Confining Stationary Light: Dirac Dynamics and Klein Tunneling

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We discuss the properties of 1D stationary pulses of light in an atomic ensemble with electromagnetically induced transparency in the limit of tight spatial confinement. When the size of the wave packet becomes comparable or smaller than the absorption length of the medium, it must be described by a two-component vector which obeys the one-dimensional two-component Dirac equation with an effective mass m^* and effective speed of light c^*. Then a fundamental lower limit to the spatial width in an external potential arises from Klein tunneling and is given by the effective Compton length $\lambda_C = \hbar/(m^* c^*)$. Since c^* and m^* can be externally controlled and can be made small, it is possible to observe effects of the relativistic dispersion for rather low energies or correspondingly on macroscopic length scales.

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When photons are confined to a volume smaller than the wavelength cubed, their interaction with atoms is dominated by quantum effects. This principle has been exploited in cavity quantum electrodynamics, where the light is confined by means of low-loss microresonators [1]. The tight confinement results in a strong coupling which can be used, e.g., to build quantum gates between photonic qubits. Yet with a decreasing resonator volume it becomes more and more difficult to maintain high Q values. However, as shown by Bajcsy *et al.* and André and Luken [2,3], it is possible to create spatially confined quasistationary pulses of light with very low losses without the need of a resonator by means of electromagnetically induced transparency (EIT) [4,5] with counterpropagating control fields. For a weak confinement in the longitudinal direction, stationary light is well described by a Schrödinger-type equation with complex mass for a normal mode of the system, the stationary dark-state polariton [6–8]. We here show that this is no longer the case for stronger spatial confinement, i.e., when the characteristic length of the weak light pulse becomes comparable or smaller than the absorption length of the medium. Here a description in terms of a two-component vector obeying a one-dimensional Dirac equation becomes necessary. The two characteristic parameters of this equation, the effective mass m^* and the effective speed of light c^*, depend on the strength of the EIT control fields and can be made many orders of magnitude smaller than the vacuum speed of light c and, respectively, the mass of the atoms forming the EIT medium. As a consequence, effects of the relativistic dispersion can arise already at rather low energy scales. On one hand this leads to a fundamental lower limit for the spatial confinement of stationary light in an external potential, as generated, e.g., by specially tailoring the Raman couplings. This limit is given by the effective Compton length $\lambda_C = \hbar/(m^* c^*)$ which due to the smallness of $m^* c^*$ can become large. On the other hand it opens the possibility to study relativistic effects such as Klein tunneling [9] and *Zitterbewegung*, which regained a lot of interest recently in connection with electronic properties of graphene [10] and ultracold atoms in light- or rotation-induced gauge potentials [11,12].

We consider here an ensemble of atoms or other quantum oscillators with a double-Λ structure of dipole transitions as shown in Fig. 1. The ground state $|g\rangle$ and the metastable state $|s\rangle$ are coupled via Raman transitions through the excited states $|e_\pm\rangle$ by two counterpropagating control lasers of opposite circular polarization and Rabi frequencies $\Omega_\pm$ and two counterpropagating probe fields $E_\pm$ again of opposite circular polarization. Both Λ schemes are assumed to be in two-photon resonance with the ground state transition, which guarantees EIT. Furthermore, the control fields are taken homogeneous, constant in time and of equal strength $\Omega_+ = \Omega_+^* = \Omega_- = \Omega_-^*$. As shown in [7] the control fields generate a quasistationary pulse of light. For this discussion we restrict ourselves to a one-dimensional dynamical model. In paraxial approximation the transverse dynamics is that of a Schrödinger particle with the transverse mass being much

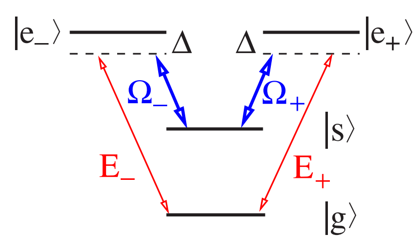

FIG. 1 (color online). Stationary light scheme: The interaction of double Λ atoms driven by two counterpropagating control fields of (equal) Rabi frequency Ω and opposite circular polarization with two counterpropagating probe fields $E_\pm$ of corresponding polarization generates a quasistationary pattern of the probe fields.

larger than the longitudinal one [13]. Thus the transverse dynamics occurs on a much longer time scale than the longitudinal one, which justifies the 1D approximation.

We introduce normalized field amplitudes that vary slowly in space and time, $E_\pm(\mathbf{r}, t) = \sqrt{\hbar\omega/2\varepsilon_0}[\mathcal{E}_\pm(\mathbf{r}, t) \times \exp\{-i(\omega t \mp kz)\} + \text{H.a.}]$, and continuous atomic-flip operators $\hat{\sigma}_{\mu\nu}(\mathbf{r}, t) = \frac{1}{\Delta N}\sum_{j\in\Delta V(\mathbf{r})}\hat{\sigma}^j_{\mu\nu}$, with $\hat{\sigma}^j_{\mu\nu} \equiv |\mu\rangle_{jj}\langle\nu|$ being the flip operator of the jth atom. The sum is over all ΔN atoms in a small volume ΔV around position $\mathbf{r}$. Then the dynamical equations read in the linear response limit, i.e., for a low probe light intensity,

$$\frac{\partial}{\partial t}\hat{\sigma}_{gs} = i\Omega_+\hat{\sigma}_{ge_+} + i\Omega_-\hat{\sigma}_{ge_-}, \tag{1}$$

$$\frac{\partial}{\partial t}\hat{\sigma}_{ge_\pm} = -(\gamma + i\Delta)\hat{\sigma}_{ge_\pm} + i\Omega_\pm\hat{\sigma}_{gs} + ig\sqrt{n}\mathcal{E}_\pm, \tag{2}$$

$$ig\sqrt{n}\hat{\sigma}_{ge_\pm} = \left[\frac{\partial}{\partial t} \pm c\frac{\partial}{\partial z}\right]\mathcal{E}_\pm. \tag{3}$$

Here we have introduced the common single-photon detuning of the upper states from the control and probe field transitions Δ. n is the atom density and $g = \frac{\wp}{\hbar}\sqrt{\hbar\omega/2\varepsilon_0}$ the common coupling constant of both probe fields with $\wp$ denoting the respective dipole matrix element. γ is the transverse decay rate of the optical dipole transitions $|e_\pm\rangle - |g\rangle$, and we have used that in the linear response limit $\hat{\sigma}_{gg} \approx 1$.

Adiabatically eliminating the atomic variables from the equations of motion leads to the shortened wave equations for the forward ($\mathcal{E}_+$) and backward ($\mathcal{E}_-$) propagating field components

$$\left[\frac{\partial}{\partial t} \pm c\frac{\partial}{\partial z}\right]\mathcal{E}_\pm = -\frac{g^2 n}{\Gamma}\mathcal{E}_\pm + \frac{g^2 n}{2\Gamma}(\mathcal{E}_+ + \mathcal{E}_-) - \tan^2\theta\left(\frac{\partial}{\partial t}\mathcal{E}_+ + \frac{\partial}{\partial t}\mathcal{E}_-\right), \tag{4}$$

where $\Gamma = \gamma + i\Delta$, and $\tan^2\theta = g^2 n/\Omega^2$, with $\Omega^2 = \Omega_+^2 + \Omega_-^2$. If the characteristic length scale of the probe fields is large compared to the absorption length of the medium, $L_{\text{abs}} = \gamma c/(g^2 n)$, it is convenient to introduce sum and difference normal modes $\mathcal{E}_S = (\mathcal{E}_+ + \mathcal{E}_-)/\sqrt{2}$ and $\mathcal{E}_D = (\mathcal{E}_+ - \mathcal{E}_-)/\sqrt{2}$. Expressing the equations of motion in terms of these normal modes and subsequently adiabatically eliminating the fast decaying difference normal mode $\mathcal{E}_D$ leads to a Schrödinger-type equation with a complex effective mass [6,7]:

$$i\hbar\frac{\partial}{\partial t}\mathcal{E}_S = -\frac{\hbar^2}{2m^*}\left(1 + i\frac{\gamma}{\Delta}\right)\frac{\partial^2}{\partial z^2}\mathcal{E}_S. \tag{5}$$

Here m^* denotes the real part of the effective mass

$$m^* = \frac{\hbar}{2L_{\text{abs}}}\frac{1}{c\cos^2\theta}\frac{\gamma}{\Delta} = m\frac{v_{\text{rec}}}{v_{\text{gr}}}\frac{\gamma}{\Delta}\frac{1}{2kL_{\text{abs}}}. \tag{6}$$

As comparative scales we have introduced in the second equation the mass m and the recoil velocity $v_{\text{rec}} = \hbar k/m$ of an atom and the group velocity of EIT $v_{\text{gr}} = c\cos^2\theta$.

On the other hand, if the characteristic length scale of the probe fields becomes comparable to the absorption length nonadiabatic couplings between the sum and difference mode are relevant and the elimination of the difference mode is no longer valid. Instead one has to keep both amplitudes which can be collected in a two-component vector $\tilde{\mathcal{E}} = (\mathcal{E}_+, \mathcal{E}_-)^\top$. The equation of motion can then be written in the compact form

$$\left(\mathsf{A}\frac{\partial}{\partial t} + \mathsf{B}\frac{\partial}{\partial z}\right)\tilde{\mathcal{E}} = \mathsf{C}\tilde{\mathcal{E}}, \tag{7}$$

where $\mathsf{A} = \mathbf{1} + \frac{1}{2}\tan^2\theta(\mathbf{1} + \sigma_x)$, $\mathsf{B} = c\sigma_z$, and $\mathsf{C} = (\sigma_x - \mathbf{1})g^2 n/(2\Gamma)$, σ_x and σ_z being the Pauli matrices. Applying the transformation

$$\underline{\mathcal{E}} = \exp\{\beta\sigma_x\}\tilde{\mathcal{E}}, \tag{8}$$

with $\tanh(2\beta) = (1 - \cos^2\theta)/(1 + \cos^2\theta)$ finally yields for large single-photon detuning $|\Delta| \gg \gamma$, i.e., for a negligible imaginary part of the effective mass:

$$i\hbar\frac{\partial}{\partial t}\underline{\mathcal{E}} = \left(-i\hbar c^*\sigma_z\frac{\partial}{\partial z} + m^* c^{*2}\sigma_x\right)\underline{\mathcal{E}}. \tag{9}$$

Here we have removed an irrelevant constant term by a gauge transformation. Equation (9) has the form of a two-component, one-dimensional, massive Dirac equation. The effective speed of light c^* in Eq. (9)

$$c^* = c\cos\theta = \sqrt{v_{\text{gr}}c}, \tag{10}$$

which in EIT media can be varied over a large range and can be much smaller than the vacuum speed of light. It should be noted that despite the formal equivalence of Eq. (9) to the two-component Dirac equation the fundamental quasiparticle excitations of the light matter interaction are bosons [7] and can, e.g., undergo Bose condensation [13]. Equation (5) is of course recovered from Eq. (9) in the low energy limit of long wavelength excitations.

For particles with a rest mass m the characteristic length scale at which relativistic effects become important is given by the Compton length $\lambda_C = \hbar/mc$. In the present case for the massive light this corresponds to

$$\lambda_C \equiv \frac{\hbar}{m^* c^*} = 2L_{\text{abs}}\frac{\Delta}{\gamma}\cos\theta. \tag{11}$$

While for electrons λ_C is on the order of picometers, for stationary light it can become rather large. It can exceed the absorption length if the EIT group velocity is sufficiently large, i.e., $v_{\text{gr}}/c > \gamma/\Delta$. Since typical values for the optical depth $\text{OD} = L/L_{\text{abs}}$ of EIT media are in the range between a few and a few hundred, λ_C can be a sizable fraction of the medium length L and thus can become macroscopic. Hence it is possible to achieve the corresponding relativistic limit of the massive light in state-of-the-art experiments.

The free evolution of a quasistationary pulse of light is quite different in the two limits $L \gg \lambda_C$, Eq. (5), and $L \lesssim \lambda_C$, Eq. (9), which allows for a simple experimental distinction of the two regimes. This is illustrated in Fig. 2, which shows the dynamics of a wave packet obtained from a numerical solution of the full Maxwell-Bloch equations, which agrees with the dynamics from the effective Schrödinger and Dirac equations. One recognizes in the first case the familiar slow dispersive spreading, while in the second case two split wave packets emerge which move outward with the effective speed of light c^*.

A relativistic wave equation does not permit a confinement of a wave packet to less than the Compton length [14]. For example, a square-well potential

$$U(z) = \begin{cases} -U_0 & |z| \le a \\ 0 & |z| > a, \end{cases} \tag{12}$$

with $a \to 0$ and $U_0 \to \infty$ such that $U_0 a = \text{const}$ has lowest energy eigensolutions with energy

$$E = \pm m^* c^{*2} \cos\left(\frac{U_0}{m^* c^{*2}} \frac{a}{\lambda_C}\right). \tag{13}$$

The corresponding eigensolutions have the form

$$\mathcal{E} = \mathcal{E}^{(\pm)} \exp\left[-\frac{m^* c^* |z|}{\hbar} \left|\sin\left(\frac{U_0}{m^* c^{*2}} \frac{a}{\lambda_c}\right)\right|\right]. \tag{14}$$

Thus the characteristic confinement length L_{conf} reads

$$L_{\text{conf}} = \frac{\lambda_C}{2} \frac{1}{|\sin(\frac{U_0}{m^* c^{*2}} \frac{a}{\lambda_C})|} \ge \frac{\lambda_C}{2}, \tag{15}$$

which is always larger than λ_C. One can show in general that any eigensolution of any (“electrostatic”) confining potential has a minimum size of $\lambda_C/2$. If solutions with an energy exceeding $\mp m^* c^{*2}$ exist they are resonances which have a finite width due to Klein tunneling into the negative (positive) energy continuum [9]. Depending on the form of the confining potential the corresponding decay rate can, however, be small. The effect of Klein tunneling is illustrated for confined stationary light in Fig. 3. Here an initial Gaussian stationary wave packet of initial width $L_p = 0.05\lambda_c$ is considered in a potential well with $a = 0.1\lambda_c$ and potential depth $U_0 = 1.875 m^* c^{*2}$ (top) and $U_0 = 3.125 m^* c^{*2}$ (bottom). Because of the mismatch of the initial wave packet with the bound states of the potential there is some initial loss. After some time the pulse shape remains rather unchanged, however, in the first case while it displays continuing decay in the second due to Klein tunneling. The plots are obtained from a solution of the full 1D Maxwell-Bloch equations with an additional potential

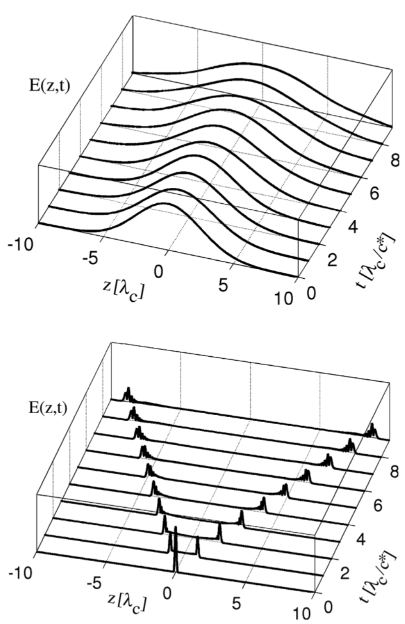

FIG. 2. Top: Diffusive expansion of an initial Gaussian stationary light pulse with width $L_p = 2.5\lambda_C$. Bottom: The same for an initial width of $L_p = 0.05\lambda_C$. Results are obtained from numerical solutions of the full Maxwell-Bloch equations which agree very well with solutions of the Schrödinger equation (5) (top plot) and the Dirac equation (9) (bottom plot). The parameters are $\gamma/\Delta = 0.01$, $\Omega_\pm/\Delta = 0.2$, $\cos\theta = 0.9975$, and hence $\lambda_C/L_{\text{abs}} = 199.5$.

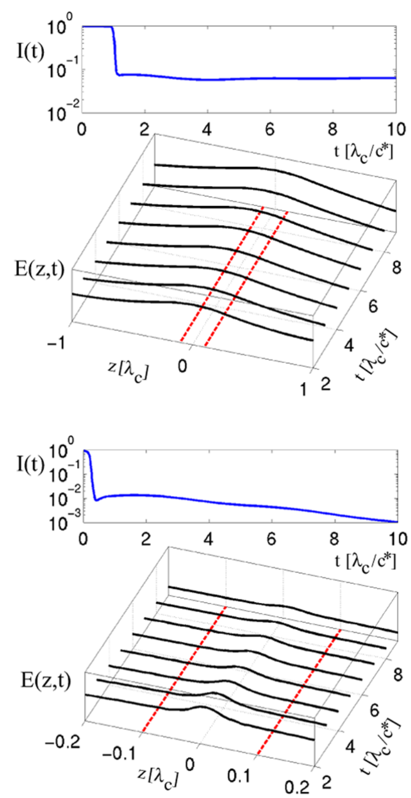

FIG. 3 (color online). Evolution of an initial Gaussian stationary light pulse with initial width $L_p = 0.05\lambda_c$ in a square-well potential (the extension of which is indicated by dashed red lines) with $a = 0.1\lambda_C$ and depth $U_0 = 1.875 m^* c^{*2}$ (top), and $U_0 = 3.125 m^* c^{*2}$ (bottom), respectively. Besides initial losses due to mismatch of the wave functions [see insets at the top, $I(t)$ is the integrated intensity], a fast decay due to Klein tunneling is apparent in the second case.

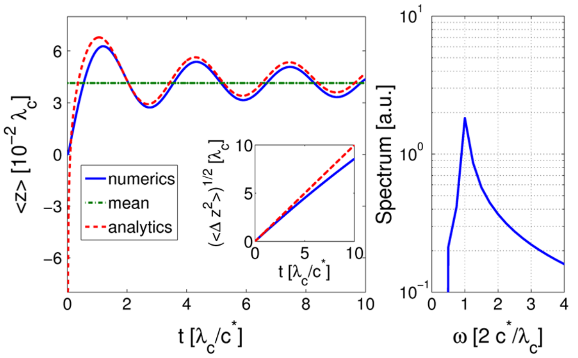

FIG. 4 (color online). Left: Center of mass of emerging left- and right-moving wave packets obtained from solution of Maxwell-Bloch equations of initial Gaussian wave packet of width $\Delta z = 10L_{\rm abs} \ll \lambda_C = 199.98L_{\rm abs}$ (solid blue line) and from solution (17) of the Dirac equation (dashed red line). At $t = 0$ a flip of the relative phase of the drive fields was assumed corresponding to $\mathcal{E}_+(0) = i\mathcal{E}_-(0)$. Parameters are as in Fig. 2. Right: Fourier-spectrum of $\langle z(t)\rangle$ showing the pronounced peak at $2m^*c^{*2}/\hbar$.

generated by a finite, space-dependent two-photon detuning which shows very good agreement with the solutions of the corresponding Dirac equation.

There is another well-known effect of the Dirac dynamics which can be observed in the present system. If we consider an initial stationary Gaussian wave packet and switch the relative phase between the two counterpropagating drive fields instantaneously from 0 to $\pi/2$, the relative motion of the two emerging wave packets is superimposed by an oscillation, known as *Zitterbewegung*. After the $\pi/2$ phase flip the initial wave packet reads in k space

$$\tilde{\mathcal{E}}(k, t = 0) = \frac{1}{\sqrt{\sigma_k}\pi^{1/4}} \exp\left\{-\frac{k^2}{2\sigma_k^2}\right\}\begin{pmatrix}1\\ i\end{pmatrix}. \qquad (16)$$

Then in the large-time limit $t \gg \hbar/(m^*c^{*2})$ one finds for the center of mass of the two wave packets (setting $\gamma/\Delta = 0$)

$$\langle z(t)\rangle = \frac{2\sqrt{\pi}}{\sigma_k}\frac{\cos^2\theta}{1+\cos^2\theta}\left[1 - \left(\pi\frac{m^*c^{*2}}{\hbar}t\right)^{-1/2} \times \cos\left(\frac{2m^*c^{*2}}{\hbar}t + \frac{\pi}{4}\right)\right], \qquad (17)$$

which shows the characteristic oscillation with frequency $2m^*c^{*2}/\hbar$.

Figure 4 shows the center of mass of a pair of left- and right-moving wave packets for an initial Gaussian pulse obtained from a numerical solution of the 1D Maxwell-Bloch equations as well as the analytic result (17) obtained from the Dirac equation. The *Zitterbewegung* can be observed, e.g., by detecting the overlap of the left- and right-moving wave packets after exiting the medium. A typical scale of the amplitude of the *Zitterbewegung* would be $L_{\rm abs}/10 \sim 0.1$ cm, which should be much easier to observe than the values in the range from 1 to 100 nm predicted for cold atoms [11], graphene [15], ions [16], and photonic crystals [17].

In summary we have shown that stationary light in the limit of tight longitudinal spatial confinement must be described by a two-component, one-dimensional Dirac equation with effective mass and effective speed of light that can be controlled externally and that can be much smaller than the corresponding values for atoms and light in vacuum. As a consequence, relativistic effects related to the Dirac dispersion can be observed at rather low energy scales or, respectively, at rather large length scales. One immediate consequence of the latter is the impossibility to spatially compress a stationary light pulse below the Compton length. Moreover, in contrast, e.g., to electrons in graphene, interactions between stationary-light polaritons are very week. Thus stationary light may be employed to observe relativistic phenomena related to the Dirac dynamics in the absence of interactions under experimentally realistic conditions.

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