Continuous-variable versus electromagnetically-induced-transparency-based quantum memories

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We discuss a general model of a quantum memory for a single light mode in a collective mode of atomic oscillators. The model includes interaction Hamiltonians that are of second order in the canonical position and momentum operators of the light and atomic oscillator modes. We also consider the possibility of measurement and feedback. We identify an interaction Hamiltonian that leads to an ideal mapping by pure unitary evolution and compare several schemes which realize this mapping using a common continuous-variable description. In particular, we discuss schemes based on the off-resonant Faraday effect supplemented by measurement and feedback and proper preparation of the atoms in a squeezed state and schemes based on off-resonant Raman coupling as well as electromagnetically induced transparency.

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I. INTRODUCTION

One of the key elements of quantum communication [1,2] and network quantum computers [3,4] is a high-fidelity quantum memory. In recent years, substantial progress has been made in the design and experimental realization of quantum memory schemes that are based on photons as information carriers and an ensemble of atoms or other quantum radiators as a storage unit. In particular, three conceptually different approaches to such a quantum light-matter interface have been put forward. One possibility is to make use of the off-resonant Faraday effect to transfer the quantum state of a polarized light field to a macroscopic atomic spin of the atomic ensemble [5–13]. This quantum nondemolition (QND) light-matter coupling gives rise to only a partial transfer of the quantum information, but that can be overcome by additional measurement and feedback [10–13] or using some more involved multipassage geometric configuration [7–9]. In contrast to the Faraday interaction, off-resonant Raman scattering allows complete transfer directly [14–16]. The quantum information coherently oscillates between the light and the atomic ensemble, thus also implementing a kind of interspecies beam splitters for some instants. Finally, electromagnetically induced transparency (EIT) can also be used to map the light field operators to the collective atomic coherences [17–31]. This process is usually interpreted as an adiabatic following of the dark polariton eigenstates, and a simple picture based on effective Hamiltonians between the light and atomic variables is missing. Currently, substantial work is devoted to the optimization of these schemes which is crucial for potential large-scale implementations. An obstacle in this effort is the rather different theoretical frameworks used to describe these approaches. We here put forward a common description of the Faraday, Raman, and EIT schemes, which can be used to compare their advantages and drawbacks. We reveal the similarities of the different approaches using a simplified, single-mode model and, for each scheme, we derive the effective Hamiltonian that directly relates the light and atomic variables to each other.

We begin with a review of general properties of a single-mode quantum memory in terms of continuous light and matter variables recovering ideal Hamiltonians for a purely unitary realization of the map. We then discuss the possibility to realize a one-way map from light to atoms using Hamiltonians that are not equivalent to the ideal one, such as the Faraday interaction Hamiltonian, by means of measurement and feedback techniques as well as proper state preparation of the atomic ensemble [5,12]. We calculate the fidelity of storing nonclassical states like the Schrödinger cat states using this setup. It is shown how the rather demanding conditions on measurement and state preparation can be substantially reduced in double-pass configurations [9] and can be totally eliminated in a triple-pass scheme.

We then discuss physical implementations of the different mapping approaches starting with a summary of the $J=-1/2\leftrightarrow J=+1/2$ scheme of Ref. [12] realizing a Faraday coupling and nonunitary quantum memory. We then present a double-Λ atomic configuration with off-resonant Raman coupling. We show that our scheme is capable of implementing both the Faraday coupling Hamiltonian and the ideal mapping Hamiltonian due to controllable interference between the pathways involving different upper levels. Finally, we discuss the quantum memory based on EIT [19]. In our formalism, we can derive an effective Hamiltonian that directly links the light and atomic variables. We show that this scheme corresponds to an ideal mapping Hamiltonian when a proper spin polarized (but not squeezed) initial state of the atomic ensemble is considered.

The paper is organized as follows. In Sec. II, we consider a simple model for quantum memories in terms of atomic and light field quadrature variables. In Sec. III, we discuss physical systems that can be used to realize Faraday-type, off-resonant Raman-type, and EIT interaction Hamiltonians. Section IV summarizes our results.

II. REALIZATIONS OF THE IDEAL MAP USING UNITARY EVOLUTION, MEASUREMENT, AND FEEDBACK

We consider an abstract model of a reversible memory for the quantum state of a light mode in an ensemble of atoms.

The light mode (system L) is described in terms of the canonical quadrature variables $\hat{X}_L$ and $\hat{P}_L$. The quantum memory (system A) to which we intend to transfer the quantum state will be precisely specified in Sec. III. Here we only assume that it can be described by a similar set of continuous variables $\hat{X}_A$ and $\hat{P}_A$ with $[\hat{X}_A,\hat{P}_A]=i$ ($\hbar=1$). This is the case, e.g., for a large ensemble of initially polarized spins, if the excitation probability of each individual spin is small. The time evolution of the two quantum systems can be described in the Heisenberg picture by a map that connects the dynamical variables of the systems at some initial time to those at a final time t: $(\hat{X}_A(0),\hat{P}_A(0),\hat{X}_L(0),\hat{P}_L(0))\mapsto(\hat{X}_A(t),\hat{P}_A(t),\hat{X}_L(t),\hat{P}_L(t))$. For an ideal quantum memory, we require the mapping to be linear in the quadrature variables and complete in the sense that the variables of one subsystem are mapped only to those of the other. That is, employing the vector notation

$$\hat{\mathbf{y}} \equiv (\hat{X}_A,\hat{P}_A,\hat{X}_L,\hat{P}_L)^T, \tag{1}$$

the map has the compact form

$$\hat{\mathbf{y}}_{\text{out}} = \mathbf{M}\hat{\mathbf{y}}_{\text{in}} \text{ with } \mathbf{M} = \begin{pmatrix} 0 & \mathbf{M}_1 \\ \mathbf{M}_2 & 0 \end{pmatrix}, \tag{2}$$

with $\mathbf{M}_i$ being 2×2 symplectic, real matrices. The matrices need to be symplectic in order to conserve commutation relations.

A. Purely unitary evolution

Let us first consider the question under what conditions an ideal quantum memory map (2) can be realized by pure unitary evolution. To ensure the linearity of the map, the Hamiltonian should be of at most second order in the quadrature variables (or in the corresponding annihilation and creation operators). Specifically, we consider pure harmonic oscillators with a quadratic interaction between them as follows:

$$\hat{H} = \hat{H}_A + \hat{H}_L + \hat{H}_{\text{int}}, \tag{3}$$

$$\hat{H}_A = \frac{\omega_A}{2}(\hat{X}_A^2 + \hat{P}_A^2) = \omega_A\left(\hat{a}_A^\dagger\hat{a}_A + \frac{1}{2}\right), \tag{4}$$

$$\hat{H}_L = \frac{\omega_L}{2}(\hat{X}_L^2 + \hat{P}_L^2) = \omega_L\left(\hat{a}_L^\dagger\hat{a}_L + \frac{1}{2}\right), \tag{5}$$

$$\hat{H}_{\text{int}} = p\hat{X}_A\hat{X}_L + q\hat{X}_A\hat{P}_L + r\hat{P}_A\hat{X}_L + s\hat{P}_A\hat{P}_L, \tag{6}$$

where ω_A and ω_L are the effective oscillator frequencies of the atomic and light systems, respectively, and the real parameters p, q, r, and s characterize the interaction. These parameters may have explicit time dependence. To further simplify the discussion, we assume that the interaction free energy $\hat{H}_0=\hat{H}_A+\hat{H}_L$ is conserved and that the interaction Hamiltonian commutes at different times,

$$[\hat{H}_0(t),\hat{H}_{\text{int}}(t)] = 0 \quad \text{and} \quad [\hat{H}_{\text{int}}(t),\hat{H}_{\text{int}}(t')] = 0. \tag{7}$$

The former implies exact resonance between the atomic and light systems, $\omega_A=\pm\omega_L$. With this restriction, the free Hamiltonian can be eliminated from the equations of motion. Indeed, let us express operators of the Heisenberg picture ($\hat{A}_H(t)$) in the frame rotating according to the free Hamiltonian (rotating wave picture): $\hat{A}(t)\equiv\hat{U}_0(t)\hat{A}_H(t)\hat{U}_0^\dagger(t)$, where $\hat{U}_0(t)$ denotes the unitary operator of the interaction free time evolution. Thus, the equation of motion for operators in the rotating frame reads

$$\frac{d}{dt}\hat{A}(t) = i[H_{\text{int}}(t),\hat{A}(t)]. \tag{8}$$

With the second assumption in Eq. (7) the time evolution operator corresponding to the interaction can be written in an exponential form with no time ordering necessary.

For nonvanishing effective oscillator frequencies $\omega_A=\omega_L$, the most general form for the interaction Hamiltonian is $\hat{H}_{\text{int}}(t)=\alpha(t)\hat{H}_1$ with

$$H_1 \equiv \sin\xi(\hat{X}_A\hat{X}_L + \hat{P}_A\hat{P}_L) + \cos\xi(\hat{P}_A\hat{X}_L - \hat{X}_A\hat{P}_L). \tag{9}$$

That is, $\hat{H}_{\text{int}}$ can have explicit time dependence only through $\alpha(t)$. We note here that the complementary resonance condition $\omega_A=-\omega_L$ would imply a slightly different interaction Hamiltonian, which would result in two-mode squeezing [32,33] rather than coherent sinusoidal oscillation of the quantum information between the light and atomic systems. Keeping in mind the objectives of quantum memory, we concentrate on the case $\omega_A=\omega_L$ and interaction Hamiltonian (9). Then Eq. (8) can be formally solved using the Baker-Campbell-Hausdorff formula,

$$\hat{A}(t) = \sum_{n=0}^{\infty}\frac{1}{n!}\left(\int_0^t d\tau\alpha(\tau)\right)^n\hat{A}_n, \tag{10}$$

where $\hat{A}_n=(-i)^n[[[\hat{A}(0),\hat{H}_1],\hat{H}_1],\ldots]$ is proportional to the n-fold commutator of $\hat{A}(0)$ with $\hat{H}_1$. It is easy to see that the commutators of the quadrature variables $\hat{\mathbf{y}}$ with the interaction Hamiltonian (9) are linear in the same set of quadratures; namely, we have

$$[\hat{\mathbf{y}},\hat{H}_1] = i\mathbf{C}\hat{\mathbf{y}} \quad \text{and} \quad \hat{\mathbf{y}}_n = \mathbf{C}\hat{\mathbf{y}}_{n-1} = \mathbf{C}^n\hat{\mathbf{y}}, \tag{11}$$

where

$$\mathbf{C} = \begin{pmatrix} \mathbf{0} & -\mathbf{R}^{-1} \\ \mathbf{R} & \mathbf{0} \end{pmatrix} \quad \text{and} \quad \mathbf{R} = \begin{pmatrix} \cos\xi & -\sin\xi \\ \sin\xi & \cos\xi \end{pmatrix}. \tag{12}$$

Using the series (10) and recognizing that $\mathbf{C}^2=-\mathbf{1}$, we find the linear relation between the quadratures at time t and the initial time

$$\hat{\mathbf{y}}(t) = \left[\cos\kappa(t)\mathbf{1} + \sin\kappa(t)\begin{pmatrix} \mathbf{0} & -\mathbf{R}^{-1} \\ \mathbf{R} & \mathbf{0} \end{pmatrix}\right]\hat{\mathbf{y}}(0), \tag{13}$$

with

$$\kappa(t) = \int_0^t d\tau\, \alpha(\tau). \tag{14}$$

We see that perfect quantum memory mapping is achieved after an interaction time T such that the area of coupling is $\kappa(T)=(2n+1)\pi/2$ (with $n \in \mathbb{Z}$). In this case one has

$$\hat{\mathbf{y}}(T) = \pm \begin{pmatrix} \mathbf{0} & -\mathbf{R}^{-1} \\ \mathbf{R} & \mathbf{0} \end{pmatrix} \hat{\mathbf{y}}(0). \tag{15}$$

Regarding the possibility of physical realizations of the interaction Hamiltonian (9), two different values of ξ are of particular importance. For $\xi=0$, we have

$$\hat{H}_{\text{int}}(t) = \alpha(t)(\hat{P}_A\hat{X}_L - \hat{X}_A\hat{P}_L). \tag{16}$$

If the envelope $\alpha(t)$ is chosen such that $\kappa(T)=\pi/2$ then we arrive at an ideal quantum memory map

$$\hat{X}_A(T) = \hat{X}_L, \quad \hat{P}_A(T) = \hat{P}_L,$$

$$\hat{X}_L(T) = -\hat{X}_A, \quad \hat{P}_L(T) = -\hat{P}_A. \tag{17}$$

We can also consider the storing process in which the $\hat{X}$ and $\hat{P}$ quadratures of the memory system are interchanged with respect to the previous transformation. This corresponds to $\xi=\pi/2$ and the interaction Hamiltonian

$$\hat{H}_{\text{int}}(t) = \alpha(t)(\hat{X}_A\hat{X}_L + \hat{P}_A\hat{P}_L) \tag{18}$$

leads to the map

$$\hat{X}_A(T) = \hat{P}_L, \quad \hat{P}_A(T) = -\hat{X}_L,$$

$$\hat{X}_L(T) = \hat{P}_A, \quad \hat{P}_L(T) = -\hat{X}_A, \tag{19}$$

which is again the map of an ideal quantum memory.

B. Single-pass scheme with feedback and initial spin squeezing

A quantum memory for light, which is not based entirely on unitary evolution but is rather an approximate simulation of the Hamiltonian (18), was proposed and experimentally demonstrated in [12]. The light-matter interaction there is due to the Faraday effect and is described by the Hamiltonian

$$\hat{H}_0 = \text{const}, \quad \hat{H}_{\text{int}}(t) = \alpha(t)\hat{H}_1, \quad \hat{H}_1 = \hat{P}_A\hat{P}_L. \tag{20}$$

See Sec. III A for a possible derivation of Eq. (20). In this case the matrix $\mathbf{C}$ of Eq. (11) cannot be represented in the form (12). The unitary evolution will simply shift the position operators by an amount proportional to the momentum of the other system as well as to the area of coupling κ, while the momenta are constants of motion.

$$\hat{X}_A(t) = \hat{X}_A + \kappa(t)\hat{P}_L, \quad \hat{P}_A(t) = \hat{P}_A,$$

$$\hat{X}_L(t) = \hat{X}_L + \kappa(t)\hat{P}_A, \quad \hat{P}_L(t) = \hat{P}_L. \tag{21}$$

One recognizes that only the momentum quadrature of the light mode is transferred to the atomic ensemble. To also map the position quadrature to the ensemble the unitary evolution was complemented in Ref. [12] by a homodyne measurement of the outgoing light quadrature $\hat{X}_L$ and the measurement result x was fed back by applying a momentum displacement of $-x/\kappa$ on system A. As a consequence of the measurement, one can formally write the c-number x in place of $\hat{X}_L(t)$ and rearrange Eq. (21) to conclude that after measurement and feedback,

$$\hat{X}_A^{\text{mem}} = \hat{X}_A + \kappa\hat{P}_L, \quad \hat{P}_A^{\text{mem}} = -\frac{1}{\kappa}\hat{X}_L. \tag{22}$$

If the atomic ensemble were initially prepared in a position eigenstate, i.e., in an infinitely squeezed state, the operator $\hat{X}_A$ in Eq. (22) could be replaced by a c number. The resulting map would in this case ideally transfer the complete state of light to the atomic ensemble.

To verify these statements in a more rigorous way, we calculate the state of the quantum memory after the storage and the storage fidelity in terms of Wigner functions. The atomic and light systems are initially disentangled, so the two-particle Wigner function is of the product form

$$W_0(x_A,p_A;x_L,p_L) = W_A(x_A,p_A)W^{\text{in}}(x_L,p_L). \tag{23}$$

After the transformation (21), the new state is given by

$$W'(x_A,p_A;x_L,p_L) = W_0(x_A-\kappa p_L,p_A;x_L-\kappa p_A,p_L). \tag{24}$$

Then the quadrature $\hat{X}_L'$ of the outgoing light is measured. For an ideal measurement, the projection corresponding to the outcome x is $\hat{\mathbb{1}}_A \otimes \hat{\Pi}_x$, where $\hat{\Pi}_x=|x\rangle_{LL}\langle x|$ and its Wigner function is $\Pi_x(x_L',p_L')=\delta(x_L'-x)$. The (unnormalized) conditional atomic state can be obtained by a von Neumann projection. The feedback is described by a shift in the atomic momentum $p_A'=p_A''+x/\kappa$ and thus the state of the memory conditioned on the measurement result x and after feedback reads

$$W_x^{\text{mem}}(x_A',p_A'') = \int dx_L'dp_L' W_A(x_A'-\kappa p_L',p_A''+x/\kappa) \times W^{\text{in}}(x_L'-\kappa p_A''-x,p_L')\Pi_x(x_L',p_L'). \tag{25}$$

The norm of Eq. (25) gives the probability distribution $P(x)$ of the measurement outcome x. In general, $P(x)$ has explicit dependence on the unknown input state $W^{\text{in}}(x_L,p_L)$. This means that the measurement yields information about the input state of the light which is, therefore, distorted. However, in the special case of an infinitely squeezed initial atomic state $W_A(x_A,p_A)=\delta(x_A-x_0)$, we have a uniform probability distribution $P(x)=1/\kappa$ and no information about the initial light state is obtained by the measurement. In this ideal case, the final state is

$$W^{\text{mem}}(x,p) = W^{\text{in}}(-\kappa p,(x-x_0)/\kappa), \tag{26}$$

which corresponds to an ideal quantum memory mapping regardless of the outcome x.

In reality, however, it is impossible to prepare the atomic ensemble in a position eigenstate, i.e., in an infinitely squeezed state. Furthermore, the measurement will be imper-

fect in general. In the following, we discuss the effects of finite initial squeezing and finite detection efficiency. For this we replace the Wigner function Π_x used above with the corresponding expression for a finite detection efficiency [34]

$$\hat{\Pi}_{x,\sigma_\eta} = (2\pi\sigma_\eta^2)^{-1/2} \int dy e^{-[(y-x)^2/2\sigma_\eta^2]} |y\rangle_{LL}\langle y|,$$

$$\Pi_{x,\sigma_\eta}(x_L', p_L') = (2\pi\sigma_\eta^2)^{-1/2} \exp\left\{-\frac{(x_L'-x)^2}{2\sigma_\eta^2}\right\}. \quad (27)$$

Here σ_η characterizes the resolution of the position measurement, with $\sigma_\eta=0$ corresponding to a noiseless, perfect measurement, while a typical experimental value is in the order of $\sigma_\eta^2=2.5\times10^{-3}$. Averaging over the measurement outcome x gives the memory state

$$W^{\text{mem}}(x,p) = \int dx'dp'dx'' W_A(x',p') \times W^{\text{in}}(-\kappa p + \sigma_\eta x'', (x-x')/\kappa)\Pi_{0,1}(x'',0). \quad (28)$$

Let us consider a Gaussian initial atomic state

$$W_A(x,p) = \frac{1}{2\pi\sigma_{X,A}\sigma_{P,A}} e^{-[(x-x_0)^2/2\sigma_{X,A}^2]-[(p-p_0)^2/2\sigma_{P,A}^2]}. \quad (29)$$

In the experiment of [12], it was actually a coherent spin state with $\sigma_{X,A}^2=\sigma_{P,A}^2=\frac{1}{2}$. The imperfect measurement (27) then results in the state

$$W^{\text{mem}}(x,p) = \frac{1}{2\pi\sigma_\eta\sigma_{X,A}} \int dx'dp' e^{-(x'^2/2\sigma_\eta^2)-(\kappa^2p'^2/2\sigma_{X,A}^2)} \times W^{\text{in}}(-\kappa p + x', (x-x_0)/\kappa + p') \quad (30)$$

for the atomic quantum memory. Compared to the ideal memory state (26), we have higher uncertainties both in the X and P quadratures due to the noisy measurement and the imperfect initial state preparation, respectively. The average fidelity of the storage process can be calculated from the overlap of the ideal (26) and real (30) output states,

$$F = 2\pi \int dxdp W^{\text{in}}(-\kappa p, (x-x_0)/\kappa) W^{\text{mem}}(x,p). \quad (31)$$

For a Gaussian state of the input light field

$$W^{\text{in}}(x,p) = \frac{1}{2\pi\sigma_{X,L}\sigma_{P,L}} e^{-[(x-x^{\text{in}})^2/2\sigma_{X,L}^2]-[(p-p^{\text{in}})^2/2\sigma_{P,L}^2]}, \quad (32)$$

the fidelity reads

$$F = [(\sigma_{X,A}^2/\kappa^2 + 2\sigma_{P,L}^2)(\sigma_\eta^2 + 2\sigma_{X,L}^2)]^{-1/2}, \quad (33)$$

which is about 82% for a coherent spin state and coherent light input ($\sigma_{X,A}^2=\sigma_{X,L}^2=\sigma_{P,L}^2=1/2$, $\sigma_\eta=0$, and $\kappa=1$), as in Ref. [12]. However, direct calculation shows that the fidelity of storage quickly decreases for nonclassical states like Schrödinger cat states (see Fig. 1).

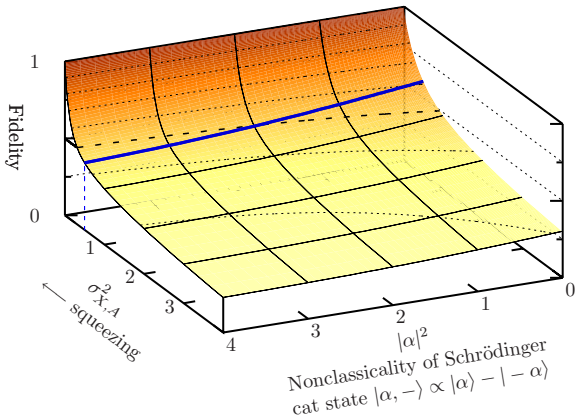

FIG. 1. (Color online) Fidelity of storing the odd Schrödinger cat states $|\alpha,-\rangle \propto |\alpha\rangle - |-\alpha\rangle$ (with real α) in the one-pass scheme of [12]. The atomic ensemble is initially prepared in a spin squeezed state: the smaller the position variance $\sigma_{X,A}$, the higher the squeezing. The thick line corresponds to the coherent spin state with no squeezing and it reaches the classical limit of fidelity 0.5 (thick dashed contour line) at about $|\alpha|^2\approx2.0$. Since the measurement alters the input state, the scheme cannot efficiently store superpositions of states with very different momenta.

We conclude that the single-pass feedback technique provides the perfect quantum memory mapping if and only if the atomic ensemble is initially prepared in an infinitely squeezed spin state $W_A(x_A,p_A)=\delta(x_A-x_0)$ and if the measurement is noiseless.

C. Double-pass schemes

The necessity of preparing the atomic ensemble in a highly squeezed state or performing a measurement with feedback can be avoided in a double-pass scheme with two successive, different unitary evolutions [9]. In this scheme, an interaction Hamiltonian identical to Eq. (20) is applied first. After a time t, the interaction is suddenly changed to $\hat{H}_2$, so we have

$$\hat{H}_1 = \hat{P}_A\hat{P}_L, \quad \hat{H}_2 = \hat{X}_A\hat{X}_L, \quad (34)$$

$$\hat{X}_A(t+t') = \hat{X}_A + \kappa\hat{P}_L,$$

$$\hat{P}_A(t+t') = (1-\kappa\kappa')\hat{P}_A - \kappa'\hat{X}_L,$$

$$\hat{X}_L(t+t') = \hat{X}_L + \kappa\hat{P}_A,$$

$$\hat{P}_L(t+t') = (1-\kappa\kappa')\hat{P}_L - \kappa'\hat{X}_A, \quad (35)$$

with $\kappa'=\int_t^{t'} d\tau\alpha(\tau)$. If the interaction times are adjusted such that $\kappa\kappa'=1$, we directly obtain a mapping like Eq. (22), however, without measurement and feedback,

$$\hat{X}_A' = \hat{X}_A + \kappa\hat{P}_L, \quad \hat{P}_A' = -\frac{1}{\kappa}\hat{X}_L,$$

$$\hat{X}'_L = \hat{X}_L + \kappa \hat{P}_A, \quad \hat{P}'_L = -\frac{1}{\kappa}\hat{X}_A. \tag{36}$$

Perfect mapping can thus be achieved if the atomic ensemble is initially prepared in an infinitely squeezed state. In such a scheme the fidelity of the memory is $F=(\sigma^2_{X,A}/\kappa^2+1)^{-1/2}$ for coherent input light.

Alternatively, as will be shown in the following, the necessity of an initial atomic squeezing can be avoided by applying measurement and feedback instead. Indeed, a measurement of the quadrature $\hat{P}'_L$ can project the initial atomic state to a squeezed state—thus appropriately performing the atomic state preparation after the interaction [13]. If the measurement of $\hat{P}'_L$ gives a value p, we can replace $\hat{X}_A$ by $-\kappa p$. Applying a position displacement of κp of the atomic position, $\hat{X}^{\mathrm{mem}}_A \equiv \hat{X}'_A + \kappa p$, we obtain an ideal quantum memory map

$$\hat{X}^{\mathrm{mem}}_A = \kappa \hat{P}_L, \quad \hat{P}^{\mathrm{mem}}_A = -\frac{1}{\kappa}\hat{X}_L. \tag{37}$$

Note that an imperfect measurement similar to Eq. (27) also introduces noise in the position quadrature,

$$W^{\mathrm{mem}}(x,p) = \frac{1}{\sqrt{2\pi\sigma^2_\eta}} \int dx' W^{\mathrm{in}}(-\kappa p, x/\kappa + x') e^{-(x'^2/2\sigma^2_\eta)}. \tag{38}$$

However, light measurement can be performed with far higher accuracy than spin squeezing. For coherent input light, the fidelity of storage reads $F=(\sigma^2_\eta+1)^{-1/2}$. Thus in the double-pass scheme one can get rid of either the measurement and feedback or the preparation of the atomic ensemble in a squeezed state.

D. Triple-pass scheme

Finally, we mention that the ideal unitary evolution given in Sec. II A can be equivalently achieved in a three-pass scheme without measurement and with no initial atomic squeezing. As it was pointed out in [33], a beam-splitter-like interaction Hamiltonian can be simulated by successively applying the above two kinds of Hamiltonians three times. In fact, if we reapply $\hat{H}_1$ for a third time, we obtain

$$\hat{H}_1 = \hat{P}_A\hat{P}_L, \quad \hat{H}_2 = \hat{X}_A\hat{X}_L, \quad \hat{H}_3 = \hat{P}_A\hat{P}_L, \tag{39}$$

$$\hat{X}_A(t+t'+t'') = (1-\kappa'\kappa'')\hat{X}_A + [\kappa + \kappa''(1-\kappa\kappa')]\hat{P}_L,$$

$$\hat{P}_A(t+t'+t'') = (1-\kappa\kappa')\hat{P}_A - \kappa'\hat{X}_L,$$

$$\hat{X}_L(t+t'+t'') = (1-\kappa'\kappa'')\hat{X}_L + [\kappa + \kappa''(1-\kappa\kappa')]\hat{P}_A,$$

$$\hat{P}_L(t+t'+t'') = (1-\kappa\kappa')\hat{P}_L - \kappa'\hat{X}_A. \tag{40}$$

Setting $\kappa=\kappa'=\kappa''=1$ we arrive at the ideal mapping (19). Let us denote with $\hat{U}$ the unitary operator describing the time evolution due to the consecutive actions of $\hat{H}_1$, $\hat{H}_2$, and $\hat{H}_3$ with areas of coupling $\kappa=1$, $\kappa'=1$, and $\kappa''=1$, respectively,

$$\hat{U} = e^{-i\hat{P}_A\hat{P}_L} e^{-i\hat{X}_A\hat{X}_L} e^{-i\hat{P}_A\hat{P}_L}, \tag{41}$$

and let us denote with $\hat{U}_{\mathrm{ideal}}$ the unitary evolution corresponding to the Hamiltonian of the ideal quantum memory (18) with an area of coupling $\pi/2$,

$$\hat{U}_{\mathrm{ideal}} = e^{-i(\pi/2)(\hat{X}_A\hat{X}_L + \hat{P}_A\hat{P}_L)}. \tag{42}$$

The two unitary operators have the same effect on the quadrature variables and they are in fact identical,

$$e^{-i\hat{P}_A\hat{P}_L} e^{-i\hat{X}_A\hat{X}_L} e^{-i\hat{P}_A\hat{P}_L} = e^{-i(\pi/2)(\hat{X}_A\hat{X}_L + \hat{P}_A\hat{P}_L)}. \tag{43}$$

III. PHYSICAL SYSTEMS REALIZING ATOMIC QUANTUM MEMORY

In this section, we analyze three kinds of configurations that can serve as collective atomic quantum memory for light. The first of them is experimentally carried out by Julsgaard *et al.* [12]. Then we discuss a scheme based on off-resonant Raman scattering that realizes the ideal interaction Hamiltonian (18) directly. Finally, we show that electromagnetically induced transparency (EIT) gives rise to an effective interaction Hamiltonian of type (16).

A. Quantum memory based on Faraday rotation

Let us analyze first a scheme in which the light-matter interaction originates in the paramagnetic Faraday effect [35]: given an ensemble of atoms with macroscopic magnetic moment and shined by a linearly polarized light beam propagating in the direction of the magnetic moment, the plane of light polarization is rotated.

To describe this atom-light interaction, consider an atomic level structure depicted in the inset of Fig. 2 where the two ground levels are off-resonantly coupled to the upper ones by the right and left circularly polarized electromagnetic field modes of the same frequencies $\omega_R=\omega_L=\omega$. In the experiment of Julsgaard *et al.* [12], levels $|1\rangle$ and $|2\rangle$ correspond to the Zeeman sublevels $M_F=\pm 4$ of the ground state $6^2S_{1/2}(F=4)$ of cesium, while the light pulses are detuned to the blue by $\Delta=700$ MHz from the $6^2S_{1/2}(F=4)\rightarrow 6^2P_{3/2}(F=5)$ transition ($\lambda=852$ nm).

If the detuning Δ from the atomic transitions is large enough ($g\langle\hat{a}^\dagger\hat{a}\rangle \ll \Delta$), we can adiabatically eliminate the off-resonant excited levels so that the system reduces to an effective two-level atom. The dynamics is then governed by the following effective Hamiltonian:

$$\hat{H}_0 = \omega(\hat{a}^\dagger_R\hat{a}_R + \hat{a}^\dagger_L\hat{a}_L + 1) + E_0\sum_{i=1}^{N_A}(|1\rangle_{ii}\langle 1| + |2\rangle_{ii}\langle 2|), \tag{44a}$$

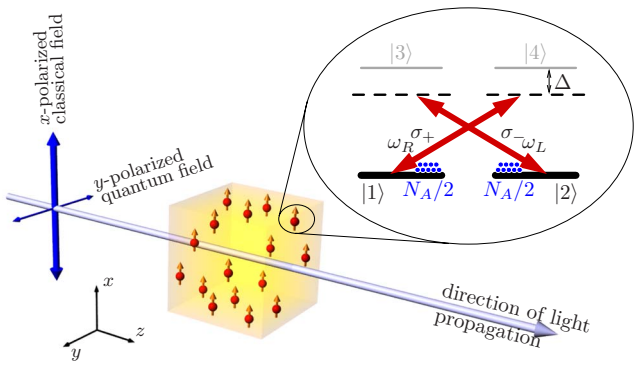

FIG. 2. (Color online) Quantum memory scheme based on Faraday rotation. The atoms with doubly degenerate ground states are polarized in the polarization direction of the strong classical light field. The inset shows a simplified atomic level structure when the quantization axis is the direction of light propagation. The atoms are in an equal-weighted coherent superposition of the ground levels. The presence of a weak y-polarized light field results in an imbalance of the dynamic Stark shifts for the two levels, developing a relative phase in the superposition. The atomic spins are, thus, coherently rotated in the xy plane as a back action of light on atoms.

$$\hat{H}_{\rm int} = \sum_{i=1}^{N_A} \frac{g^2}{\Delta} [\hat{a}_R^\dagger \hat{a}_R |1\rangle_{ii}\langle 1| + \hat{a}_L^\dagger \hat{a}_L |2\rangle_{ii}\langle 2|], \tag{44b}$$

where N_A is the number of atoms and the electric dipole coupling constant for the single-photon transitions is

$$g = \sqrt{\frac{\omega}{2V}} |\langle 1|\hat{\mathbf{d}}\cdot\boldsymbol{\epsilon}_R|4\rangle| = \sqrt{\frac{\omega}{2V}} |\langle 2|\hat{\mathbf{d}}\cdot\boldsymbol{\epsilon}_L|3\rangle|. \tag{45}$$

Note that Eq. (44b) is nothing more than the dynamic Stark shift (light shift) of the energy of the lower levels caused by virtual transitions to the off-resonant upper levels.

Let us introduce the quantum mechanical Stokes parameters to describe the polarization state of light,

$$\hat{S}_x \equiv (\hat{a}_R^\dagger \hat{a}_L + \hat{a}_L^\dagger \hat{a}_R)/2 = (\hat{a}_x^\dagger \hat{a}_x - \hat{a}_y^\dagger \hat{a}_y)/2,$$

$$\hat{S}_y \equiv (\hat{a}_R^\dagger \hat{a}_L - \hat{a}_L^\dagger \hat{a}_R)/(2i) = (\hat{a}_x^\dagger \hat{a}_y + \hat{a}_y^\dagger \hat{a}_x)/2,$$

$$\hat{S}_z \equiv (\hat{a}_R^\dagger \hat{a}_R - \hat{a}_L^\dagger \hat{a}_L)/2 = (\hat{a}_x^\dagger \hat{a}_y - \hat{a}_y^\dagger \hat{a}_x)/(2i),$$

$$\hat{N}_S \equiv (\hat{a}_R^\dagger \hat{a}_R + \hat{a}_L^\dagger \hat{a}_L)/2 = (\hat{a}_x^\dagger \hat{a}_x + \hat{a}_y^\dagger \hat{a}_y)/2. \tag{46}$$

$\hat{S}_x$ is half the photon number (intensity) difference of the x and y linearly polarized light components, $\hat{S}_y$ is that of the diagonally polarized ones, $\hat{S}_z$ corresponds to the difference in the right and left circularly polarized components, while $\hat{N}_S$ is half of the total photon number in the two modes. It is easy to see that they satisfy the standard angular momentum commutation relations $[\hat{S}_\alpha, \hat{S}_\beta] = i\epsilon_{\alpha\beta\gamma}\hat{S}_\gamma$ and $[\hat{N}_S, \hat{\mathbf{S}}] = \mathbf{0}$. The annihilation operators $\hat{a}_x = (\hat{a}_R + \hat{a}_L)/\sqrt{2}$ and $\hat{a}_y = (\hat{a}_R - \hat{a}_L)/(i\sqrt{2})$ correspond to the linearly polarized components of the light beam. Note that $\hat{\mathbf{S}}$ is not a vector operator, i.e., it does not transform as a vector under rotations.

For the atomic magnetic moment, we can use Schwinger's representation and introduce the collective atomic quasispin variables,

$$\hat{\sigma}_x \equiv \sum_i (|2\rangle_{ii}\langle 1| + |1\rangle_{ii}\langle 2|)/2,$$

$$\hat{\sigma}_y \equiv \sum_i (|2\rangle_{ii}\langle 1| - |1\rangle_{ii}\langle 2|)/2i,$$

$$\hat{\sigma}_z \equiv \sum_i (|2\rangle_{ii}\langle 2| - |1\rangle_{ii}\langle 1|)/2,$$

$$\hat{N}_\sigma \equiv \sum_i (|2\rangle_{ii}\langle 2| + |1\rangle_{ii}\langle 1|)/2, \tag{47}$$

where $2\hat{N}_\sigma$ is equal to the number of atoms in the subspace spanned by $|1\rangle$ and $|2\rangle$, i.e., $\hat{N}_\sigma$ counts the atoms contributing to the phenomenon. Note that unless we have a real spin system, the quasispin vector is not a vector operator either because $[\hat{\sigma}_\alpha, \hat{F}_\beta] \neq i\epsilon_{\alpha\beta\gamma}\hat{\sigma}_\gamma$ with $\hat{\mathbf{F}}$ being the total angular momentum. By definition, the components satisfy the commutation relations

$$[\hat{\sigma}_\alpha, \hat{\sigma}_\beta] = i\epsilon_{\alpha\beta\gamma}\hat{\sigma}_\gamma. \tag{48}$$

We are now ready to express the effective interaction Hamiltonian (44b) in terms of the Stokes and quasispin vectors,

$$\hat{H}_0 = \omega(2\hat{N}_S + 1) + 2E_0\hat{N}_\sigma, \tag{49a}$$

$$\hat{H}_{\rm int} = \frac{2g^2}{\Delta}(\hat{N}_S\hat{N}_\sigma - \hat{S}_z\hat{\sigma}_z). \tag{49b}$$

The operators $\hat{N}_S$, $\hat{N}_\sigma$, and $\hat{N}_S\hat{N}_\sigma$ all commute with both $\hat{\mathbf{S}}$ and $\hat{\boldsymbol{\sigma}}$ (and thus, also with $\hat{S}_z\hat{\sigma}_z$) and they have nothing to do with the dynamics. Therefore, we can safely omit them and write

$$\hat{H}_0 = \text{const}, \quad \hat{H}_{\rm int} = -\frac{2g^2}{\Delta}\hat{S}_z\hat{\sigma}_z. \tag{50}$$

The remaining term explains paramagnetic Faraday rotation [35]. If the z component of the collective atomic quasispin has a macroscopic expectation value, it results in an interaction Hamiltonian proportional to $\hat{S}_z$, which in turn introduces rotation of the Stokes vector along the z axis—thus turning the linear polarization in the xy plane. Conversely, as a back action of light on atoms, if the z component of the Stokes vector has a macroscopic expectation value, it will rotate coherently each atomic spin along the z axis.

Now we show—following Ref. [12]—that Eq. (50) gives rise to a Hamiltonian of the form (20). Consider an atomic ensemble in which all atoms are initially prepared in the superposition state $(|1\rangle + |2\rangle)/\sqrt{2}$. In this coherent spin state (CSS), the x component of the quasispin vector has a macroscopic expectation value $\langle\hat{\sigma}_x\rangle_0 = N_A/2$. This state is actually a quasi-spin-$\frac{N_A}{2}$ state, i.e., an eigenstate of $\hat{\boldsymbol{\sigma}}^2$ with eigenvalue $\frac{N_A}{2}(\frac{N_A}{2}+1)$. Since the Hamiltonian (49) commutes with

$\hat{\boldsymbol{\sigma}}^2$, the atomic state will always remain a quasi-spin-$\frac{N_A}{2}$ state. Moreover, as long as the interaction introduces small perturbation to $\hat{\boldsymbol{\sigma}}$, the y and z components of $\hat{\boldsymbol{\sigma}}$ stay small with respect to the x component. Therefore, we can use the Holstein-Primakoff approximation [36] and introduce the position and momentumlike quadrature operators

$$\hat{X}_A \equiv \hat{\sigma}_y/\sqrt{\langle\hat{\sigma}_x\rangle_0}, \quad \hat{P}_A \equiv \hat{\sigma}_z/\sqrt{\langle\hat{\sigma}_x\rangle_0}. \tag{51}$$

As long as the number of atomic excitations $\hat{n}_A=\frac{1}{2}(\hat{X}_A^2+\hat{P}_A^2-1)$ is small compared to the number of atoms, the deviation of the quasispin vector from the CSS stays in the tangent plane of the spin sphere. Then Eq. (48) ensures that we have the approximately correct commutation relation

$$[\hat{X}_A,\hat{P}_A]=i-\frac{2i}{N_A}\hat{n}_A \approx i. \tag{52}$$

In the experiment of [12], the quantum information is represented in the y-polarized weak signal beam propagating in the z direction and it is mixed with the copropagating x-polarized strong classical control field having coherent amplitude α (Fig. 2). Writing the c-number α in place of $\hat{a}_x$, we find that the x component of the Stokes vector (46) has a macroscopic expectation value, $\langle\hat{S}_x\rangle=|\alpha|^2/2$ in first order. Therefore, we can introduce the light quadrature variables as

$$\hat{X}_L \equiv \hat{S}_y/\sqrt{\langle\hat{S}_x\rangle_0}=(e^{-i\varphi}\hat{a}_y+e^{i\varphi}\hat{a}_y^\dagger)/\sqrt{2},$$

$$\hat{P}_L \equiv \hat{S}_z/\sqrt{\langle\hat{S}_x\rangle_0}=(e^{-i\varphi}\hat{a}_y-e^{i\varphi}\hat{a}_y^\dagger)/i\sqrt{2}, \tag{53}$$

where $\varphi=\arg(\alpha)$ is the complex phase of α. In summary we express the Stokes and quasispin vectors in terms of the quadrature operators [cf. Eqs. (46) and (47)],

$$\hat{S}_x=|\alpha|^2/2-\hat{n}_L, \quad \hat{\sigma}_x=N_A/2-\hat{n}_A,$$

$$\hat{S}_y=\hat{X}_L\sqrt{|\alpha|^2/2}, \quad \hat{\sigma}_y=\hat{X}_A\sqrt{N_A/2},$$

$$\hat{S}_z=\hat{P}_L\sqrt{|\alpha|^2/2}, \quad \hat{\sigma}_z=\hat{P}_A\sqrt{N_A/2},$$

$$\hat{N}_S=|\alpha|^2/2, \quad \hat{N}_\sigma=N_A/2. \tag{54}$$

Note that these operators exactly satisfy the angular momentumlike commutation relations $[\hat{N}_\sigma,\hat{\boldsymbol{\sigma}}]=\mathbf{0}$, $[\hat{\sigma}_\alpha,\hat{\sigma}_\beta]=i\epsilon_{\alpha\beta\gamma}\hat{\sigma}_\gamma$, $[\hat{N}_S,\hat{\mathbf{S}}]=\mathbf{0}$, and $[\hat{S}_\alpha,\hat{S}_\beta]=i\epsilon_{\alpha\beta\gamma}\hat{S}_\gamma$, despite the fact that we used an oscillator approximation for the atomic system and a classical model for the x-polarized light.

Putting Eqs. (51) and (53) into Eq. (50) we arrive at the interaction Hamiltonian in terms of the atomic and light quadrature operators,

$$\hat{H}_{\rm int}=-\frac{g^2|\alpha|\sqrt{N_A}}{\Delta}\hat{P}_L\hat{P}_A. \tag{55}$$

We recover the nonideal interaction Hamiltonian (20). As a consequence, the atomic position quadrature $\hat{X}_A$ is displaced by an amount proportional to $\hat{P}_L$ [cf. Eq. (21)], thus rotating the quasispin vector towards the y axis.

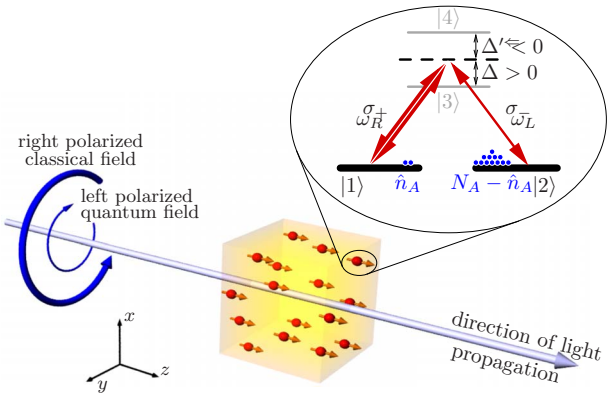

FIG. 3. (Color online) Quantum memory scheme based on off-resonant Raman scattering. The inset shows the Λ-type atomic configuration. The degenerate ground states $|1\rangle$ and $|2\rangle$ are off-resonantly coupled to the intermediate levels $|3\rangle$ and $|4\rangle$ via right and left circularly polarized monochromatic light beams. Light shifts of the lower levels are canceled by tuning the laser fields right between the two upper levels. The classical control field is the right polarized one. Only level $|2\rangle$ is macroscopically populated, which corresponds to a quasispin polarization in the direction of light propagation. Absorption of a signal photon results in a collective atomic excitation in level $|1\rangle$.

A quantum memory mapping without additional squeezing can be achieved by setting $\kappa=1$ in Eq. (21). Since the interaction strength is proportional to the amplitude $\alpha(t)$ of the classical control field, the area of coupling κ can be directly controlled in an experiment. In our simplified model involving only a single light mode and a single spin wave mode,

$$\kappa(t)=-\frac{g^2\sqrt{N_A}}{\Delta}\int_0^t d\tau|\alpha(\tau)|. \tag{56}$$

To compare with the experiment of [12], for a control pulse of length $L\sim c\times 1$ ms containing $N_P\sim 10^{12}$ photons (corresponding to an optical power of 0.5 mW), the integral in Eq. (56) gives $L\sqrt{N_P}/c$. We can express the coupling constant as $g^2=\frac{3}{8\pi}c\lambda^2\gamma/V$, where $\gamma\sim 5$ MHz is the natural linewidth of the excited levels and $V=LA$ is the light quantization volume. Taking $A\sim 2$ cm^2 as the cross-section area of the beam (which is comparable to the size of the gas cells) and $N_A\sim 10^{11}$ atoms, we obtain an area of coupling of unit magnitude,

$$\kappa=-\frac{3}{8\pi}\frac{\lambda^2\gamma}{\Delta A}\sqrt{N_A N_P}\sim 1. \tag{57}$$

B. Off-resonant Raman scheme

Now we discuss another physical system which directly realizes the ideal interaction Hamiltonian (18). Our quantum memory scheme is based on off-resonant Raman scattering by Λ-type atoms [14].

Consider an ensemble of N_A atoms depicted in Fig. 3 that is shined by a monochromatic light beam consisting of copropagating phase-locked right and left circularly polarized

components. Levels $|1\rangle$ and $|2\rangle$ can be, for example, the $m_F=\pm 1$ Zeeman sublevels of the $^2S_{1/2}(F=1)$ ground state of sodium or rubidium (with nuclear spin $I=3/2$), while levels $|3\rangle$ and $|4\rangle$ are the $m_F=0$ Zeeman sublevels of the hyperfine levels $^2P_{1/2}(F=1)$ and $^2P_{1/2}(F=2)$. Although the Zeeman sublevels $M_F=\pm 2$ of the uppermost level $^2P_{1/2}(F=2)$ also contribute to the light shifts, the principle of the model is not changed.

If the atomic transitions are far off-resonant, the four-level atom is reduced to an effective two-level one. After adiabatic elimination of the upper levels, the dynamics is governed by an effective interaction Hamiltonian that consists of dynamic Stark shifts and two-photon processes between the ground states,

$$\hat{H}_{\rm int}=\sum_{i=1}^{N_A}\left\{\left(\frac{|g_{3R}^i|^2}{\Delta}+\frac{|g_{4R}^i|^2}{\Delta'}\right)\hat{a}_R^\dagger\hat{a}_R|1\rangle_{ii}\langle 1|+\left(\frac{|g_{3L}^i|^2}{\Delta}+\frac{|g_{4L}^i|^2}{\Delta'}\right)\hat{a}_L^\dagger\hat{a}_L|2\rangle_{ii}\langle 2|+\left[\left(\frac{g_{3R}^i g_{3L}^{i*}}{\Delta}+\frac{g_{4R}^i g_{4L}^{i*}}{\Delta'}\right)\hat{a}_L^\dagger\hat{a}_R|2\rangle_{ii}\langle 1|+\text{H.c.}\right]\right\}, \tag{58}$$

where the electric dipole coupling constants for the single-photon transitions are

$$g_{3R}^i=\sqrt{\frac{\omega_R}{2V}}\langle 1|\hat{\mathbf{d}}\cdot\boldsymbol{\epsilon}_R|3\rangle e^{i(\mathbf{k}_R\cdot\mathbf{r}_i+\phi_R)}, \tag{59}$$

and similarly for g_{3L}^i, g_{4R}^i, and g_{4L}^i, with $\mathbf{r}_i$ being the position of the ith atom. For symmetry reasons, we have $|g_{3R}^i|=|g_{3L}^i|\equiv g$ and $|g_{4R}^i|=|g_{4L}^i|\equiv g'$. Furthermore, since the frequencies and propagation directions of the two polarized light beams coincide, we can choose the relative phase ϕ_R-ϕ_L of the mode functions of the light modes so that $g_{3R}^i g_{3L}^{i*}=g^2$ is real and positive. However, in our example of sodium or rubidium, g_{4R}^i and g_{4L}^i have opposite signs for this choice of phases, so $g_{4R}^i g_{4L}^{i*}=-g'^2$.

We can introduce the photonic Stokes vector $\hat{\mathbf{S}}$ (46) and the collective atomic quasispin vector $\hat{\sigma}$ (47). If the ground states are $F=1$ hyperfine sublevels, the components of the quasispin expressed by the total angular momentum $\hat{\mathbf{F}}$ are

$$\hat{\sigma}_x=\sum_i\frac{1}{2}(\hat{F}_x^{(i)2}-\hat{F}_y^{(i)2}),$$

$$\hat{\sigma}_y=\sum_i\frac{1}{2}(\hat{F}_x^{(i)}\hat{F}_y^{(i)}+\hat{F}_y^{(i)}\hat{F}_x^{(i)}),$$

$$\hat{\sigma}_z=\hat{F}_z/2. \tag{60}$$

We are now ready to simplify the interaction Hamiltonian (58). We realize that the first two terms therein are proportional to $\hat{N}_S\hat{N}_\sigma-\hat{S}_z\hat{\sigma}_z$ and they are responsible for paramagnetic Faraday rotation. Since $\hat{N}_S\hat{N}_\sigma$ commutes with both $\hat{\mathbf{S}}$ and $\hat{\sigma}$ and has nothing to do with the dynamics, we will omit it in the following. The last term in Eq. (58) is proportional to $\hat{S}_x\hat{\sigma}_x+\hat{S}_y\hat{\sigma}_y$. All in all, we have

$$\hat{H}_{\rm int}=-2\left(\frac{g^2}{\Delta}+\frac{g'^2}{\Delta'}\right)\hat{S}_z\hat{\sigma}_z+2\left(\frac{g^2}{\Delta}-\frac{g'^2}{\Delta'}\right)(\hat{S}_x\hat{\sigma}_x+\hat{S}_y\hat{\sigma}_y). \tag{61}$$

We note that the effective Hamiltonian is essentially of the same form even for more general configurations involving multiple atomic levels [37,38]. The meaning of the first term is explained in Sec. III A. By tuning the laser fields right between the two upper levels so that $g'^2/\Delta'=-g^2/\Delta$, we can cancel the first term. The remaining term then describes the two-photon processes of the atoms making transitions between the two ground states. However, this term can also be interpreted as Faraday rotation. Suppose, for example, that the y component of the Stokes vector has a macroscopic expectation value, that is, the atomic ensemble is irradiated by a single 45° linear polarized classical beam. Then the interaction Hamiltonian (61) reduces to a term proportional to $\hat{\sigma}_y$ corresponding to a coherent rotation of the quasispin along the y axis. Indeed, if we choose y as the quantization axis, such a diagonally polarized light will induce virtual atomic transitions from the superposition state $(|1\rangle+i|2\rangle)/\sqrt{2}$ to the off-resonant upper levels, thus shifting the energy level of this superposition state with respect to the orthogonal state $(|1\rangle-i|2\rangle)/\sqrt{2}$. As a consequence, all the atomic quasispins are rotated along the y axis. Appropriately choosing the polarization and detuning of a single classical light field, one can realize rotation of the quasispin vector along an *arbitrary* axis.

To use the present configuration as a quantum memory, the polarizations of the control and signal fields and the initial atomic state should be chosen differently from that of Sec. III A. Let us represent the quantum information in the weak left circularly polarized light beam (signal) and let the right circularly polarized light be the strong classical field (control) with coherent amplitude α (though not too strong so that it remains off-resonant, $|\alpha|\ll|\Delta/g|$, $|\Delta'/g'|$). We can write the c-number α instead of the right circularly polarized annihilation operator $\hat{a}_R$ and we find that the z component of the Stokes vector is a classical variable $\langle\hat{S}_z\rangle_0=|\alpha|^2/2$. This enables us to introduce the light quadrature variables as

$$\hat{X}_L\equiv\hat{S}_x/\sqrt{|\alpha|^2/2}=(e^{-i\varphi}\hat{a}_L+e^{i\varphi}\hat{a}_L^\dagger)/\sqrt{2},$$

$$\hat{P}_L\equiv\hat{S}_y/\sqrt{|\alpha|^2/2}=(e^{-i\varphi}\hat{a}_L-e^{i\varphi}\hat{a}_L^\dagger)/i\sqrt{2}, \tag{62}$$

where $\varphi=\arg(\alpha)$ is the phase of α. Similarly, we can introduce quadratures for the atomic ensemble if the collective atomic state stays close to the coherent spin state (CSS) in which all atoms are in level $|2\rangle$. The z component of the quasispin has a macroscopic expectation value, $\langle\hat{\sigma}_z\rangle_0=N_A/2$ in zeroth order, and we can approximate it with $\hat{\sigma}_z=N_A/2-\hat{n}_A$. Then we can write the atomic quadrature variables as

$$\hat{X}_A\equiv\frac{\hat{\sigma}_x}{\sqrt{N_A/2}},\quad \hat{P}_A\equiv\frac{\hat{\sigma}_y}{\sqrt{N_A/2}}. \tag{63}$$

In order to keep the correct angular momentum commutation relations of the Stokes and quasispin vectors, we write

$$\hat{S}_x = \hat{X}_L\sqrt{|\alpha|^2/2}, \quad \hat{\sigma}_x = \hat{X}_A\sqrt{N_A/2},$$

$$\hat{S}_y = \hat{P}_L\sqrt{|\alpha|^2/2}, \quad \hat{\sigma}_y = \hat{P}_A\sqrt{N_A/2},$$

$$\hat{S}_z = |\alpha|^2/2 - \hat{n}_L, \quad \hat{\sigma}_z = N_A/2 - \hat{n}_A,$$

$$\hat{N}_S = |\alpha|^2/2, \quad \hat{N}_\sigma = N_A/2. \tag{64}$$

Note that the number of atomic excitations $\hat{n}_A = \frac{1}{2}(\hat{X}_A^2 + \hat{P}_A^2 - 1)$ is equal to the population of level $|1\rangle$, $\hat{n}_A = \sum_i |1\rangle_{ii}\langle 1|$. Putting all together while canceling the term $\hat{S}_z\hat{\sigma}_z$ in Eq. (61), we obtain the interaction Hamiltonian

$$\hat{H}_{\text{int}} = \frac{2g^2|\alpha|\sqrt{N_A}}{\Delta}(\hat{X}_L\hat{X}_A + \hat{P}_L\hat{P}_A). \tag{65}$$

The Hamiltonian (65) results in an oscillation of the excitations between modes A and L. Indeed, with the creation and annihilation operators of systems A and L we recognize the beam splitter Hamiltonian

$$\hat{X}_L\hat{X}_A + \hat{P}_L\hat{P}_A = \hat{a}_L^\dagger\hat{a}_A + \hat{a}_A^\dagger\hat{a}_L. \tag{66}$$

Absorption of a left polarized photon makes an atomic transition from level $|2\rangle$ to level $|1\rangle$, and conversely, emission of such a photon causes a decrease in the population $\hat{n}_A$ of level $|1\rangle$.

The state of the two systems can be exchanged completely by appropriately adjusting the amplitude $\alpha(t)$ of the classical right circularly polarized control field, so that we have a $\pi/2$ pulse,

$$\kappa = \frac{2g^2\sqrt{N_A}}{\Delta}\int |\alpha(t)|dt = \frac{\pi}{2}. \tag{67}$$

This leads to the ideal quantum memory mapping (19).

C. Resonant EIT scheme

In quantum memories based on EIT [19], an intense classical radiation field (control) and a weak quantum field to be stored (signal) are adjusted on or near resonance with the transitions of the Λ-type atoms (see Fig. 4). In this subsection we show how to adiabatically eliminate the resonant excited level and we derive a beam-splitter-like effective Hamiltonian between the signal light field and the collective coherences of the lower atomic levels. In order to simplify the discussion, we will restrict ourselves to single-mode radiation fields for the control and signal with exact two-photon resonance and standing polariton wave. The Hamiltonian in the rotating frame then reads

$$\hat{H} = \sum_{i=1}^{N_A}[-\Delta|3\rangle_{ii}\langle 3| + (g_i\hat{a}_L|3\rangle_{ii}\langle 2| + \Omega_i|3\rangle_{ii}\langle 1| + \text{H.c.})], \tag{68}$$

where $\hat{a}_L$ is the bosonic operator of the signal field, and Δ is the one-photon detuning for both transitions. Furthermore,

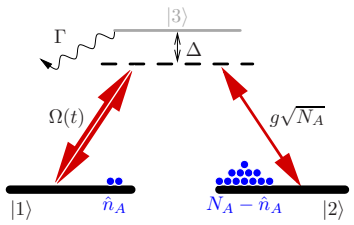

FIG. 4. (Color online) Λ-type atomic configuration in the EIT scheme. Levels $|1\rangle$ and $|3\rangle$ are coupled by the classical field of Rabi frequency $\Omega(t)$ which is controlled externally. Levels $|2\rangle$ and $|3\rangle$ are coupled by the weak signal field. Initially, only level $|2\rangle$ is populated.

we disregard atomic motion and assume that the coupling constants g_i are real and the same for all atoms, and so are the Rabi frequencies Ω_i. This allows us to introduce collective spin operators

$$\hat{\sigma}_{ab} \equiv \sum_i |a\rangle_{ii}\langle b| \quad (a,b = 1,2,3). \tag{69}$$

Due to the symmetry of the Hamiltonian (68), only the totally symmetric Dicke states are coupled to the light fields. The totally symmetric state containing n atoms on level $|1\rangle$, m atoms on level $|2\rangle$, and l on level $|3\rangle$ is defined as

$$|n_1,m_2,l_3\rangle \equiv [n!m!l!(n+m+l)!]^{-1/2} \times \sum |1{:}i_1,\ldots,i_n;2{:}j_1,\ldots,j_m;3{:}k_1,\ldots,k_l\rangle, \tag{70}$$

where the summation is over the $N_A = n+m+l$ mutually different variables $i_1,\ldots,i_n,j_1,\ldots,j_m,k_1,\ldots,k_l$ going from 1 to N_A. The ket vector at the right of Eq. (70) represents the atomic product state in which atoms indexed by $i_1,\ldots,i_n$ are in state $|1\rangle$, etc. It is easy to verify that the spin-flip operators (69) keep the symmetry of these states,

$$\hat{\sigma}_{ab}|n_a,m_b,l_c\rangle = \sqrt{(n+1)m}|(n+1)_a,(m-1)_b,l_c\rangle,$$

$$\hat{\sigma}_{aa}|n_a,m_b,l_c\rangle = n|n_a,m_b,l_c\rangle, \tag{71}$$

with $\{a,b,c\}=\{1,2,3\}$. We can see that for each value of $n=0,\ldots,N_A$, the set

$$\mathcal{S}_n \equiv \text{Span}\{|(k-l)_1,(N_A-k)_2,l_3\rangle \otimes |n-k\rangle_L | k=0,\ldots,n, \text{ and } l=0,\ldots,k\} \tag{72}$$

is an invariant subspace of the Hamiltonian (68) so that Eq. (68) is block diagonal. (The subscript L means that the corresponding Fock state refers to the signal light mode.)

Now we make use of the fact that only the atomic level $|2\rangle$ is macroscopically populated, so $\hat{\sigma}_{22}$ can be substituted by the c-number N_A. We observe that only the coherences $\hat{\sigma}_{21}$ and $\hat{\sigma}_{23}$ are important, as we can express the other spin operators as $\hat{\sigma}_{11} = \hat{\sigma}_{12}\hat{\sigma}_{21}/N_A$, $\hat{\sigma}_{33} = \hat{\sigma}_{32}\hat{\sigma}_{23}/N_A$, and $\hat{\sigma}_{13} = \hat{\sigma}_{12}\hat{\sigma}_{23}/N_A$. Compared to the single atomic oscillator in the off-resonant Raman process investigated in Sec. III B, now we have two atomic oscillator modes with annihilation operators $\hat{\sigma}_{21}/\sqrt{N_A}$ and $\hat{\sigma}_{23}/\sqrt{N_A}$, respectively. Within the subspace $\mathcal{S} \equiv \oplus_{n\ll N_A}\mathcal{S}_n$ of photonic and totally symmetric atomic

states with an arbitrary but small number of excitations n, the Hamiltonian can be written in the matrix form [39]

$$\hat{H} = \hat{\mathbf{z}}^\dagger \mathbf{H} \hat{\mathbf{z}}, \quad \mathbf{H} \equiv \begin{pmatrix} 0 & 0 & g\sqrt{N_A} \\ 0 & 0 & \Omega \\ g\sqrt{N_A} & \Omega & -\Delta \end{pmatrix}. \tag{73}$$

Here we have introduced the vector notation $\hat{\mathbf{z}}^\dagger \equiv (\hat{a}_L^\dagger, \hat{\sigma}_{12}/\sqrt{N_A}, \hat{\sigma}_{32}/\sqrt{N_A})$ for the creation operators of the photonic mode and the two atomic oscillator modes. Note that in the limit of small atomic excitation, components of $\hat{\mathbf{z}}$ and $\hat{\mathbf{z}}^\dagger$ approximately satisfy bosonic commutation relations. The matrix $\mathbf{H}$ can be brought to a block diagonal form. The corresponding transformation matrix $\mathbf{R}$ defines the quantum field variables (annihilation operators) of the so-called dark- and bright-state polaritons [18,19,40],

$$\hat{\mathbf{z}}' \equiv \begin{pmatrix} \hat{\Psi} \\ \hat{\Phi} \\ \hat{\Xi} \end{pmatrix} \equiv \mathbf{R}\hat{\mathbf{z}}, \quad \mathbf{R} = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{74}$$

where $\hat{\Psi}$, $\hat{\Phi}$, and $\hat{\Xi}$ stand for the dark-polariton, the bright-polariton, and excited-state modes, respectively. The mixing angle θ is defined as

$$\tan\theta \equiv \frac{g\sqrt{N_A}}{\Omega}. \tag{75}$$

Note that in the limit of small atomic excitations, the polariton operators also satisfy the bosonic commutation relations,

$$[\hat{\Psi}, \hat{\Psi}^\dagger] \approx [\hat{\Phi}, \hat{\Phi}^\dagger] \approx [\hat{\Xi}, \hat{\Xi}^\dagger] \approx 1,$$

$$[\hat{\Psi}, \hat{\Phi}^\dagger] \approx [\hat{\Psi}, \hat{\Xi}^\dagger] \approx [\hat{\Phi}, \hat{\Xi}^\dagger] \approx 0,$$

$$[\hat{\Psi}, \hat{\Phi}] = [\hat{\Psi}, \hat{\Xi}] = [\hat{\Phi}, \hat{\Xi}] = 0. \tag{76}$$

Actually, the rotation (74) corresponds to switching from the Heisenberg picture to a rotating axes representation. The generator of the unitary transformation is the Hermitian operator

$$\hat{K} \equiv i(\hat{\Psi}^\dagger \hat{\Phi} - \hat{\Phi}^\dagger \hat{\Psi}) = i(\hat{a}_L^\dagger \hat{\sigma}_{21} - \hat{a}_L \hat{\sigma}_{12})/\sqrt{N_A}. \tag{77}$$

Operators in the rotating axes representation are obtained from those in the Heisenberg picture in the following way:

$$\hat{\mathbf{z}}' \equiv e^{-i\theta\hat{K}} \hat{\mathbf{z}} e^{i\theta\hat{K}}. \tag{78}$$

The equations of motion for the rotating polariton variables are

$$\frac{d}{dt}\hat{\mathbf{z}}' = \frac{d}{dt}(\mathbf{R}\hat{\mathbf{z}}) = i[\hat{H}, \mathbf{R}\hat{\mathbf{z}}] + \left(\frac{d}{dt}\mathbf{R}\right)\hat{\mathbf{z}} = i[\hat{\tilde{H}}, \hat{\mathbf{z}}'], \tag{79}$$

with $\hat{\tilde{H}} \equiv \hat{\mathbf{z}}'^\dagger \tilde{\mathbf{H}} \hat{\mathbf{z}}'$,

$$\hat{\tilde{H}} = -\Delta\hat{\Xi}^\dagger\hat{\Xi} + W(\hat{\Phi}^\dagger\hat{\Xi} + \hat{\Xi}^\dagger\hat{\Phi}) - i\dot{\theta}(\hat{\Psi}^\dagger\hat{\Phi} - \hat{\Phi}^\dagger\hat{\Psi}),$$

$$\tilde{\mathbf{H}} = \mathbf{R}\mathbf{H}\mathbf{R}^{-1} + i\dot{\mathbf{R}}\mathbf{R}^{-1} = \begin{pmatrix} 0 & -i\dot{\theta} & 0 \\ i\dot{\theta} & 0 & W \\ 0 & W & -\Delta \end{pmatrix}, \tag{80}$$

where $W \equiv \sqrt{g^2 N_A + \Omega^2}$.

We immediately recognize that in the adiabatic limit $\dot{\theta} \ll W$, the dark-state polaritons are decoupled from the dynamics. Moreover, they do not involve the excited atomic state $|3\rangle$ and hence are immune to spontaneous emission. Therefore, if the initial state of the system consists of dark-state polaritons only and the mixing angle θ varies slowly enough, then the quantum state of the system adiabatically follows the smoothly changing dark eigenstates of the Hamiltonian, so the system stays in the same superposition of dark states as it was initially in. (Note that the zero eigenvalue is nondegenerate in each invariant subspace $\mathcal{S}_n$, so there is no level crossing.) Thus, adiabatic rotation of the mixing angle θ from 0 to $\pi/2$ leads to a complete transfer of the photonic state to collective atomic excitations between levels $|1\rangle$ and $|2\rangle$. Indeed, for $\theta=0$, the dark-state polariton mode $\hat{\Psi}(\theta)$ coincides with the signal mode $\hat{a}_L$, while it solely corresponds to the collective spin excitation $\hat{\sigma}_{21}/\sqrt{N_A}$ for $\theta = \pi/2$. This is the standard way of interpreting the adiabatic EIT storage process [19,20].

The bright-state polaritons are coupled to the excited-state mode that decays by spontaneous emission with rate Γ. Now we identify two important limiting cases when these modes can be adiabatically eliminated.

1. Far off-resonant regime

In the first, the one-photon resonance is far detuned with respect to the Rabi frequency Ω of the control field, to the ensemble-enhanced vacuum Rabi frequency $g\sqrt{N_A}$ of the signal field, and also to the decay rate Γ of the excited states ($W \ll \Delta$ and $\Gamma \ll \Delta$). When we eliminate the excited-state mode, we find an effective decay γ_B of the bright-state polaritons and a shift ω_B in their energy,

$$\gamma_B \approx \frac{W^2\Gamma}{\Delta^2} \ll \omega_B \approx \frac{W^2}{\Delta}, \tag{81}$$

so the Hamiltonian (80) in the rotating axes representation reduces to

$$\hat{\tilde{H}}_{\text{eff}} = \omega_B \hat{\Phi}^\dagger\hat{\Phi} - i\dot{\theta}(\hat{\Psi}^\dagger\hat{\Phi} - \hat{\Phi}^\dagger\hat{\Psi}). \tag{82}$$

Switching back to the Heisenberg picture with nonrotating light and atomic variables, we obtain the following Hamilton operator:

$$\hat{H} = \frac{g^2 N_A}{\Delta}\hat{a}_L^\dagger\hat{a}_L + \frac{\Omega^2}{\Delta}\frac{\hat{\sigma}_{12}\hat{\sigma}_{21}}{N_A} + \frac{g\sqrt{N_A}\Omega}{\Delta}\left(\hat{a}_L^\dagger\frac{\hat{\sigma}_{21}}{\sqrt{N_A}} + \frac{\hat{\sigma}_{12}}{\sqrt{N_A}}\hat{a}_L^\dagger\right). \tag{83}$$

The first two terms are ac Stark shifts for the signal mode interacting with the atoms in level $|2\rangle$ and for atoms in level $|1\rangle$ interacting with the intense control field, respectively. These terms were eliminated in Sec. III B by considering a

manifold of excited states and tuning the laser fields to a point where their ac Stark contributions exactly compensate. The third term in Eq. (83) coincides with Eq. (65) if we take $\Omega = g|\alpha|$ (the factor 2 is due to the fact that there we had two upper levels). Finally, we mention that the characteristic time T of the write process is determined by Eq. (67) and is of the order $T \sim \Delta / W^2$. Equation (81) then implies that accumulated losses are of the order $\gamma_B T \sim \Gamma / \Delta$ and can be safely neglected.

2. Resonant regime

The ensemble-enhanced coupling between bright-state polaritons and excited states—given by the effective Rabi frequency W—is usually the dominating term in Eq. (80) near one-photon resonance ($\Delta \ll W$). The decay of the bright-state polaritons thus become fast on the time scale of the dark-state polariton dynamics. Even if the initial state contained some bright polaritons—that is, the atomic ensemble was only partially polarized at $\theta = 0$ in the beginning of the write process, or the signal light mode was not in the vacuum state at $\theta = \frac{\pi}{2}$ in the beginning of the readout—these bright polaritons would rapidly vanish. It can be interpreted as optical pumping towards the dark states for the write process, or as absorption of the probe light by the not-yet-transparent medium for the readout. It is, therefore, enough to consider a restricted subspace of the Hilbert space which contains excitations neither in the bright-state polariton mode nor in the excited-state mode.

We can also formally eliminate both these modes, if the dark-bright coupling is much smaller than the inverse time scale of the bright dynamics, $\dot{\theta} \ll W$ (condition of adiabaticity). Then we obtain an energy shift ω_D and an effective decay γ_D for the dark-state polaritons,

$$\omega_D = -\frac{\Delta \dot{\theta}^2}{W^2}, \quad \gamma_D = \frac{\Gamma \dot{\theta}^2}{W^2}. \tag{84}$$

These nonadiabatic corrections can be neglected on the characteristic time $T \sim \dot{\theta}^{-1}$ of the write (or readout) process. As a consequence, the dynamics in the rotating axes representation is entirely described by the Hamiltonian $\tilde{\hat{H}}_{\text{eff}} = 0$. However, when we change from the rotating axes representation back to the Heisenberg picture, the term neglected in the adiabatic approximation reappears. In the original basis of light and matter operators, this translates to

$$\mathbf{H}_{\text{eff}} = \mathbf{R}^{-1} \tilde{\mathbf{H}}_{\text{eff}} \mathbf{R} - i \dot{\mathbf{R}} \mathbf{R}^{-1} = \begin{pmatrix} 0 & i\dot{\theta} \\ -i\dot{\theta} & 0 \end{pmatrix},$$

and thus to

$$\hat{H}_{\text{eff}} = i\dot{\theta}(\hat{a}_L^\dagger \hat{\sigma}_{21} - \hat{a}_L \hat{\sigma}_{12}) / \sqrt{N_A}. \tag{85}$$

Introducing the atomic and light quadrature variables as

$$\hat{X}_A \equiv (\hat{\sigma}_{21} + \hat{\sigma}_{12}) / \sqrt{2N_A}, \quad \hat{X}_L \equiv (\hat{a}_L + \hat{a}_L^\dagger) / \sqrt{2},$$

$$\hat{P}_A \equiv (\hat{\sigma}_{21} - \hat{\sigma}_{12}) / i\sqrt{2N_A}, \quad \hat{P}_L \equiv (\hat{a}_L - \hat{a}_L^\dagger) / i\sqrt{2}, \tag{86}$$

we find that the effective Hamiltonian is

$$\hat{H}_{\text{eff}} = \dot{\theta}(\hat{X}_A \hat{P}_L - \hat{P}_A \hat{X}_L), \tag{87}$$

and we recover the ideal mapping Hamiltonian (16) with the restriction of no initial bright polaritons. Complete mapping from light to atoms or vice versa is achieved if the time dependence of the mixing angle is adjusted such that

$$\kappa \equiv \int_0^T d\tau \dot{\theta}(\tau) = \theta(T) - \theta(0) = \pm \frac{\pi}{2}. \tag{88}$$

In the experiment of [27], a kilometer-long signal pulse was first slowed down and spatially compressed to a few centimeters, so that the pulse completely fit into the room temperature rubidium gas cell. This corresponds to a group velocity reduction of $v_g / c = \cos^2 \theta_0 \sim 10^{-5}$ and—although the light had been almost completely mapped to the atomic spin wave at that moment—this is the starting point of the adiabatic quantum memory mapping. The mixing angle was then rotated from θ_0 to $\frac{\pi}{2}$ during a characteristic time of $T \sim 3\ \mu$s, corresponding to an angular velocity of the mixing angle $\dot{\theta} \sim 1$ kHz. Taking the experimental value of $W \approx g\sqrt{N_A} \sim 2$ GHz, we find that the condition of adiabaticity is well justified.

In a similar experiment [26] carried out using a cold cloud of sodium atoms trapped in a magnetic trap, the compression of the input pulse was more significant, $v_g / c = \cos^2 \theta_0 \sim 10^{-7}$. Then in the memory write stage, the control field was turned off during $T \sim 1\ \mu$s, corresponding to an angular velocity of the mixing angle $\dot{\theta} \sim 0.3$ kHz. From the initial pump Rabi frequency $\Omega_0 \sim 16$ MHz, we estimate an effective Rabi frequency $W = \Omega_0 / \cos \theta_0 \sim 50$ GHz, so the condition of adiabaticity is justified here as well.

IV. CONCLUSIONS

In the present paper we have investigated quantum memories for light in atomic ensemble using a continuous-variable description, i.e., position and momentum variables for light and matter degrees of freedom. In particular, we have studied in detail two types of off-resonant quantum memories: one based on the quantum Faraday effect supplemented by measurement and feedback, and another involving double-Λ type atoms and Raman scattering. For the first scheme we analyzed the effect of inefficiencies in the initial atomic state preparation and imperfections in the light measurement. We found that the fidelity of the one-pass memory scheme for storing superpositions of light states with very different momenta is rather low unless atomic state preparation can be performed with very high accuracy. In a two-pass scheme the necessity of an initial atomic spin squeezing can be avoided by light measurement and feedback. In a triple-pass configuration, atomic state preparation as well as feedback can be avoided altogether. Secondly, we have investigated an off-resonant Raman scheme and proposed a configuration in which unwanted light shifts can be canceled. Finally, we have discussed near-resonant quantum memories based on electromagnetically induced transparency in terms of position and momentum operators of light and matter. We have shown that this memory can be described by a Hamiltonian

corresponding to an ideal map provided that before the write and readout processes the atomic ensemble or, respectively, the radiation mode are in the appropriate initial vacuum state. In contrast to the common approach involving wave equations for the propagating polaritons, we have developed a Hamiltonian formalism for the atomic and photonic quadrature variables similar to the formalism of the first two families. Our results allow a straightforward comparison of the various continuous-variable quantum memory schemes in the same framework.

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