Stationary Source of Nonclassical or Entangled Atoms

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A scheme for generating continuous beams of atoms in nonclassical or entangled quantum states is proposed and analyzed. For this the recently suggested transfer technique of quantum states from light fields to collective atomic excitation by stimulated Raman adiabatic passage [M. Fleischhauer and M. D. Lukin, Phys. Rev. Lett. **84**, 5094 (2000)] is employed and extended to matter waves.

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Since the experimental realization of Bose-Einstein condensation (BEC) of atoms [1] much effort has been put into the generation of coherent matter waves (atom lasers) [2]. Besides being of fundamental interest, coherent matter waves are anticipated to lead to a substantial increase of interferometer sensitivities as compared to their optical counterparts due to the much smaller wavelength. On the other hand, particle sources have a significantly smaller brightness than lasers and consequently a much higher level of shot noise. By now experimental techniques have advanced to the point that shot-noise limited operation is achievable in the laboratory [3] and quantum fluctuations represent a true sensitivity limit for atom interferometer. Furthermore, recent advances in quantum information science lead to an increased interest in sources of entangled massive particles. Some time ago Kitagawa and Ueda [4] as well as Wineland *et al.* [5] proposed to use atom beams in spin-squeezed states to reduce the noise level in interferometers and atomic clocks. While optical techniques to generate nonclassical light are by now well developed [6], less progress has been made for massive particles. We here propose a novel scheme for generating *continuous beams of atoms in nonclassical quantum states,* extending the recently suggested quantum-state transfer between light and atoms to matter waves [7–9].

Several proposals for generating atomic beams with nonclassical quantum correlations have been put forward and in part experimentally implemented. Hald *et al.* [10] reported about spin squeezing of trapped atoms created by squeezed-light pumping. The transfer was, however, accompanied by spontaneous emission and only a limited amount of spin squeezing could be achieved. A higher degree of noise reduction was obtained by Kuzmich *et al.* [11] by continuous quantum nondemolition measurements on an atomic beam. Recently Pu and Meystre [12] and Duan *et al.* [13] have proposed to make use of collisional interactions in a Bose condensate to create squeezing or entangled pairs of atoms. Conceptually related is the squeezing and entanglement generation by dissociating diatomic molecules [14]. Finally, the generation of number squeezing in BEC in an array of weakly linked traps was reported by Orzel *et al.* [15].

Recently we have proposed a technique to transfer the quantum state of photon wave packets to collective Raman excitations of atoms in a loss-free and reversible manner [7]. First experiments [8,9] have confirmed important aspects of this proposal. It is based on the adiabatic rotation of dark-state polaritons, which are quasiparticles associated with electromagnetically induced transparency (EIT), from a light field to a stationary spin excitation. When an optically thick sample of three-level atoms in Raman configuration is irradiated by a strong coherent Stokes field, the absorption of the pump field is suppressed. (The notion of pump and Stokes is introduced here to distinguish the field coupling to the initially occupied and nonoccupied lower levels in the Raman scheme. No special relation between the frequencies is implied, however.) Associated with this is a substantial reduction of the group velocity of the pump pulse [16] which corresponds to a temporary storage of its quantum state in atomic spins. A complete and persistent transfer can, however, be achieved only by a *dynamical* reduction of the group velocity due to a decrease of the Stokes field intensity in time. Although an application of this technique to generate entangled or squeezed samples of atoms has been proposed in [17], it requires an explicit time dependence and is thus limited to *pulsed* light. For many applications, in particular, subshot noise, matter-wave interferometry, and continuous teleportation, a *stationary* source of nonclassical atoms is desired. We here show that a complete transfer is possible under stationary conditions in a setup where atoms move through a spatially varying Stokes field creating an explicitly time-dependent interaction in their rest frame. In this way a simple and robust cw source of atoms in nonclassical or entangled states can be built.

We consider the one-dimensional model shown in Fig. 1. A beam of Λ-type atoms with two (meta)stable lower levels interacts with a quantum pump and a classical Stokes field. Atoms in different internal states are described by three bosonic fields $\Psi_\mu(z,t)$ ($\mu = 1,2,3$). The Stokes field is characterized by the Rabi-frequency $\Omega(z,t) = \Omega_0(z)e^{-i\omega_s(t-z/c')}$ with Ω_0 taken real, and the quantum pump field by the dimensionless positive frequency component $\hat{E}^{(+)}(z,t) = \mathcal{E}(z,t)e^{-i\omega_p(t-z/c)}$,

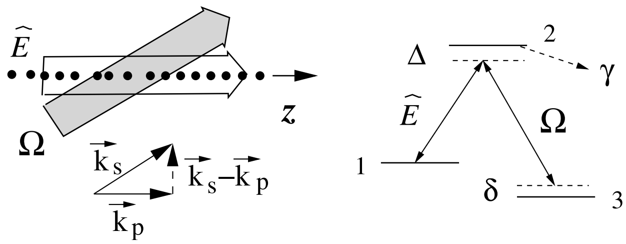

FIG. 1. Beam of three-level Λ-type atoms coupled to a classical field with Rabi-frequency $\Omega(z,t)$ and quantum field $\hat{E}(z,t)$. To minimize effect of Doppler broadening, geometry is chosen such that $(\vec{k}_s - \vec{k}_p)\cdot\vec{e}_z \approx 0$.

where c' denotes the phase velocity projected onto the z axis. The atoms are assumed to enter the interaction region in state $|1\rangle$.

It is convenient to introduce slowly varying amplitudes, and a decomposition into velocity classes $\Psi_1 = \sum_l \Phi_1^l e^{i(k_l z - \omega_l t)}$, $\Psi_2 = \sum_l \Phi_2^l e^{i[(k_l+k_p)z-(\omega_p+\omega_l)t]}$, $\Psi_3 = \sum_l \Phi_3^l e^{i[(k_l+k_p-k_s)z-(\omega_p-\omega_s+\omega_l)t]}$, where $\hbar k_l$ is the momentum of the atoms and $\hbar\omega_l = \hbar^2 k_l^2/2m$ the corresponding kinetic energy in the lth velocity class. k_p and k_s are the wave-vector projection of pump and Stokes to the z axis. The atoms shall have a narrow velocity distribution around $v_0 = \hbar k_0/m$ with $k_0 \gg |k_p - k_s|$. All fields are assumed to be in resonance for the central velocity class. The equations of motion for the matter fields read

$$\left(\frac{\partial}{\partial t} + \frac{\hbar k_l}{2m}\frac{\partial}{\partial z}\right)\Phi_1^l = -ig\mathcal{E}^+\Phi_2^l, \tag{1}$$

$$\left(\frac{\partial}{\partial t} + \frac{\hbar k_l}{2m}\frac{\partial}{\partial z}\right)\Phi_3^l = -i\Omega_0\Phi_2^l - i\delta_l\Phi_3^l, \tag{2}$$

$$\begin{aligned}\left(\frac{\partial}{\partial t} + \frac{\hbar(k_l+k_p)}{2m}\frac{\partial}{\partial z}\right)\Phi_2^l = &-(\gamma + i\Delta_l)\Phi_2^l \\ &- i\Omega_0\Phi_3^l - ig\mathcal{E}\Phi_1^l \\ &+ F_2^l,\end{aligned} \tag{3}$$

where g is the atom-field coupling constant and $\Delta_l \approx \hbar k_l k_p/m + (\omega_{21} - \omega_p)$ and $\delta_l \approx \hbar k_l(k_p - k_s)/m + (\omega_{31} - \omega_p + \omega_s)$ are the single and two-photon detunings. Here second derivatives of the slowly varying amplitudes were neglected and sufficiently slow spatial variations of $\mathcal{E}$ and Ω_0 assumed. γ denotes the loss rate out of the excited state and F_2^l the corresponding Langevin noise operator. The propagation equation for the electromagnetic field reads

$$\left(\frac{\partial}{\partial t} + c\frac{\partial}{\partial z}\right)\mathcal{E}(z,t) = -ig\Psi_1^+(z,t)\Psi_2(z,t). \tag{4}$$

The classical Stokes field is taken to be much stronger than the quantum pump and is assumed undepleted.

In the following we will omit all Langevin noise terms F_2^l as well as homogeneous contributions to the solutions. Thus the operator relations derived are valid only when taken in normal-ordered correlation functions.

Consider a stationary input of atoms in state $|1\rangle$, i.e., $\Psi_1(0,t) = \sqrt{n}$, where n is the constant total density of atoms. In the limit of a weak quantum field and weak atomic excitation one finds

$$\Phi_1^l(z,t) \approx \Phi_1^l(0, t - 2mz/\hbar k_l) = \sqrt{n}\,\xi_l e^{-i\varphi_l(z,t)}, \tag{5}$$

where $\sum_l \xi_l e^{-i\varphi_l(0,t)}$ and $\varphi_l \equiv (k_l z - \omega_l t)$. Furthermore,

$$\begin{aligned}\Phi_3^l(z,t) = &-\frac{g\mathcal{E}}{\Omega_0}\sqrt{n}\,\xi_l e^{-i\varphi_l(z,t)} \\ &+ \frac{i}{\Omega_0(z)}\left(\frac{\partial}{\partial t} + \frac{\hbar(k_l+k_p)}{2m}\frac{\partial}{\partial z}\right. \\ &\left. + i\Delta_l + \gamma\right)\Phi_2^l(z,t),\end{aligned} \tag{6}$$

$$\Phi_2^l(z,t) = \frac{i}{\Omega_0(z)}\left(\frac{\partial}{\partial t} + \frac{\hbar k_l}{2m}\frac{\partial}{\partial z} + i\delta_l\right)\Phi_3^l(z,t). \tag{7}$$

First the case of perfect two-photon resonance for all atoms shall be discussed, i.e., $\delta_l \equiv 0$. Here one can invoke an adiabatic approximation, leading to

$$\Phi_3^l(z,t) = -\frac{g\mathcal{E}(z,t)}{\Omega_0(z)}\sqrt{n}\,\xi_l e^{-i\varphi_l(z,t)}, \tag{8}$$

$$\Phi_2^l(z,t) = -i\frac{g\sqrt{n}}{\Omega_0(z)}\xi_l e^{-i\varphi_l(z,t)}\left(\frac{\partial}{\partial t} + v_l\frac{\partial}{\partial z}\right)\frac{\mathcal{E}(z,t)}{\Omega_0(z)}, \tag{9}$$

with $v_l = \frac{\hbar k_l}{2m}$.

Substituting the latter result into the equation of motion for the radiation field yields

$$\left[\left(1 + \frac{g^2 n}{\Omega_0^2(z)}\right)\frac{\partial}{\partial t} + c\left(1 + \frac{g^2 n}{\Omega_0^2(z)}\frac{v_0}{c}\right)\frac{\partial}{\partial z}\right]\mathcal{E} = \frac{g^2 n}{\Omega_0^2(z)}v_0\left(\frac{\partial}{\partial z}\ln\Omega_0(z)\right)\mathcal{E}, \tag{10}$$

with $v_0 \equiv \sum_l \xi_l v_l$.

Thus the quantum field propagates with a group velocity

$$v_{\rm gr} = c\,\frac{\left(1 + \frac{g^2 n}{\Omega_0^2(z)}\frac{v_0}{c}\right)}{\left(1 + \frac{g^2 n}{\Omega_0^2(z)}\right)}, \tag{11}$$

which approaches v_0 if $\Omega_0 \rightarrow 0$. Note that $v_0 > 0$ was assumed here. In the case of atoms moving against the direction of light propagation, i.e., for a negative v_0, a vanishing and even a negative value of $v_{\rm gr}$ can arise. Moving the medium against the very small group velocity would effectively freeze or redirect the light pulse [18] (note that Galilean laws apply since $|v_{\rm gr}| \ll c$). In reality, however, reflection as well as nonadiabatic effects and associated losses prevent the latter from happening [19].

The right-hand side of Eq. (10) describes a reduction/enhancement due to stimulated Raman adiabatic passage in a spatially varying Stokes field. It can be seen that this is possible only if $v_0 \neq 0$ in accordance with the observation in [7] that a unidirectional transfer of excitation requires an explicit time dependence. For nonvanishing v_0 a space-dependent Stokes field in the laboratory frame is equivalent to a time-dependent field in the rest frame of the atoms.

Equation (10) has the simple solution

$$\mathcal{E}(z,t) = \mathcal{E}[0, t - \tau(z)] \frac{\cos\theta(z)}{\cos\theta(0)}, \tag{12}$$

where $\tau(z) = \int_0^z dz'\, v_{\rm gr}^{-1}(z')$ and we have introduced the mixing angle $\theta(z)$ according to $\tan^2\theta(z) \equiv \frac{g^2 n}{\Omega_0^2(z)} \frac{v_0}{c}$. If $\Omega_0(z)$ is a sufficiently slowly, monotonically decreasing function of z which approaches zero, the amplitude of the pump field decreases to zero as well. At the same time one finds from (8) for $\tilde{\Psi}_3 \equiv \Psi_3 e^{-i\{(k_p - k_s)z - (\omega_p - \omega_s)t\}}$

$$\tilde{\Psi}_3(z,t) = -\sqrt{\frac{c}{v_0}} \tan\theta(z) \mathcal{E}(z,t). \tag{13}$$

If at the input of the interaction region $\theta(0) = 0$ and at the output $\theta(L) = \pi/2$ this yields

$$\tilde{\Psi}_3(L,t) = -\sqrt{\frac{c}{v_0}} \mathcal{E}(0, t - \tau), \tag{14}$$

with $\tau = \int_0^L dz\, v_{\rm gr}^{-1}(z)$. The factor $\sqrt{c/v_0}$ accounts for the fact that the input light propagates with velocity c while the output matter field propagates only with v_0. As can easily be seen, the input flux of photons is thus equal to the output flux of atoms in state $|3\rangle$: $c\langle \mathcal{E}^+ \mathcal{E}\rangle_{\rm in} = v_0 \langle \Psi_3^+ \Psi_3\rangle_{\rm out}$. Equation (14) is the main result of the paper. It shows that in the present setup the quantum properties of an input electromagnetic field can be completely transferred to an atomic beam. This is illustrated in Fig. 2, where the average value and fluctuations of the photon number $\hat{n}(z)$ and the number of atoms in state 3, $\hat{m}_3(z)$ passing a plane at position z during a certain time interval, are shown. The exchange of photons into state-3 atoms is apparent. Because of the incomplete transfer of excitations, atom-number fluctuations reach a maximum for intermediate z, but $\langle \Delta m_3^2\rangle_{\rm out} \to \langle \Delta n_{\rm ph}\rangle_{\rm in}$.

Thus continuous matter waves with nonclassical quantum correlations can be generated out of cw nonclassical light. Since the mapping technique can be applied to separate Raman transitions at the same time, it is also possible to transfer continuous entanglement from a pair of cw light beams as generated, for example, in parametric down-conversion to a pair of atomic beams.

In the derivation of the above result several approximations have been invoked. In the following, the validity of those will be discussed in more detail. The first approximations made are those of a weak quantum pump field and

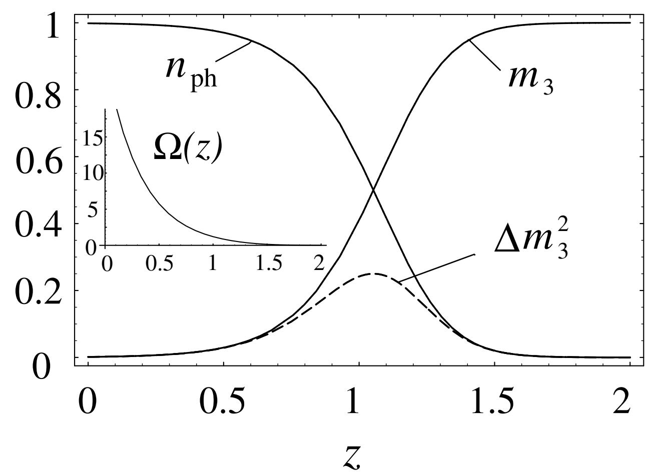

FIG. 2. Average values and fluctuations of photons and atoms in state 3 traversing a plane at position z in a given time interval normalized to input photon number. Input light is in Fock state. $\hat{n}_{\rm ph} \equiv c \int_T dt\, \mathcal{E}^\dagger(z,t)\mathcal{E}(z,t)$, $\hat{m}_3 \equiv v_0 \int_T dt\, \Psi_3^\dagger(z,t)\Psi_3(z,t)$. Inset shows $\Omega(z)$ in units of $g\sqrt{n v_0/c}$.

weak atomic excitation. It can easily be seen from Eq. (8) that the ratio of the intensity of the pump to the Stokes field is given by the ratio of the atomic number density in state $|3\rangle$ to the total number density, even when the Stokes field goes to zero: $g^2\langle \mathcal{E}^+ \mathcal{E}\rangle/\Omega_0^2 = \langle \Psi_3^+ \Psi_3\rangle/n$. It is thus sufficient to fulfill the condition of weak atomic excitation, which requires that the input flux of atoms is much larger than the input flux of pump photons. This condition requires some experimental efforts and may not be easy to satisfy. On the other hand, rather high flux densities of atoms can be achieved in supersonic beam configurations or (with narrow velocity distributions) in atom lasers.

A second assumption made is that of perfect two-photon resonance. This condition can be fulfilled only if either the velocity spread of the atoms is extremely small or if the relative wave vector of pump and Stokes fields projected onto the z axis vanishes. Both conditions are not easy to satisfy. They require either a substantial level of longitudinal cooling, i.e., a coherent source of input atoms or a careful design of pump and Stokes field geometry. For a quantitative analysis a nonvanishing but constant value of $\delta_l = \delta$ is considered in the following. In this case there is a contribution to Φ_2^l even in lowest order of the adiabatic expansion.

$$\Phi_2^l \to \Phi_2^l + \frac{\delta \Omega_0}{\Omega_0^2 - \delta(\Delta - i\gamma)} \frac{g\mathcal{E}}{\Omega_0} \sqrt{n}\, \xi_l e^{-i(k_l z - \omega_l t)}, \tag{15}$$

which gives rise to an additional dissipative loss term in the equation for $\mathcal{E}$

$$\left[\left(1 + \frac{g^2 n}{\Omega_0^2(z)}\right)\frac{\partial}{\partial t} + c\left(1 + \frac{g^2 n}{\Omega_0^2(z)} \frac{\bar{v}}{c}\right)\frac{\partial}{\partial z}\right]\mathcal{E} = \frac{g^2 n}{\Omega_0^2(z)} \bar{v}\left(\frac{\partial}{\partial z} \ln\Omega_0(z)\right)\mathcal{E} - \frac{g^2 n}{\Omega_0^2(z)}\left(\frac{\delta^2 \Omega_0 \gamma}{(\Omega_0^2 - \delta\Delta)^2 + \delta^2\gamma^2}\right)\mathcal{E} + \dots, \tag{16}$$

where the dots indicate additional imaginary terms that affect only the phase of $\mathcal{E}$.

To estimate the influence of the dissipative term $\Delta = 0$ is assumed. Integrating the field equation yields for the loss factor η of the field amplitude ($\mathcal{E} \to \eta\mathcal{E}$)

$$\eta = \exp\left\{-\alpha \int_0^1 d\zeta \, \frac{\cos^2\theta(\zeta)x^2}{\cot^4\theta(\zeta) + x^2}\right\}, \tag{17}$$

with $\zeta = z/L$. Here $\alpha \equiv g^2 nL/\gamma c$ is the opacity of the medium in the absence of EIT and $x \equiv \delta\gamma/g^2 n \frac{v_0}{c}$ is a dimensionless quantity characterizing the two-photon detuning. The cosine of the mixing angle is monotonically decreasing from some initial value to zero over the interaction length L. Assuming $x \ll 1$ one can give an upper limit to the integral in Eq. (17), by replacing the integrand by its maximum value, which is achieved when $\cos^2\theta \approx |x| \ll 1$. This gives the very good estimate $\eta \geq \exp\{-\alpha|x|/2\}$. Thus in order to neglect the influence of dissipative losses, it is sufficient that

$$|\delta| \frac{L}{v_0} \ll 1 . \tag{18}$$

A two-photon detuning can result, for example, from a residual Doppler shift of the 1–3 transition. If Δv denotes the difference of the velocity in z direction with respect to the resonant velocity class, the corresponding two-photon detuning reads $\delta = \Delta v(\vec{k}_p - \vec{k}_s) \cdot \vec{e}_z$. In this case (18) translates into

$$\frac{|\Delta v|}{v_0} \ll \frac{1}{(k_p - k_s)L} . \tag{19}$$

It can be seen from (19) that the geometry of the setup should be chosen in such a way that the beat-note wave vector has a minimal projection to the z axis. Combining this with the requirement of a spatially decreasing Stokes field $\Omega_0(z)$ is experimentally difficult, but possible in principle. Furthermore, other schemes of quantum-state transfer that use adiabatic sweeping of the two-photon detuning through resonance rather than an adiabatically varying Stokes intensity may be employed [20].

For a monochromatic quantum pump field the conditions for adiabaticity can easily be obtained from those in a stationary medium with time-dependent Stokes field [21] by a simple frame transformation. This yields

$$\gamma \int_0^L dz \, \frac{v_0[\theta'(z)]^2}{g^2 n + \Omega_0^2(z)} \ll 1 . \tag{20}$$

Setting $\theta'(z) \sim 1/L$ this results in a lower limit for the beam opacity $\alpha = g^2 nL/\gamma c$ in the absence of EIT

$$\alpha \gg \frac{v_0}{c} . \tag{21}$$

Since $v_0 \ll c$, a value of α much less than unity will be sufficient to guarantee total adiabatic transfer. Note that v_0 cannot be arbitrarily small, however, since the atom flux at the input has to be much larger than the input photon flux.

In summary, we have shown that a complete and loss-free transfer of quantum properties from a cw light field to a continuous beam of atoms is possible using a recently proposed technique based on Raman adiabatic transfer. The combination of a space-dependent Stokes field with a finite momentum of the input matter wave leads to a time-varying Stokes field in the rest frame of the atoms. In this way continuous and monochromatic matter waves in nonclassical or entangled quantum states can be generated out of light fields with corresponding properties.

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