

# Dark-state polaritons for multicomponent and stationary light fields

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We present a general scheme to determine the loss-free adiabatic eigensolutions (dark-state polaritons) of the interaction of multiple probe laser beams with a coherently driven atomic ensemble under conditions of electromagnetically induced transparency. To this end we generalize the Morris-Shore transformation to linearized Heisenberg-Langevin equations describing the coupled light-matter system in the weak excitation limit. For the simple lambda-type coupling scheme the generalized Morris-Shore transformation reproduces the dark-state polariton solutions of slow light. Here we treat a closed-loop dual- $V$  scheme wherein two counter-propagating control fields generate a quasistationary pattern of two counterpropagating probe fields—so-called stationary light. We show that contrary to previous predictions, there exists a single unique dark-state polariton; it obeys a simple propagation equation.

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## I. INTRODUCTION

The adiabatic interaction of probe light pulses with coherently driven three-level systems under conditions of electromagnetically induced transparency (EIT) [1] can most conveniently be described in terms of so-called dark-state polaritons [2]. These are the eigensolutions of the Raman interaction of probe and control fields with the atomic ensemble in the absence of atomic losses. They provide a simple theoretical framework for phenomena such as slow light [3], the storage and retrieval of coherent light pulses in atomic ensemble [4,5], and quantum memories for photons [2,6]. Most importantly they also fully incorporate the essential physics of adiabatic pulse propagation for time-varying control fields together with the associated coherent transfer of excitation and quantum state from light to matter and vice versa. This transfer cannot be understood in terms of electromagnetic field equations alone.

Recently it has been shown that the simultaneous presence of two counterpropagating control fields of comparable strength can lead to a quasistationary pattern of slow light consisting of two counterpropagating probe field components [7–10]. Such stationary light pulses hold particular interest as examples of efficient nonlinear optical processes. In contrast to the three-level coupling scheme, there exists no unique description of stationary light in terms of dark-state polaritons. Reference [8,11] introduced separate polariton fields for forward and backward propagating modes. However, as we will show here, these are not the quasi-loss-less eigensolutions of the system.

We here determine the adiabatic eigensolutions of a coupled multilevel system using a generalization of the stationary-light scheme. As with simpler systems, the eigensolutions are superpositions of collective atomic operators and operators of the electromagnetic field. Our procedure relies on the possibility of casting the linearized Maxwell-Bloch equations into a matrix differential equation similar to that used in treating the time-dependent Schrödinger equation (TDSE) describing coherent excitation of a multilevel

atom. For that equation there exists a well-known formalism to determine possible nondecaying eigenstates, the Morris-Shore transformation [12]. Generalizing this transformation to the linear Heisenberg-Langevin equations of the coupled light-matter system we can easily determine their loss-free eigensolutions, when they exist. We apply this procedure to the stationary-light equations to find a dark-state polariton solution, and to derive its equation of propagation.

## II. THE DUAL-V STATIONARY-LIGHT SCHEME

Let us consider one-dimensional propagation in the  $z$  direction of two counterpropagating *probe fields*, circularly polarized orthogonally, having electric fields operators  $\hat{E}_{\pm}(z,t)$  and propagation vectors  $k_{\pm}$ . These interact with an ensemble of stationary four-state atoms via linkages between the ground state  $|g\rangle$  and excited states  $|e_{\pm}\rangle$ . A second pair of counterpropagating fields, also circularly polarized orthogonally and termed *control fields*, link these excited states to a fourth state  $|s\rangle$ , through interactions characterized by Rabi frequencies  $\hat{\Omega}_{\pm}(z,t)$ . Figure 1 illustrates the linkage pattern of the four fields with a four-state atom. The control fields are assumed to be sufficiently strong that they are not influ-

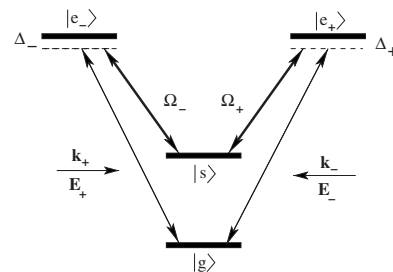


FIG. 1. The dual- $V$  linkage pattern shown together with the depicted field configuration of counter-propagating control ( $\Omega_{\pm}$ ) and induced probe fields ( $\hat{E}_{\pm}$ ) leads to stationary probe pulses.

enced by the atoms; unlike the probe fields, they are not considered as dynamical variables.

We assume that the control fields generate electromagnetically induced transparency (EIT) for the probe fields, by requiring that their Rabi frequencies are locked to those of the control fields. Thus if  $|\tilde{\Omega}_+|=|\tilde{\Omega}_-|$  the control fields form a standing-wave pattern and the group velocity of the probe fields vanish; these form a standing-wave pattern known as *stationary light* [7,9].

A stationary-light probe-field pulse can be created from a propagating-light probe field by adiabatically changing the relative intensity of the two control fields from zero to unity, thereby storing the probe pulse in a collective spin excitation [10]. The stored excitation can then be retrieved as a stationary pulse by using two counterpropagating control fields of equal intensity.

Stationary-light pulses were suggested and experimentally demonstrated for a  $\Lambda$ -type three-state configuration by Lukin and co-workers [7,9]. In contrast to the system discussed here, there the counterpropagating fields, whether probe or control field, acted upon the same transitions. A simple theoretical description of the  $\Lambda$ -type system of [9] is possible, by neglecting components of the atomic spin coherence that are rapidly oscillating in space. Such a secular approximation is only justifiable for a vapor at sufficiently high temperature, such that thermal motion of the atoms leads to a rapid dephasing of the rapidly oscillating components [8,11]. Here we avoid this dephasing approximation and consider the four-level generalization shown in Fig. 1. This model also produces stationary light but does not require any secular approximation.

We assume that the control fields have carrier frequencies  $\omega_{\pm}^{(c)}$  and wave vectors  $k_{\pm}^{(c)} = \pm k^{(c)}$  directed along the  $z$  axis. We take the carrier frequencies and wave vectors of the probe-field components to be  $\omega_{\pm}^{(p)}$  and  $k_{\pm}^{(p)} = \pm k^{(p)}$ , again along the  $z$  axis. We replace the basic parameters of the control fields  $\tilde{\Omega}_{\pm}(z, t)$  and the operators  $\hat{E}_{\pm}(z, t)$  of the probe fields by quantities  $\Omega_{\pm}(z, t) \equiv \tilde{\Omega}_{\pm}(z, t)e^{-i(k_{\pm}^{(c)}z - \omega_{\pm}^{(c)}t)}$  and  $\hat{E}_{\pm}(z, t) \equiv \hat{E}_{\pm}(z, t)e^{-i(k_{\pm}^{(p)}z - \omega_{\pm}^{(p)}t)}$  that vary slowly with  $z$  and  $t$ . We further require four-photon resonance and phase matching, as expressed by the constraints

$$\begin{aligned}\omega_+^{(c)} - \omega_+^{(p)} &= \omega_-^{(c)} - \omega_-^{(p)}, \\ k_+^{(c)} - k_+^{(p)} &= k_-^{(c)} - k_-^{(p)}.\end{aligned}\quad (1)$$

Electromagnetically induced transparency requires that the control and probe fields for both the forward and the backward propagation directions are in two-photon resonance with the atomic system, i.e.,

$$\omega_+^{(p)} - \omega_+^{(c)} = \omega_-^{(p)} - \omega_-^{(c)} = \omega_{sg}, \quad (2)$$

where  $\omega_{sg}$  is the transition frequency between the ground state  $|g\rangle$  and the spin state  $|s\rangle$ .

To describe the atomic system we use collective atomic variables  $\tilde{\sigma}_{\mu\nu}(z, t)$ , generalizations of the traditional single-atom transition operators  $\hat{\sigma}_{\mu\nu} = |\mu\rangle\langle\nu|$  between states  $|\nu\rangle$  and  $|\mu\rangle$  [2,6]. We take the two probe fields to have the same field-atom coupling strength, denoted  $g$ , and we denote by  $N$  the one-dimensional density of atoms, taken to be uniform. Then the interaction with the light fields is described, in the rotating-wave approximation, by the interaction Hamiltonian

$$\hat{H}^{\text{int}} = \hbar \sum_{j=\pm} \int dz (\omega_{e_jg} \hat{\sigma}_{e_j e_j} - \Omega_j e^{i(k_j^{(c)}z - \omega_j^{(c)}t)} \tilde{\sigma}_{e_j s} - g \sqrt{N} \hat{E}_j e^{i(k_j^{(p)}z - \omega_j^{(p)}t)} \tilde{\sigma}_{e_j g} + \text{H.a.}). \quad (3)$$

The rapidly oscillating exponents in Eq. (3) can be eliminated by introducing slowly varying amplitudes of the atomic variables

$$\hat{\sigma}_{e_{\pm}s}(z, t) = \tilde{\sigma}_{e_{\pm}s}(z, t) e^{i(k_{\pm}^{(c)}z - \omega_{\pm}^{(c)}t)}, \quad (4)$$

$$\hat{\sigma}_{e_{\pm}g}(z, t) = \tilde{\sigma}_{e_{\pm}g}(z, t) e^{i(k_{\pm}^{(p)}z - \omega_{\pm}^{(p)}t)}, \quad (5)$$

$$\hat{\sigma}_{sg}(z, t) = \tilde{\sigma}_{sg}(z, t) e^{i[(k_{\pm}^{(p)} - k_{\pm}^{(c)})z - (\omega_{\pm}^{(p)} - \omega_{\pm}^{(c)})t]}. \quad (6)$$

In contrast to the original stationary-light scheme [9,10], the conditions (1) allow the introduction of a slowly varying amplitude of the ground state coherence  $\hat{\sigma}_{gs}$  without a secular approximation. The coherent interaction Hamiltonian can now be rewritten as

$$\hat{H}^{\text{int}} = -\hbar \sum_{j=\pm} \int dz [\Delta_j \hat{\sigma}_{e_j e_j} + \Omega_j \hat{\sigma}_{e_j s} + g \sqrt{N} \hat{E}_j \hat{\sigma}_{e_j g} + \text{H.a.}]. \quad (7)$$

Here  $\Delta_{\pm} \equiv \omega_{\pm}^{(p)} - \omega_{e_{\pm}g}$  are the one-photon detunings of the forward (+) and backward (−) propagating probe fields, respectively.

The atomic system is also subject to losses and decoherence. These we treat by a set of Heisenberg-Langevin equations for the collective atomic operators and the operators for the probe fields [15]. We assume that the control fields are sufficiently strong to remain undepleted—they are not dynamical variables—and we consider only the linear response of the atoms to the probe fields. In this limit we treat the probe fields as perturbations in the Heisenberg-Langevin equations. In zeroth order all atoms will be optically pumped into the ground state  $|g\rangle$ . The only nonzero atomic variable in this limit is  $\hat{\sigma}_{gg}^{(0)} = 1$ ; other operators vanish,  $\hat{\sigma}_{\mu\nu}^{(0)} = 0$ .

In first order one finds a linear set of Heisenberg-Langevin equations for the atomic variables. These separate into two uncoupled sets. Those relevant for this work are

$$\partial_t \hat{\sigma}_{ge_{\pm}} = -[i\Delta_{\pm} + \gamma_{ge_{\pm}}] \hat{\sigma}_{ge_{\pm}} + ig\sqrt{N} \hat{\mathcal{E}}_{\pm} + i\Omega_{\pm} \hat{\sigma}_{gs} + \hat{F}_{ge_{\pm}}, \quad (8)$$

$$\partial_t \hat{\sigma}_{gs} = +i\Omega_{+}^* \hat{\sigma}_{ge_{+}} + i\Omega_{-}^* \hat{\sigma}_{ge_{-}}, \quad (9)$$

where the quantities  $\gamma_{ge_{\pm}}$  parametrize the losses and decoherence rates and the  $\hat{F}_{ge_{\pm}}$  are Langevin noise forces [15,16] associated with the decays. In Eq. (9) we have assumed that the ground-state coherence  $\hat{\sigma}_{gs}$  is stable. This is appropriate for hyperfine states in cold and hot atomic vapors for which the coherence lifetimes, mainly caused by elastic collisions, are of the order of milliseconds [4,5,9]. As a consequence of this stability the equation includes no corresponding Langevin noise operator.

The Langevin noise operators  $\hat{F}_A(t)$  are necessary to preserve the commutation relations of quantum variables. We assume, as is customary, that the associated decay is exponential in time, so that the noise operators are delta correlated in time,

$$\langle \hat{F}_A(t) \hat{F}_B(t') \rangle = D_{AB}(t) \delta(t-t'). \quad (10)$$

The diffusion coefficient  $D_{AB}(t)$  appearing here is related to the dissipative part of the dynamics through the dissipation-fluctuation theorem [15,16],

$$D_{AB}(t) = \frac{d}{dt} \langle \hat{A}(t) \hat{B}(t) \rangle_{\text{loss}} - \left\langle \frac{d}{dt} \hat{A}(t) \right|_{\text{loss}} \hat{B}(t) \rangle - \left\langle \hat{A}(t) \frac{d}{dt} \hat{B}(t) \right|_{\text{loss}} \rangle.$$

One easily verifies that, because the quantum noise originates from spontaneous emission processes, the relevant diffusion coefficients of the present system are proportional to the excited-state populations. However, in the linear response limit considered here this population is negligible and thus we can ignore the Langevin noise terms altogether.

To complete the description of the atom-field system we require equations for the probe fields. These we take to be wave equations for the slowly varying probe field amplitudes

$$[\partial_t \pm c\partial_z] \hat{\mathcal{E}}_{\pm} = ig\sqrt{N} \hat{\sigma}_{ge_{\pm}}. \quad (11)$$

These, together with the atomic Eqs. (8) and (9), form the set of self-consistent set of Maxwell-Bloch equations which are the basis of the following considerations.

We note that Eqs. (8) and (9) are formally identical to the time-dependent Schrödinger equation (or TDSE) for the  $\Lambda$  system after the secular approximation [see, e.g., Eqs. (9) and (10) in [10]]. No such approximation has been used here, however. Furthermore, in contrast to the system studied in [9,10] the single-photon detunings  $\Delta_{\pm}$  can be chosen to be different for the forward and backward directions, which adds another degree of control.

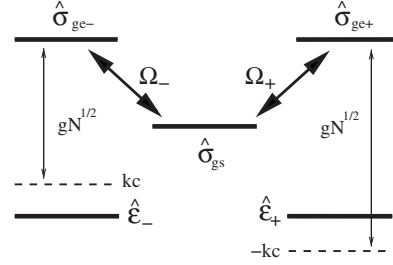


FIG. 2. The  $M$  scheme corresponding to the dual- $V$  coupling scheme depicted in Fig. 1. The matter and field amplitudes are treated as basis states of the new system.

We now assume that the slowly varying amplitudes of the control field Rabi frequencies are constant in space, as is the atom density  $N$ . We introduce a spatial Fourier transformation, through variables  $X(z,t) = \int dk \exp(-ikz) X(k,t)$ , thereby reducing the Heisenberg equations and the shortened wave equation to coupled differential equations in time alone. We write these in matrix form as

$$\frac{d}{dt} \mathbf{X} = -i\mathbf{H}\mathbf{X}, \quad (12)$$

where the column vector of dynamical variables has the elements  $\mathbf{X}^T = \{\hat{\mathcal{E}}_+, \hat{\mathcal{E}}_-, \hat{\sigma}_{gs}, \hat{\sigma}_{ge_+}, \hat{\sigma}_{ge_-}\}$ , and the matrix of (slowly varying) coefficients is

$$\mathbf{H} = \begin{bmatrix} -kc & 0 & 0 & -g\sqrt{N} & 0 \\ 0 & +kc & 0 & 0 & -g\sqrt{N} \\ 0 & 0 & 0 & -\Omega_+ & -\Omega_- \\ -g\sqrt{N} & 0 & -\Omega_+^* & -i\Gamma_{ge_+} & 0 \\ 0 & -g\sqrt{N} & -\Omega_-^* & 0 & -i\Gamma_{ge_-} \end{bmatrix}. \quad (13)$$

To simplify typography we have introduced the symbol

$$\Gamma_{ge_{\pm}} \equiv i\Delta_{\pm} + \gamma_{ge_{\pm}}. \quad (14)$$

The linkage pattern of these equations is shown in Fig. 2. It corresponds to the  $M$ -type linkage found with the TDSE for light fields interacting with atoms, as was extensively studied in the literature [14]. In the following we draw on this analogy with the conventional TDSE to identify the loss-free adiabatic eigensolutions of the system through use of the Morris-Shore transformation, a method widely used to find adiabatic dark states of the interaction of atomic multilevel systems with near-resonant laser fields.

### III. THE MORRIS-SHORE TRANSFORMATION

Let us first summarize the basic properties of the Morris-Shore transformation [12], before considering the two examples most relevant for us. Consider a first-order ODE, of the form (12), in which  $\mathbf{X}$  is an  $N$ -dimensional column vector that separates into two classes of variables: a set  $A$  of  $N_A$  variables and a set  $B$  of  $N_B$  variables. We suppose that the  $N \times N$  matrix  $\mathbf{H}$  has elements only between the  $A$  and  $B$  sets, not within them.  $\mathbf{X}$  and  $\mathbf{H}$  therefore have the forms

$$\mathbf{X} = \begin{bmatrix} \mathbf{X}_A \\ \mathbf{X}_B \end{bmatrix}, \quad \mathbf{H} = \begin{bmatrix} 0 & \mathbf{V} \\ \mathbf{V}^\dagger & 0 \end{bmatrix}, \quad (15)$$

where  $\mathbf{V}$  is an  $N_A \times N_B$  matrix. We further assume that initially (at  $t=0$ ) all variables of the  $B$  set vanish,  $\mathbf{X}_B(t=0)=0$ . Such equations can, by means of the Morris-Shore (MS) transformation  $\mathbf{Y} = \mathbf{M}^\dagger \mathbf{X} \mathbf{M}$  [12], be rewritten as a set of independent equations involving only pairs of variables (one being a combination of the  $A$  set variables, the other a combination from the  $B$  set) or unlinked single variables. The number of unlinked (“dark”) variables  $N_D$  is the difference  $N_D = |N_A - N_B|$ . In the new basis the equation of motion reads

$$\frac{d}{dt} \mathbf{Y} = -i \mathbf{H}^{MS} \mathbf{Y}, \quad (16)$$

where the matrix of coefficients has the structure

$$\mathbf{H}^{MS} \equiv \mathbf{M}^\dagger \mathbf{H} \mathbf{M} = \begin{bmatrix} \mathbf{w}^{(1)} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{w}^{(2)} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{w}^{(3)} & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}. \quad (17)$$

Here the  $\mathbf{w}^{(i)}$  are matrices of dimension  $2 \times 2$ ,

$$\mathbf{w}^{(j)} = \begin{bmatrix} 0 & v^{(j)} \\ v^{(j)} & 0 \end{bmatrix}. \quad (18)$$

There are  $\min(N_A, N_B)$  of these, each linking pairs of “bright” variables. The remaining “dark” variables remain constant, i.e.,

$$\frac{d}{dt} Y_n = 0. \quad (19)$$

For the  $\Lambda$  and dual- $V$  linkages considered here the  $A$  set includes variables representing both field and atom degrees of freedom. The resulting MS dark variables, combining field and atom properties, are dark-state polaritons.

### A. The $\Lambda$ system

For the  $\Lambda$  system ( $N=3$  with  $N_A=2, N_B=1$  and hence  $N_D=1$  dark state) the original matrix, when expressed in a basis  $\{X_1, X_2, X_3\}$ , has the form (apart from reordering)

$$\begin{bmatrix} 0 & 0 & V_1 \\ 0 & 0 & V_2 \\ V_1 & V_2 & 0 \end{bmatrix}. \quad (20)$$

The single dark variable, constructed entirely from  $A$  variables, is

$$Y_D = [V_2 X_1 - V_1 X_2] / \mathcal{N}, \quad (21)$$

where  $\mathcal{N}$  is a normalization factor. Note that  $Y_D$  remains a “dark” variable even when a finite diagonal element is added to the third row of the matrix in Eq. (20).

To connect this formalism with that of dark-state polaritons we let  $\mathbf{X}$  represent the amplitudes of the wave vector of

the atomic  $\Lambda$  system, with  $X_1$  and  $X_2$  being the amplitudes of the stable (i.e., nondecaying) lower levels. Then  $Y_D$  is the well-known dark state of that system.

### B. The $M$ system

The  $M$  system ( $N=5$  with  $N_A=3, N_B=2$ ) has also only one dark variable. The original matrix has the form (apart from reordering)

$$\begin{bmatrix} 0 & 0 & 0 & V_1 & 0 \\ 0 & 0 & 0 & 0 & V_2 \\ 0 & 0 & 0 & V_3 & V_4 \\ V_1 & 0 & V_3 & 0 & 0 \\ 0 & V_2 & V_4 & 0 & 0 \end{bmatrix}. \quad (22)$$

The single dark variable is

$$Y_D = [V_2 V_3 X_1 + V_1 V_4 X_2 - V_1 V_2 X_3] / \mathcal{N}, \quad (23)$$

where  $\mathcal{N}$  is a normalization constant. As with the  $\Lambda$  system,  $Y_D$  remains a “dark” state if diagonal elements are added to the two lowest rows of the matrix.

## IV. APPLICATION OF THE MORRIS-SHORE TRANSFORMATION TO THE DUAL- $V$ SYSTEM

We now apply this mathematical tool to actual physical problems. The dual- $V$  system depicted in Fig. 1 can be described in the Heisenberg picture as the  $M$  system shown in Fig. 2. Applying the Morris-Shore transformation one can immediately construct the dark variable. Neglecting the ( $\pm kc$ ) diagonal elements we find

$$Y_D = [\Omega_+ g \sqrt{N} \hat{\mathcal{E}}_+ + \Omega_- g \sqrt{N} \hat{\mathcal{E}}_- - g^2 N \hat{\sigma}_{gs}] / \mathcal{N}. \quad (24)$$

To express this more simply we define mixing angles by means of the relationships  $\tan^2 \theta = g^2 N / \Omega^2$  and  $\tan^2 \varphi = \Omega_-^2 / \Omega_+^2$ , where  $\Omega^2 = \Omega_+^2 + \Omega_-^2$ . The construction then reads

$$Y_D = (\cos \varphi \hat{\mathcal{E}}_+ + \sin \varphi \hat{\mathcal{E}}_-) \cos \theta - \hat{\sigma}_{gs} \sin \theta. \quad (25)$$

This is the Fourier transform of the dark-state polariton (DSP) for a dual- $V$ -system. To obtain the adiabatic limit we did set  $kc \rightarrow 0$  in Eqs. (13), thereby neglecting all spatial variations; that limit produces the equation

$$\frac{d}{dt} Y_D = 0. \quad (26)$$

In addition to the usual adiabatic condition  $\Omega_{\text{eff}} T \gg 1$ , where  $\Omega_{\text{eff}} = \sqrt{g^2 N + \Omega^2}$  and  $T$  characterizes the pulse duration, one must impose a further adiabatic condition on the spatial Fourier frequencies to justify the neglect of the terms  $\pm kc$ . As has been discussed in detail in Ref. [6] the corresponding condition is

$$L_p \gg \sqrt{L_{\text{abs}}} \sqrt{\frac{c}{\gamma}}, \quad (27)$$

where  $L_{\text{abs}} = c\gamma/g^2 N$  is the absorption length of the medium, and  $L_p$  is the characteristic length scale over which changes

occur in the amplitude of the dark-state polariton in the medium.

The adiabatic limit, Eq. (26), does not account properly for propagation effects [13], e.g., it neglects consequences of nonzero group velocity. In order to describe the propagation dynamics of the DSPs one needs at least the lowest order corrections to the spatial Fourier components. These can be calculated in a very simple perturbative way. The introduction of nonzero  $kc \neq 0$  amounts to adding the terms  $\{-kc, +kc, 0, 0, 0\}$  to the diagonal elements of  $\mathbf{H}$ , leading to the equation

$$\frac{d}{dt}\mathbf{Y} = -i\mathbf{H}^{MS}\mathbf{Y} - i\mathbf{M}^\dagger \begin{bmatrix} -kc & 0 & 0 & 0 & 0 \\ 0 & +kc & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \mathbf{M}\mathbf{Y}. \quad (28)$$

From this we deduce the dispersion relation for the dark-state polariton

$$\omega(k) = kC_1 + k^2C_2. \quad (29)$$

The coefficients appearing here are obtained as the first two nonvanishing corrections to the eigenvalue of the dark-state polariton due to the terms proportional to  $\pm kc$ ,

$$C_1 = v_{gr} \cos 2\varphi \quad (30a)$$

$$C_2 = v_{gr}L_{\text{abs}} \left( \frac{\Delta}{\gamma} - i \right) (\sin^2 2\varphi + \cos^2 2\varphi \sin^4 \theta). \quad (30b)$$

Here  $v_{gr} = c \cos^2 \theta$  is the group velocity of standard slow light [2] and we have assumed that  $\Gamma_{ge_+} = \Gamma_{ge_-} = \Gamma = \gamma + i\Delta$ . We see that the effective velocity of the dark-state polariton in the stationary light case is then given by

$$v = \frac{d\omega}{dk} = c \cos^2 \theta \cos 2\varphi \quad (31)$$

and that the DSP gains an effective complex-valued mass

$$\frac{1}{m^*} \equiv \frac{1}{\hbar} \frac{d^2\omega}{dk^2} = \frac{4\pi v_{gr} L_{\text{abs}}}{m v_{rec} \lambda} \left( \frac{\Delta}{\gamma} - i \right) (\sin^2 2\varphi + \cos^2 2\varphi \sin^4 \theta). \quad (32)$$

Here  $\lambda = 2\pi/k^{(p)}$  is the wavelength of the probe field,  $m$  is the atomic mass, and  $mv_{rec} = \hbar k^{(p)}$  the recoil momentum of a probe-field photon. In general the effective mass of the DSPs of stationary light is much smaller than the real mass of the atoms, even if the group velocity  $v_{gr}$  is of the order of the recoil velocity  $v_{rec}$ , because the wavelength  $\lambda$  is much smaller than the absorption length  $L_{\text{abs}}$ .

From the above considerations we obtain a very simple equation of motion for the dark-state polaritons including the lowest order corrections. The equation for Fourier components reads

$$\left[ \frac{d}{dt} + ikv - i \frac{\hbar}{2m^*} k^2 \right] Y_D = 0. \quad (33)$$

Transformed to real space this equation becomes

$$\left[ \frac{\partial}{\partial t} - v \frac{\partial}{\partial z} + i \frac{\hbar}{2m^*} \frac{\partial^2}{\partial z^2} \right] \hat{\Psi}_D = 0. \quad (34)$$

The effective group velocity of the DSP, Eq. (31), is fully determined by the mixing angles. The DSP comes to rest,  $v=0$ , when at least one of the two trigonometric functions,  $\cos \theta$  or  $\cos 2\varphi$ , vanishes. It is interesting to note that the sign of  $v$  can be changed by altering the mixing angle  $\varphi$ , i.e., by changing the ratio of the Rabi frequencies of the two counterpropagating control fields. If both these fields have the same strength then  $v=0$  and the field pattern has no net motion. If the forward (backward) propagating control field is stronger than the other one, the net motion of the probe field pattern is also in the forward (backward) direction.

One also notices that when  $\cos 2\varphi=0$  the first-order term  $C_1$  vanishes, and thus the dynamics is dominated by either a Schrödinger-type evolution or a weak diffusive spreading of the probe field, depending on the relative size of imaginary and real parts of  $m^*$ . On the other hand, if only one of the two control lasers is present then  $\cos 2\varphi = \pm 1$  and we recover the equation of motion for the slow light polaritons found in [2].

It should also be noted that as soon as one knows the form of the dark state one is able to calculate all other states of this system, also termed bright-state polaritons. This can be done by defining mutually orthogonal eigensolutions, starting with the dark state. One is able also to derive all nonadiabatic couplings arising, e.g., from time dependent mixing angles, leading to first order corrections to the adiabatic approximation.

Let us finally address some limiting factors of the considered system and approach, in particular phase fluctuations of the control fields, collisional dephasing of the ground-state transition, and Doppler broadening due to atomic motion. In the above discussion we have disregarded processes which destroy either the amplitude or the phase of the ground-state coherence  $\hat{\sigma}_{gs}$ . While the amplitude of ground-state superpositions is usually quite robust, its phase can be destroyed by two processes: atomic collisions and fast phase fluctuations of the coupling laser [17]. If the laser phase fluctuations are sufficiently slow, such that adiabaticity holds true, probe and control fields are phase correlated and their difference phase does not change [18]. Phase destroying collisions as well as fast laser phase fluctuations lead to a decay of dark-state polaritons into bright polaritons which are subsequently absorbed. Thus the concept of the dark polaritons is only useful if the latter process is sufficiently slow on the time scales of interest.

Second, let us introduce the Doppler width  $\Delta_D$  of the  $|g\rangle - |e_{\pm}\rangle$  transitions and the factor  $r_{\pm} = (\omega_{e_{\pm}g} - \omega_{e_{\pm}s}) / \omega_{e_{\pm}g}$ . Using these we note that if the conditions  $\Omega^2 \gg \gamma_{gs}\Delta_D$ ,  $\Omega^2 \gg \Delta_D r_{\pm}$ , and  $\Omega^2 \gg (\gamma_{ge_{\pm}} + \gamma_{gs_{\pm}})\Delta_D r_{\pm}$  are met, the influence of Doppler broadening onto the absorptive and dispersive properties of a  $\Lambda$ -type, three-level system is negligible as long as the Doppler-free configuration of co-propagating

control and probe fields is used [19]. The dual- $V$  stationary light scheme we have introduced here is essentially a double  $\Lambda$  scheme with the two linkages coupling to the same coherence  $\hat{\sigma}_{gs}$ . Both  $\Lambda$ -type subsystems work in the necessary Doppler-free configuration. If the lower states  $|g\rangle$  and  $|s\rangle$  are hyperfine levels, the above conditions are practically always fulfilled. Hence Doppler broadening is not a limiting factor here.

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