λ-type atoms. The two cavity modes replace the collective states of the ensemble of two-level systems and the Λ-type atoms form the modes of the reservoir. In contrast, e.g., to the radiation field as a reservoir, the atomic ensemble can be easily modified and controlled dynamically, which can be used for reservoir engineering [2]. E.g., the density of states of the reservoir can be tailored by application of magnetic or electric fields and thus it should be possible to implement, e.g., the quantum Zeno [3] and anti-Zeno [4–6] effects, which are otherwise difficult to realize. Moreover the reservoir of Λ atoms can be prepared in different initial states. E.g., coherent ensemble states can be created by using electromagnetically induced transparency [7] or methods of adiabatic population transfer [8]. Also nonclassical states can be prepared [9–12] which can be used to simulate a squeezed-vacuum reservoir [13–15]. If the atomic ensemble is prepared in only one internal state, serving as the vacuum of reservoir excitations, the analogue of spontaneous decay can be observed, where the photons of one cavity mode are transferred irreversibly, i.e., nonunitarily, to the second mode. This effect can have a variety of applications: e.g., the creation of new quantum states, transfer of photons of optical frequency to the microwave domain and vice versa, or a photon diode, i.e., a device where a single photon injected into anyone of two inputs ports leads to a single-photon emission from the same output port. The system considered here can easily be constructed with current technology and is available in several labs. E.g., strong coupling of a cavity mode with a Bose-Einstein condensate of atoms [16–18] or a cold atomic cloud [19,20] was achieved. We emphasize that as the two modes one can use orthogonal polarizations of the same frequency and the required spectral broadening of the atomic ensemble can be achieved, e.g., by application of an inhomogeneous magnetic field.

Let us consider the interaction of two light modes described by the annihilation operators $\hat{a}_1$ and $\hat{a}_2$ with an ensemble of three-level Λ atoms (see Fig. 1). $\hat{a}_1$ and $\hat{a}_2$ couple the ground state $|g\rangle$, respectively, a metastable lower state $|s\rangle$ to a common excited state $|e\rangle$ in a Raman transition. The two-photon transition between $|g\rangle$ and $|s\rangle$ shall be inhomogeneously broadened, as indicated in Fig. 1. For simplicity we assume a discrete spectrum consisting of f energy levels. In each spectral class there are N atoms, so that the total number of atoms is fN.

To describe the quantum properties of the medium, we use collective atomic operators for each spectral compo-

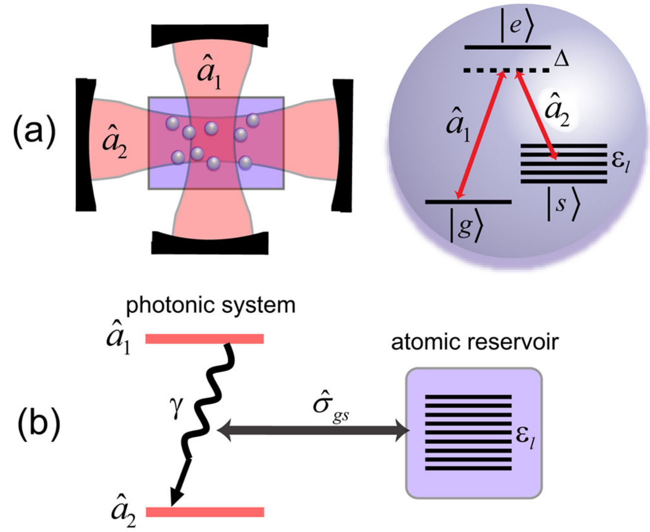

FIG. 1 (color online). (a) Schematic setup: Two quantized cavity modes $\hat{a}_1$ and $\hat{a}_2$ interact with an ensemble of three-level Λ type atoms with inhomogeneously broadened two-photon transition $|g\rangle - |s\rangle$. (b) For a large number of three-level atoms and a sufficiently large spectral width of the Raman transition the system resembles a collection of two-level systems coupled to a bosonic reservoir.

nent $\hat{\sigma}^l_{ij} = \frac{1}{\sqrt{N}}\sum_{k=1}^N |i\rangle^{ll}_{kk}\langle j|$, where i, $j \in e$, g, s, and k labels the atom. The dynamics of the system is then governed by the Hamiltonian

$$\hat{H} = \hbar\sqrt{N}\sum_{l=1}^{f}[\Delta\hat{\sigma}^l_{ee} + \varepsilon_l\hat{\sigma}^l_{ss} + (g_1\hat{\sigma}^l_{eg}\hat{a}_1 + g_2\hat{\sigma}^l_{es}\hat{a}_2 + \mathrm{H.c.})], \tag{1}$$

where Δ is the one-photon detuning of $\hat{a}_1$, and ε_l is the two-photon detuning of the lth spectral class. $g_{1,2}$ are the coupling strength of both modes, which are assumed to be the same for all spectral components. The factor $\sqrt{N}$ is due to the collective coupling of the atoms of each spectral class to the cavity modes [21,22]. If the vacuum Rabi frequencies $g_{1,2}$ and the two-photon detunings ε_l are significantly smaller than Δ, we can adiabatically eliminate the upper level $|e\rangle$. Let us further assume that the atomic ensemble is prepared in a state which is close to the collective ground state and that the total number of photons is much less than the total number of atoms. Then the population of $|s\rangle$ remains small and the atomic operators $\hat{\sigma}^l_{gs}$ and $\hat{\sigma}^l_{sg}$ obey approximately bosonic commutation relations $[\hat{\sigma}^l_{sg}, \hat{\sigma}^p_{sg}] \approx 0$, $[\hat{\sigma}^l_{gs}, \hat{\sigma}^p_{sg}] \approx \delta_{l,p}$; i.e., we can set $\hat{\sigma}^l_{gs} = \hat{\beta}_l$, $\hat{\sigma}^l_{sg} = \hat{\beta}^\dagger_l$. Thus passing to an interaction picture we arrive at an effective Hamiltonian

$$\hat{H}_{\mathrm{eff}} = -\hbar\sqrt{N}\sum_{l=1}^{f}\{\eta\hat{\beta}_l\hat{a}^\dagger_1\hat{a}_2 e^{i\omega_l t} + \mathrm{H.c.}\}, \tag{2}$$

where $\eta = (g_1)^* g_2/\Delta$ and $\omega_l = \varepsilon_l + |g_1|^2/\Delta$ is an effective detuning containing ac-Stark shift contributions.

$\hat{H}_{\mathrm{eff}}$ is similar to the Hamiltonian that describes the interaction of an ensemble of two-level systems consisting of states $|1\rangle$ and $|2\rangle$ with a reservoir of bosonic modes $\hat{\beta}_l$. $\hat{a}^\dagger_1\hat{a}_2$ destroys an "atom" in state $|2\rangle$ and creates an "atom" in state $|1\rangle$.

The dynamics of a two-level system that interacts with a reservoir becomes irreversible when the number of modes of the bath tends to infinity. A well-known consequence of this is spontaneous decay and a similar dynamics can be obtained here for photons. To see this we derive an effective equation of motion for the photon modes by tracing out the atomic degrees of freedom using second order perturbation theory. This yields for the density operator $\hat{\rho}$ of the photon modes $\dot{\hat{\rho}}(t) = -\int_0^t d\tau g(t-\tau)\times[\hat{a}^\dagger_1(t)\hat{a}_2(t)\hat{a}_1(\tau)\hat{a}^\dagger_2(\tau)\hat{\rho}(\tau) - \hat{a}_1(\tau)\hat{a}^\dagger_2(\tau)\hat{\rho}(\tau)\hat{a}^\dagger_1(t)\hat{a}_2(t)] + \mathrm{H.c.}$, where $g(t) = N\sum_{l=1}^f |\eta|^2 e^{i\omega_l t}$. The behavior of the system depends on the reservoir response function $g(t)$ which is determined by the effective light-field coupling constants η and the resonance frequencies ω_l of the atomic ensemble [23]. We assume f to be large enough so that recurrence can be disregarded. For simplicity we also assume that the ensemble has equidistant spectral lines with $\omega_l = \varepsilon_{\max}(2l-1-f)/f$, where $2\varepsilon_{\max}$ is the spectral width of the atomic ensemble. Under these conditions, $g(t)$ can be approximated as a delta function $g(t) \approx \pi(Nf/\varepsilon_{\max})\delta(t)|\eta|^2$, which constitutes the Born-Markov approximation. Note that we assumed that the spectrum of the reservoir is symmetric and thus the imaginary part of $g(t)$ vanishes. In this limit we obtain

$$\frac{\partial\hat{\rho}}{\partial t} = -\frac{\gamma}{2}(\hat{a}^\dagger_1\hat{a}_1\hat{a}_2\hat{a}^\dagger_2\hat{\rho} + \hat{\rho}\hat{a}^\dagger_1\hat{a}_1\hat{a}_2\hat{a}^\dagger_2 - 2\hat{a}_1\hat{a}^\dagger_2\hat{\rho}\hat{a}^\dagger_1\hat{a}_2), \tag{3}$$

with $\gamma = \pi Nf|\eta|^2/(\varepsilon_{\max})$. Note furthermore that similar results could be obtained using a continuous unstructured reservoir spectrum.

The Lindblad equation (3) describes the irreversible transfer of excitations from mode 1 to 2 with rate γ. If there are many photons in the system the decay will be enhanced due to stimulated Raman emission into mode 2. The atomic analogue of this process is the collective decay of atoms (superfluorescence) [22].

(i) Irreversible photon transfer.—Let us first consider the irreversible transfer of photons from mode 1 to mode 2. We assume an initially mixed state of both modes $\hat{\rho}_0 = \sum_{m,m',n,n'} A^{n,n'}_{m,m'}|n\rangle_{11}\langle n'| \otimes |m\rangle_{22}\langle m'|$. In this case the final state reads $\hat{\rho}_{\mathrm{fin}} = \sum_{p,p'} B_{p,p'}|0\rangle_{11}\langle 0| \otimes |p\rangle_{22}\langle p'|$ where $B_{p,p'} = \sum_{q=0} A^{q,q}_{p-q,p'-q}$. This state is a pure state $|\phi\rangle_2 = \sum_m \beta_m|m\rangle_2$ if $B_{p,p'} = \beta^*_p\beta_{p'}$. This implies that all initial states with $A^{m,m'}_{n,n'} = \beta_{m+n}\beta^*_{m'+n}\alpha_n\delta_{n,n'}$, where α_n is an arbitrary function of n, evolve into a pure state. E.g., assume that is completely mixed and contains only one photon $\hat{\rho}_0 = \frac{1}{2}|1\rangle_{11}\langle 1| \otimes |0\rangle_{22}\langle 0| + \frac{1}{2}|0\rangle_{11}\langle 0| \otimes |1\rangle_{22}\langle 1|$. Then the final state will be a pure single-photon state $\hat{\rho}_{\mathrm{fin}} = |0\rangle_{11}\langle 0| \otimes |1\rangle_{22}\langle 1|$. I.e., an initially mixed states of light can be purified with *conservation* of photon number. Another related example is the case when initially we have $\hat{\rho}_0 = |n\rangle_{11}\langle n| \otimes |m\rangle_{22}\langle m|$. The final state is again a pure Fock state $\hat{\rho}_{\mathrm{fin}} = |0\rangle_{11}\langle 0| \otimes |n+m\rangle_{22}\langle n+m|$, that contains the sum of initial photon numbers. The latter process can be realized also by unitary operations, but for that it is necessary to know the exact number of photons in each mode [24,25].

(ii) Reservoir engineering.—As originally formulated by Mishra and Sudarshan a decaying quantum system that is continuously observed in a specific state does not decay, which they called the quantum Zeno effect [3]. In practice a continuous observation is approximated by a periodic sequence of measurements. In order to observe the decay suppression the period of the repeated projections has to be shorter than a characteristic time determined by the spectral structure of the reservoir coupling. The latter makes the observation of the effect in the decay of a two-level atom to the free-space electromagnetic vacuum rather difficult. In this case the spectrum of the reservoir coupling is flat and the frequency of measurements has to be comparable to the transition frequency in order to see the quantum Zeno

suppression of decay. If the reservoir spectrum is structured also the opposite effect, called anti-Zeno effect is possible [4,6]. The periodic interaction between system and measurement device shifts the effective resonance frequency of the system and moves it to a different part of the reservoir spectrum. This can increase (decrease) the system-reservoir coupling, and thus lead to an increase (decrease) of the decay rate. Similar effects can be observed by keeping the resonant frequency of the system unperturbed but shifting the spectrum of the reservoir. The latter can be realized by a measurement of the reservoir [5,6]. In our system the reservoir is atomic; thus, one can easily tailor the reservoir spectrum and measure it by application of an electromagnetic field. Depending on the reservoir response function $f(t)$ the measurement will either accelerate (anti-Zeno effect) or slow down (Zeno effect) the spontaneous photon transfer from one mode into another.

Another potential application of reservoir engineering is the possibility to prepare the reservoir in a certain quantum state, e.g., in multimode squeezed states [9–12]. With this it should be possible to study the decay in a squeezed vacuum [13–15]. The latter has been proposed and analyzed theoretically for atoms coupled to a squeezed reservoir of radiation modes. An experimental verification is, however, extremely difficult due to the requirement of a broad-band and isotropic squeezed-vacuum radiation field.

(iii) Photon diode.—In the last part of this Letter we discuss an interesting application of the irreversible photon transfer, a diode for photons, i.e., a four-port device where single-photon pulses injected into any of the two input ports will be directed to the same output port. To model the input-output processes we introduce a continuum of free-space modes with field operators $\hat{b}_{1q}$ and $\hat{b}_{2q}$ which are coupled to the cavity modes $\hat{a}_1$ and $\hat{a}_2$, respectively. For simplicity we assume that the coupling constants κ_1 and κ_2 are the same for all relevant modes. The corresponding interaction is described by the following Hamiltonians ($m = 1, 2$)

$$\hat{V}_m = \hbar\kappa_m \sum_q (\hat{a}_m^\dagger \hat{b}_{mq} + \text{H.c.}) + \sum_q \hbar\Delta_m^q \hat{b}_{mq}^\dagger \hat{b}_{mq}.$$

Here Δ_m^q are the detunings of free-field modes from the cavity resonance. Let us consider input fields in a single-photon state $|\psi_{\text{in}}\rangle_m = \sum_q P_q^{\text{in}}(t)\hat{b}_{mq}^\dagger|0\rangle$. All properties of the fields are then described by the single-photon wave function $\Phi_{\text{in}}^m(z,t) = \sum_q \langle 0|\hat{b}_{mq}e^{iqz}|\psi_{\text{in}}\rangle_m$. Since all atoms of the atomic ensemble are initially in the ground state, the field in state $|\psi_{\text{in}}\rangle_2$ sees an empty cavity and will be reflected from it with some time delay. In the following we want to prove that $|\psi_{\text{in}}\rangle_1$ is transferred to $|\psi_{\text{out}}\rangle_2$, i.e., to a state where the excitation is in the orthogonal output channel. In general the state of the system can be written in the following form

$$|\psi(t)\rangle = \Big(\sum_q P_q(t)\hat{b}_{1q}^\dagger + Q(t)\hat{a}_1^\dagger + \sum_{l=1}^{f} R_l(t)\hat{a}_2^\dagger\hat{\beta}_l^\dagger + \sum_{l=1}^{f}\sum_q S_{ql}(t)\hat{b}_{2q}^\dagger\hat{\beta}_l^\dagger\Big)|\mathbf{gs}\rangle,$$

with $|\mathbf{gs}\rangle$ denoting the ground state of the atomic ensemble, where all atoms are in state $|g\rangle$. Since initially only mode 1 is excited and all atoms are in the ground state $Q(t) = R_l(t) = S_{ql}(t) = 0$ if $t < 0$, where $t = 0$ is the beginning of interaction. The evolution of the system is described by the Schrödinger equation

$$\dot{P}_q(t) = -i\Delta_1^q P_q(t) - i\kappa_1 Q(t), \tag{4}$$

$$\dot{Q}(t) = -i\kappa_1 \sum_q P_q(t) + i\eta\sqrt{N}\sum_{l=1}^{f} e^{i\omega_l t}R_l(t), \tag{5}$$

$$\dot{R}_l(t) = i\eta^*\sqrt{N}e^{-i\omega_l t}Q(t) - i\kappa_2\sum_q S_{ql}(t), \tag{6}$$

$$\dot{S}_{ql}(t) = -i\Delta_2^q S_{ql}(t) - i\kappa_2 R_l(t). \tag{7}$$

Substituting the formal solution of Eq. (7) into (6) and assuming the Markov limit yields

$$\dot{R}_l(t) = i\eta^*\sqrt{N}e^{-i\omega_l t}Q(t) - \frac{\gamma_2}{2}R_l(t), \tag{8}$$

where $\gamma_2 = \kappa_2^2 L/c$ is the cavity loss rate of mode 2 and L is the quantization length of $\hat{b}$ modes. (Note that $\kappa_m \sim 1/\sqrt{L}$, so that the dependence on L drops.) Furthermore substituting the formal solutions of Eqs. (4) and (8) into (5) we find again using the Markov approximation

$$\dot{Q}(t) = -\frac{(\gamma + \gamma_1)}{2}Q(t) - i\kappa_1\sum_q P_q(0)e^{-i\Delta_1^q t}, \tag{9}$$

where we have used the photon decay rate γ and introduced the cavity loss rate of mode 1, $\gamma_1 = \kappa_1^2 L/c$. Upon integrating Eq. (9) we finally find the input-output relation for port 1, i.e., for the modes $\hat{b}_1^q$.

$$\Phi_{\text{out}}^1(t) = \Phi_{\text{in}}^1(t) - \gamma_1 F(t), \tag{10}$$

where $F(t) = \int_0^t d\tau \Phi_{\text{in}}^1(\tau)e^{-((\gamma+\gamma_1)/2)(t-\tau)}$. In order to achieve a maximum transfer of free-field photons into the cavity, the outgoing component should be minimized. According to Eq. (10) this can be realized by requiring impedance matching. If $\gamma_1 = \gamma$ and the pulse is much longer than the relaxation rates, the two terms on the right-hand side of Eq. (10) cancel each other and there is no output into the modes $\hat{b}_{1q}$.

Because of the dissipative nature of the coupling between the two cavity modes the output field will be in general in a mixed state when tracing out the degrees of freedom of the atomic ensemble. Only if the input is an eigenstate of the total excitation number, the cavity output can be in a pure state. We will show now that in this case the final state indeed factorizes into a single photon distributed over many modes $\hat{b}_{2q}$ and a single collective excitations of the atoms. Following similar steps as above we find for the output wave function for the modes $\hat{b}_{2q}$ when the atomic excitation is in mode l, $\Phi_{\rm out}^{2,l} \equiv \sum_q \langle 0|\langle 1_l|\hat{b}_{2q}|\psi\rangle_{\rm out}$ where $|1_l\rangle$ denotes a single excitation in the lth spectral class of the atomic ensemble, reads

$$\Phi_{\rm out}^{2,l}(t) = -i\frac{\kappa_1}{\kappa_2}\gamma_2\eta^*\sqrt{N}\int_0^t d\tau_1 e^{-i\omega_l\tau_1}e^{-(\gamma_2/2)(t-\tau_1)}F(\tau_1).$$

Thus the probability of having a photon in output port 2, obtained by tracing out the atomic part, reads

$$\rho_{\rm out}(t) = \sum_l |\Phi_{\rm out}^{2,l}|^2 = \gamma_1\gamma_2\gamma\int_0^t d\tau_1 e^{-\gamma_2(t-\tau_1)}|F(\tau_1)|^2.$$

If the cavity decay rate of mode 2, γ_2, is much larger than γ and γ_1, the integral over τ_1 can easily be evaluated. In this case $\rho_{\rm out}$ can be expressed as a product of two single-photon wave packets $\rho_{\rm out}(t) = |\Phi_{\rm out}^2(t)|^2$, where

$$\Phi_{\rm out}^2(t) = \sqrt{\gamma_1\gamma}F(t).$$

Thus indeed a single-photon input wave packet results into an asymptotic final state which is a product of a collective atomic excitation and a single-photon wave packet in the output mode 2. It should be noted that since the physical mechanism that leads to the diode function is irreversible, input superposition states will in general not be mapped to pure output states. In conclusion we have shown that a two-mode system interacting with a spectrally broadened ensemble of Λ-type atoms behaves as a collection of two-level systems interacting with a bosonic reservoir. The analogy between these two systems allows the observation and simulations of several interesting phenomena of dissipative processes in engineered reservoirs. In particular we have shown that similarly to spontaneous decay of atoms one can irreversibly transfer photons from one mode to another. The possibility to tailor the reservoir spectrum and to prepare collective quantum states of the ensemble of Λ atoms can be used to observe the quantum Zeno- and anti-Zeno effects, to study the decay in a squeezed reservoir and to observe destructive interference of spontaneous decay channels. As a particular application we have discussed in more detail a photon diode, i.e., a device where a single-photon input in any of the two input ports is always emitted in only one of the two outputs. Besides being interesting in its own right the diode may be used for the implementation of a classical logical OR.

G. N. thanks Jyrki Piilo for helpful discussion. The work of G. N. was supported by Alexander von Humboldt Foundation.

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