Phase-noise squeezing in electromagnetically induced transparency

M. Fleischhauer, U. Rathe, and M. O. Scully
Max-Planck-Institut für Quantenoptik, W-8046 Garching, Germany
and Department of Physics, Texas A&M University, College Station, Texas 77843
(Received 20 April 1992)

Light transmitted through a resonant atomic system with electromagnetically induced transparency displays reduced phase-noise fluctuations. For the case of the medium being inside a cavity, a 50% squeezing of the out-of-phase compoennt is possible outside the cavity.

PACS number(s): 42.50.Dv, 42.50.Lc

I. INTRODUCTION

Quantum coherence and interference can completely change the optical properties of resonant atomic systems. They can lead, for instance, to a large dip in the absorption spectrum of a noninverted system. Such a nonabsorbing resonance has been observed for a Λ scheme in the experiments of Alzetta *et al.* [1] and for a V scheme in the experiment of Harris and co-workers [2]. In the case where there is a small amount of population in the upper level of the optical transition, lasing without inversion [3–6] and a transparent medium with an ultralarge index of refraction are possible [7–9].

On the other hand, interesting quantum properties originate from atomic coherence and interference. For a quantum-beat laser, for instance, the spontaneous emissions from two upper to a lower level are correlated, leading to a quenching of the fluctuations in the difference phase of the two corresponding field modes [10–12], which has been observed in the experiments of Winters, Hall, and Toscheck [13]. The concept of correlated spontaneous emission has been applied to different laser configurations and has predicted noise reduction and even squeezing [14,15]. To obtain bright squeezed light, however, more sophisticated methods like a combination of two-photon CEL action and lasing without inversion [15] are necessary.

We propose a simple scheme, in which the phase fluctuations of a (bright) input light source can be reduced up to 50% below the standard quantum limit. We consider the three-level scheme depicted in Fig. 1. A resonant driving field establishes coherence between a pair of upper levels which leads to an almost perfect cancellation of the absorption from the ground state b to the upper level a [2,3]. While the linear part of the corresponding polarization is very small, the nonlinear polarization contributions are considerably large at resonance. As we will show in this paper, these nonlinearities lead to a squeezing of the phase fluctuations of the transmitted light.

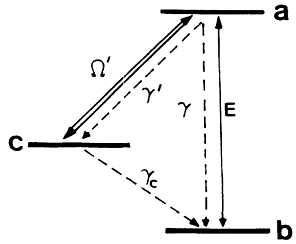

FIG. 1. Atomic level scheme investigated. The two upper levels in a Λ-type three-level system are driven by a resonant field, producing a nonabsorbing resonance.

In Sec. II we derive quantum-Langevin equations for the atomic and field variables and transform the quantum into c-number equations. From an adiabatic elimination of the atomic variables, an effective equation of motion for the field mode is obtained. In Sec. III the corresponding effective diffusion coefficients are calculated to all orders in the test field and the squeezing spectrum is derived for the out-of-phase component. Section IV is devoted to a summary and conclusion.

II. LANGEVIN EQUATIONS AND ADIABATIC ELIMINATION OF ATOMIC VARIABLES

We consider the case of a cavity containing a three-level medium in a Λ configuration (Fig. 1), which is pumped by an external classical field. The field which drives the transition between the two upper levels is also described classically. The interaction Hamiltonian of the system reads

$$H_{\rm int}=-\hbar\sum_i[g(a^\dagger\sigma_0^i+\sigma_0^{i\dagger}a)+(\Omega'^*\sigma_2^i+\Omega'\sigma_2^{i\dagger})] \ . \tag{1}$$

Here a and $a^\dagger$ denote the annihilation and creation operators of the field mode, Ω' is the Rabi frequency of the driving field, and the atomic operators of the ith atom are defined as

$$\begin{aligned}\sigma_0^i&=|b\rangle\langle a|_i \ ,\\ \sigma_1^i&=|b\rangle\langle c|_i \ ,\\ \sigma_2^i&=|c\rangle\langle a|_i \ ,\\ \sigma_\alpha^i&=|\alpha\rangle\langle\alpha|_i \ .\end{aligned} \tag{2}$$

From Eq. (1) we obtain the following quantum-Langevin equations:

$$\dot\sigma_a^i=-(\gamma+\gamma')\sigma_a^i-i(\Omega'^*\sigma_2^i-\Omega'\sigma_2^{i\dagger}) -ig(a^\dagger\sigma_0^i-a\sigma_0^{i\dagger})+F_a^i \ , \tag{3a}$$

$$\dot{\sigma}^i_b=\gamma\sigma^i_a+\gamma_c\sigma^i_c+ig(a^\dagger\sigma^i_0-a\sigma^{i\dagger}_0)+F^i_b\ , \tag{3b}$$

$$\dot{\sigma}^i_0=-[i\Delta+\tfrac{1}{2}(\gamma+\gamma')]\sigma^i_0+iga(\sigma^i_b-\sigma^i_a)+i\Omega'\sigma^i_1+F^i_{\sigma_0}\ , \tag{3c}$$

$$\dot{\sigma}^i_1=-(i\Delta+\tfrac{1}{2}\gamma_c)\sigma^i_1-iga\sigma^{i\dagger}_2+i\Omega'^*\sigma^i_0+F^i_{\sigma_1}\ , \tag{3d}$$

$$\dot{\sigma}^i_2=-\tfrac{1}{2}(\gamma+\gamma'+\gamma_c)\sigma^i_2+i\Omega'(\sigma^i_c-\sigma^i_a)+iga\sigma^{i\dagger}_1+F^i_{\sigma_2}\ , \tag{3e}$$

where Δ is the detuning between the atomic transition ω_{ab} and the test field ν, $\Delta=\omega_{ab}-\nu$. We have assumed that collisions can be neglected so that all decays are purely radiative (the required atomic densities are small enough that this assumption is justified). Note that we have included a decay from c to b with rate γ_c, which is small compared to γ and γ', since the corresponding transition is dipole forbidden. The fluctuation operators F_x in Eqs. (3) have zero mean value and are δ correlated. This corresponds to a Markov approximation of the related decay processes. Using the generalized Einstein relations [16], the diffusion coefficients can be calculated easily. Their explicit form is given in the Appendix.

We apply a c-number approach and transform the equations of motion and the diffusion coefficients into their c-number analog, after having defined an ordering procedure. The ordering we chose is

$$a^\dagger,\sigma^{i\dagger}_2,\sigma^{i\dagger}_1,\sigma^{i\dagger}_0,\sigma^i_a,\sigma^i_b,\sigma^i_c,\sigma^i_0,\sigma^i_1,\sigma^i_2,a\ , \tag{4}$$

which we will call "normal ordering." The c-number variables of the atomic system will be labeled with the same symbols as used above for the quantum variables; the classical field variable is denoted by α. The equations of motion are already normally ordered, so we only need to replace the variables in Eq. (3) by the corresponding c-number quantities. The c-number diffusion coefficients are again listed in the Appendix.

We now assume that the decay rate of the a-b transition γ is large compared to the cavity damping rate γ_0. In this case, we may adiabatically eliminate σ^i_a, σ^i_0, and σ^i_2, i.e., we neglect the time derivatives of these variables in Eqs. (3). Inserting the adiabatic expressions for σ^i_0 and σ^i_2 into the equations for σ^i_1 and σ^i_b, we realize that under the condition $2|\Omega'|^2/(\gamma+\gamma')\gg\gamma_0$, σ^i_1 and σ^i_b can also be adiabatically eliminated. The driving field is assumed to be resonant with the a-c transition and we consider the case where $\Delta=0$. In this case we can further simplify the calculations by introducing the new variables Σ^i_0, Σ^i_1, and Σ^i_2 and the corresponding fluctuation operators:

$$\begin{aligned}\sigma^i_0&=ig\alpha\Sigma^i_0\ ,\\ \sigma^i_1&=-g\Omega'^*\alpha\Sigma^i_1\ ,\\ \sigma^i_2&=i\Omega'\Sigma^i_2\ .\end{aligned} \tag{5}$$

In the adiabatic limit, we thus obtain the following set of algebraic equations:

$$0=-(\gamma+\gamma')\sigma^i_a+|\Omega|^2(\Sigma^i_0+\Sigma^{i*}_0)+|\Omega'|^2(\Sigma^i_2+\Sigma^{i*}_2)+F^i_a\ , \tag{6a}$$

$$0=\gamma\sigma^i_a+\gamma_c\sigma^i_c-|\Omega|^2(\Sigma^i_0+\Sigma^{i*}_0)+F^i_b\ , \tag{6b}$$

$$0=-\tfrac{1}{2}(\gamma+\gamma')\Sigma^i_0+(\sigma^i_b-\sigma^i_a)-|\Omega'|^2\Sigma^i_1+F^i_{\Sigma_0}\ , \tag{6c}$$

$$0=-\frac{\gamma_c}{2}\Sigma^i_1+\Sigma^{i*}_2+\Sigma^i_0+F^i_{\Sigma_1}\ , \tag{6d}$$

$$0=-\tfrac{1}{2}(\gamma+\gamma'+\gamma_c)\Sigma^i_2+\sigma^i_c-\sigma^i_a-|\Omega|^2\Sigma^{i*}_1+F^i_{\Sigma_2}\ , \tag{6e}$$

where $|\Omega|^2=g^2\alpha^*\alpha$.

The equation of motion of the field mode reads

$$\dot{\alpha}=-\gamma_0\alpha+r-g^2\alpha\sum_i\Sigma^i_0\ , \tag{7}$$

where r describes the pumping of the cavity mode by an external field and γ_0 is the cavity damping rate.

III. STEADY-STATE VALUES AND PHASE-NOISE SQUEEZING

In order to calculate the semiclassical steady-state values, we neglect the fluctuating operators in Eqs. (6). In this case all variables are real. Thus, we immediately obtain from Eqs. (6a) and (6b),

$$\langle\sigma^i_a\rangle=\frac{2|\Omega|^2}{\gamma+\gamma'}\langle\Sigma^i_0\rangle+\frac{2|\Omega'|^2}{\gamma+\gamma'}\langle\Sigma^i_2\rangle\ , \tag{8a}$$

$$\langle\sigma^i_c\rangle=\frac{2|\Omega|^2}{\gamma_c}\langle\Sigma^i_0\rangle-\frac{\gamma}{\gamma_c}\langle\sigma^i_a\rangle\ . \tag{8b}$$

From Eq. (6d) it follows that

$$\langle\Sigma^i_1\rangle=\frac{2}{\gamma_c}(\langle\Sigma^i_0\rangle+\langle\Sigma^i_2\rangle)\ . \tag{9}$$

Together with these expressions we obtain from Eqs. (6c) and (6e)

$$\langle\Sigma^i_0\rangle=\left[\tfrac{1}{2}(\gamma+\gamma'+\gamma_c)+\frac{2|\Omega|^2}{\gamma_c}+\frac{2|\Omega'|^2}{\gamma+\gamma'}+\frac{2\gamma|\Omega'|^2}{\gamma_c(\gamma+\gamma')}\right]\frac{1}{D}\ , \tag{10a}$$

$$\langle\Sigma^i_2\rangle=-2|\Omega|^2\left[\frac{1}{(\gamma+\gamma')}+\frac{\gamma}{\gamma_c(\gamma+\gamma')}\right]\frac{1}{D}\ , \tag{10b}$$

where the determinant D is given by

$$D=\left[\tfrac{1}{2}(\gamma+\gamma')+\frac{2\gamma'|\Omega|^2}{\gamma_c(\gamma+\gamma')}+\frac{4|\Omega|^2}{\gamma+\gamma'}+\frac{2|\Omega'|^2}{\gamma_c}\right]\left[\tfrac{1}{2}(\gamma+\gamma'+\gamma_c)+\frac{2\gamma|\Omega'|^2}{\gamma_c(\gamma+\gamma')}+\frac{2|\Omega'|^2}{\gamma+\gamma'}+\frac{2|\Omega|^2}{\gamma_c}\right]$$
$$-\left[\frac{2|\Omega|^2}{\gamma+\gamma'}+\frac{2\gamma|\Omega|^2}{\gamma_c(\gamma+\gamma')}\right]\left[\frac{2\gamma'|\Omega'|^2}{\gamma_c(\gamma+\gamma')}+\frac{4|\Omega'|^2}{\gamma+\gamma'}\right]. \tag{11}$$

If the driving field is sufficiently strong, such that $|\Omega'|^2 \gg \gamma\gamma_c, \gamma'\gamma_c$, we have

$$\langle \Sigma_0^i \rangle \approx \frac{\gamma_c}{2}\frac{(|\Omega|^2+|\Omega'|^2)\gamma+|\Omega|^2\gamma'}{(\gamma'|\Omega|^2+\gamma|\Omega'|^2)(|\Omega|^2+|\Omega'|^2)}. \tag{12}$$

Linearizing the semiclassical version of Eqs. (3) for the atomic variables and Eq. (7) for the field around the steady-state solutions, we find only eigenvalues with negative real part, i.e., stability of the steady state.

We now proceed to calculate the phase noise of the output field. The phase-diffusion coefficient of the cavity field α is determined by the effective fluctuation operator of Σ_0^i according to Eq. (7). From Eq. (6), we find the adiabatic expression

$$\Sigma_0^i=\frac{\gamma_c}{2}\frac{(|\Omega|^2+|\Omega'|^2)\gamma+|\Omega|^2\gamma'}{(\gamma'|\Omega|^2)+\gamma|\Omega'|^2)(|\Omega|^2+|\Omega'|^2)}+F_\Sigma^i , \tag{13}$$

so that

$$F_\alpha=-g^2\alpha\sum_i F_\Sigma^i=-g^2\alpha\sqrt{N}F_\Sigma . \tag{14}$$

The phase-diffusion coefficient $\langle F_\phi F_\phi\rangle$ depends only on the correlation of the imaginary part of F_Σ

$$\langle F_\phi F_\phi\rangle=-\frac{1}{4}\left\langle\left[\frac{F_\alpha}{\alpha}-\frac{F_\alpha^*}{\alpha^*}\right]^2\right\rangle$$
$$=g^4N\langle F_\Sigma^I F_\Sigma^I\rangle , \tag{15}$$

which simplifies the calculations. F_Σ^I can again be derived from Eqs. (6) and has the form

$$F_\Sigma^I=\frac{\gamma_c}{(\gamma+\gamma')(|\Omega'|^2+|\Omega|^2)}\times\left[\left[\tfrac{1}{2}(\gamma+\gamma'+\gamma_c)+2\frac{|\Omega|^2}{\gamma_c}\right]F_{\Sigma_0}^I+\frac{2|\Omega'|^2}{\gamma_c}F_{\Sigma_2}^I-\frac{|\Omega'|^2(\gamma+\gamma'+\gamma_c)}{\gamma_c}F_{\Sigma_1}^I\right], \tag{16}$$

where we applied the same approximations as before to the denominator. Making use of the diffusion coefficients given in the Appendix, we finally find for the phase-diffusion coefficient

$$\langle F_\phi F_\phi\rangle=\frac{g^4N\gamma_c}{2}\times\frac{[4|\Omega|^4-\gamma(\gamma+\gamma')|\Omega'|^2]}{(\gamma+\gamma')(|\Omega|^2\gamma'+|\Omega'|^2\gamma)(|\Omega|^2+|\Omega'|^2)^2}. \tag{17}$$

Here we have neglected noise-induced contributions to the drift terms.

The squeezing spectrum [17] of the out-of-phase component of the field is determined by the phase-diffusion coefficient and the drift coefficient for the imaginary part of $\delta\alpha/\alpha_0$, which according to Eqs. (7) and (12) is given by $\gamma_0+\gamma_1$, where $\gamma_1=g^2N\langle\Sigma_0\rangle$ is the absorption rate of the medium,

$$S(\omega)=1+\frac{8\gamma_0\alpha_0^*\alpha_0}{\omega^2+(\gamma_0+\gamma_1)^2}\langle F_\phi F_\phi\rangle . \tag{18}$$

Thus we have

$$S(\omega)=1-\frac{8\gamma_0\gamma_1}{\omega^2+(\gamma_0+\gamma_1)^2}\frac{\kappa^2}{(1+\kappa^2)[1+(1+\gamma'/\gamma)\kappa^2]}\times\left[1-\frac{4\kappa^2|\Omega|^2}{\gamma(\gamma+\gamma')}\right], \tag{19}$$

where $\kappa\equiv|\Omega|/|\Omega'|$. The optimum phase noise reduction in the output field obviously occurs with $\kappa=1$, i.e., in the strongly nonlinear regime, and if the internal losses γ_1 are equal to the external cavity losses γ_0. Moreover, if $\Omega' \ll \gamma$, we can neglect the second term in the brackets in Eq. (19) and obtain the output squeezing spectra depicted in Fig. 2 for different values of γ'/γ. For $\gamma' \ll \gamma$ we recognize at $\omega=0$ a maximum noise reduction of 50% below the standard noise limit. Note, that for increasing values of γ' the noise reduction decreases. Since in the optimum case the internal losses are equal to the cavity losses, half of the input intensity is transmitted. We have therefore found a scheme in which the phase noise of bright input laser light can be strongly reduced, resulting in a bright squeezed light source.

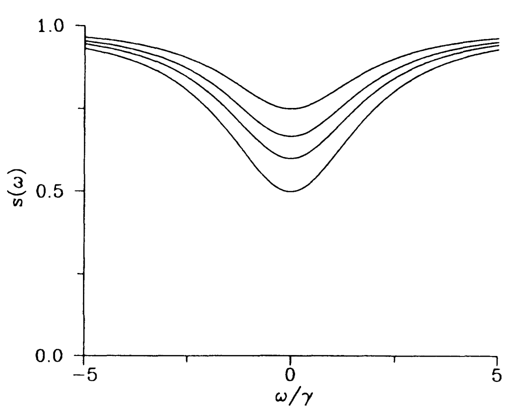

FIG. 2. Squeezing spectra of the out-of-phase component in the output field. From bottom to top we have $\gamma'=0$, $\gamma'=\gamma/2$, $\gamma'=\gamma$, $\gamma'=2\gamma$. The frequency is given in units of γ.

IV. SUMMARY

We have investigated the quantum properties of three-level systems, in which a coherent driving field resonant to the transition between the two upper levels establishes atomic coherence. The large nonlinearities at the resulting nonabsorbing resonance squeeze the phase fluctuations of a bright input light field. The maximum achievable squeezing in the output is for optimum conditions 50% below the standard quantum limit.

ACKNOWLEDGMENT

The support of the U.S. Office of Naval Research is gratefully acknowledged.

APPENDIX

Diffusion coefficients of the Λ scheme

The operator diffusion coefficients of the Λ configuration, obtained from the Langevin equations (3), are

$$\langle F_a^i F_a^i \rangle = (\gamma+\gamma')\langle \sigma_a^i \rangle ,$$
$$\langle F_b^i F_b^i \rangle = \gamma \langle \sigma_a^i \rangle + \gamma_c \langle \sigma_c^i \rangle ,$$
$$\langle F_a^i F_b^i \rangle = -\gamma \langle \sigma_a^i \rangle ,$$
$$\langle F_b^i F_{\sigma_2}^i \rangle = -\gamma_c \langle \sigma_2^i \rangle ,$$
$$\langle F_{\sigma_2}^{i\dagger} F_{\sigma_2}^i \rangle = \gamma_c \langle \sigma_a^i \rangle ,$$
$$\langle F_{\sigma_1}^i F_{\sigma_2}^i \rangle = \gamma_c \langle \sigma_0^i \rangle ,$$
$$\langle F_{\sigma_1}^{i\dagger} F_{\sigma_1}^i \rangle = \gamma' \langle \sigma_a^i \rangle .$$

The corresponding nonzero c-number coefficients are

$$\langle F_a^i F_a^i \rangle = (\gamma+\gamma')\langle \sigma_a^i \rangle + (ig\alpha\langle \sigma_0^{i*} \rangle + \text{c.c.}) + (i\Omega'\langle \sigma_2^{i*} \rangle + \text{c.c.}) ,$$
$$\langle F_b^i F_b^i \rangle = \gamma \langle \sigma_a^i \rangle + \gamma_c \langle \sigma_c^i \rangle + (ig\alpha\langle \sigma_0^{i*} \rangle + \text{c.c.}) ,$$
$$\langle F_a^i F_b^i \rangle = -\gamma \langle \sigma_a^i \rangle - (ig\alpha\langle \sigma_0^{i*} \rangle + \text{c.c.}) ,$$
$$\langle F_a^i F_{\sigma_1}^i \rangle = -ig\alpha\langle \sigma_2^{i*} \rangle + i\Omega'^*\langle \sigma_0^i \rangle ,$$
$$\langle F_b^i F_{\sigma_2}^i \rangle = -ig\alpha\langle \sigma_1^{i*} \rangle - \gamma_c\langle \sigma_2^i \rangle ,$$
$$\langle F_{\sigma_0}^i F_{\sigma_0}^i \rangle = -2ig\alpha\langle \sigma_0^i \rangle ,$$
$$\langle F_{\sigma_2}^i F_{\sigma_2}^i \rangle = -2i\Omega'\langle \sigma_2^i \rangle ,$$
$$\langle F_{\sigma_1}^{i*} F_{\sigma_1}^i \rangle = \gamma'\langle \sigma_a^i \rangle + (ig\alpha\langle \sigma_0^{i*} \rangle + \text{c.c.}) ,$$
$$\langle F_{\sigma_2}^{i*} F_{\sigma_2}^i \rangle = \gamma_c\langle \sigma_a^i \rangle ,$$
$$\langle F_{\sigma_0}^i F_{\sigma_1}^i \rangle = -ig\alpha\langle \sigma_1^i \rangle ,$$
$$\langle F_{\sigma_0}^i F_{\sigma_2}^i \rangle = -ig\alpha\langle \sigma_2^i \rangle - i\Omega'\langle \sigma_0^i \rangle ,$$
$$\langle F_{\sigma_1}^i F_{\sigma_2}^i \rangle = \gamma_c\langle \sigma_0^i \rangle + ig\alpha(\langle \sigma_b^i \rangle - \langle \sigma_c^i \rangle) + i\Omega'\langle \sigma_1^i \rangle .$$

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