Comment on "Electromagnetically Induced Left Handedness in Optically Excited Four-Level Atomic Media"

In a recent Letter [Phys. Rev. Lett. 96, 053601 (2006)], Thommen and Mandel discussed a novel scheme to induce left handedness and negative refraction in an atomic fourlevel scheme [1]. The proposal is based on a coherent cross coupling of electric and magnetic dipole transitions, which couple to the corresponding components of the probe field. A very important feature of the scheme is that the two transitions do not have to involve common states, which greatly enhances the freedom of choice of levels and makes the scheme much more applicable than previous proposals [2]. We here show that although the main conclusion of [1]—the possibility to create negative refraction in the driven four-level scheme-remains valid, the results obtained are quantitatively not correct. In particular we show that there is never gain in the scheme and that the ratio of negative refraction to absorption is only on the order of unity.

Let us consider the four-level scheme of Ref. [1] shown in Fig. 1. In order to calculate the complex index of refraction, Thommen and Mandel solved the densitymatrix equations in the weak-excitation limit, i.e., setting $\rho_{11} = 1$. The imaginary part of the permittivity ε obtained from this attains negative values for certain parameter values, corresponding to gain, which is, however, impossible for the considered scheme. This is a result of the weak-excitation assumption, which for the values of Ω_{13} used in [1] is no longer valid.

Furthermore the coherent cross coupling between electric and magnetic dipole transitions leads to chirality, where the magnetic component **H** of the probe field couples to the electric polarization **P** and, correspondingly, the electric component **E** to the magnetization **M**: $\mathbf{P} = \chi_e \mathbf{E} + \xi_{EH} \mathbf{H}$, and $\mathbf{M} = \xi_{HE} \mathbf{E} + \chi_m \mathbf{H}$. Here ξ_{EH} and ξ_{HE} are the chirality coefficients. In a chiral medium the expression for the index of refraction reads [3]

$$n = \sqrt{\varepsilon \mu - \frac{(\xi_{EH} + \xi_{HE})^2}{4}} + \frac{i}{2}(\xi_{EH} - \xi_{HE}).$$
(1)

Rather than applying Eq. (1), Thommen and Mandel followed the approach of Oktel and Müstecaplioglu [2] and used the relation $\mathbf{B} = \mathbf{k} \times \mathbf{E}/(\omega c)$ to calculate the permeability μ from the matrix element $\rho_{12}(\mathbf{E})$ and from that the refractive index $n = \sqrt{\varepsilon \mu}$. Although this captures the most important contributions, it neglects the modification of the electric polarization by the magnetic component of the probe field.



FIG. 1. Left: 4-level scheme of [1], Ω_{13} and Ω_{42} denote Rabifrequencies of (strong) external drive fields. Electric (*E*) and magnetic (*B*) components of probe field couple to $|3\rangle - |4\rangle$ and $|1\rangle - |2\rangle$, respectively. Right: Real (full line) and imaginary part (dashed line) of *n*. Levels $|3\rangle$ and $|4\rangle$ are assumed to decay with rate $\gamma = 10^7 \text{ s}^{-1}$, while the upper level $|2\rangle$ of the magnetic transition decays with rate $\gamma_2 = \gamma/(137)^2$. $\lambda = 600 \text{ nm}$, $\Omega_{13} = \gamma$, $\Omega_{42} = 10^{-2}\gamma$.

We have calculated the index of refraction for the scheme of Fig. 1 from Eq. (1) using the stationary solutions of the density-matrix equations without the linear-response approximation. The results are shown on the right half of Fig. 1, where we have plotted Re[n] and Im[n] as function of the detuning $\delta = \omega - \omega_{34} = \omega - \omega_{12}$, with ω being the probe-field frequency and $\omega_{12} = \omega_{34}$ the atomic transition frequencies. The parameters differ from those of Ref. [1] in order to obtain a negative Re[n]. We find that Im[n] is always positive, corresponding to absorption, and that the ratio of refraction to absorption, |Re[n]/Im[n]|, is only on the order of unity. We note, however, that the idea of Mandel and Thommen can be employed to attain negative refraction with suppressed absorption when combined with interference effects [4].

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