Nonperturbative approach to multimode photodetection

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A multimode photodetection theory is developed that includes both the presence of radiation sources and the damping of the field due to the measurement. Making use of Green's-function techniques and the rotating-wave approximation, the counting statistics is calculated in terms of input field quantities. The treatment of the detector-field interaction yields expressions that essentially differ from the results of nonperturbative single-mode theories but are very similar to those of the well-known perturbative photodetection theories. The field damping due to the detection of photons is found to lead to a renormalized detector response function that approaches a maximum value if the number of detector atoms tends to infinity. It is worth noting that our result may be closely related to those of Mandel's heuristic description of photodetection based on the concept of configuration-space photon number operators.

I. INTRODUCTION

The quantum theory of photodetection is of fundamental interest in quantum optics since it predicts the measurable quantities of the radiation field. Much attention has been paid to the different aspects of this matter. From the semiclassical point of view, the statistics of the photodetection process was described by Mandel more than 30 years ago.[1] A perturbative quantum approach to this problem was first made by Glauber[2] and under more general conditions (i.e., in the presence of sources) by Kelley and Kleiner.[3] The resulting distribution of the photocounts is given by the well-known (quantum) Mandel formula[1-3]

$$P_M(m;t,T)=\langle \Omega \frac{\hat{\Phi}^m}{m!}\exp(-\hat{\Phi})\rangle \ , \tag{1.1a}$$

$$\hat{\Phi}=\beta\int_t^{t+T} d\tau\, \hat{I}(\tau) \ . \tag{1.1b}$$

Here β describes the detector efficiency, which is proportional to the number N of detector atoms, $\hat{I}(t)$ is the intensity operator of the radiation field, and Ω means normal and time ordering of the field operators. Although this formula has been successfully applied to a variety of problems, it is restricted by some limitations, which were the origin of some objections and controversial discussions.[4] For instance, applying Eq. (1.1) to the case of a single-mode field yields

$$P_M(m;t,T)=\sum_{n=m}^{\infty}\begin{bmatrix} n \\ m\end{bmatrix}(1-\lambda T)^{n-m}(\lambda T)^m\rho_{nn} \ , \qquad (\lambda\sim\beta\sim N) \ . \tag{1.2}$$

It is clearly seen that for $\lambda T>1$, Eq. (1.2) becomes unphysical. In particular, the number of counts $\langle m\rangle$ would exceed the number of photons $\langle n\rangle$ if the detection time T or the number N of detector atoms becomes sufficiently large:

$$\langle m\rangle=\lambda T\langle n\rangle \ . \tag{1.3}$$

The origin of these unphysical features has been seen in the perturbative approach, which neglects the damping of the field due to photon absorption during the detection process. For the case of a single-mode field this damping has been taken into account by Mollow,[5] Scully and coworkers,[6] Selloni *et al.*,[7] Srinivas and Davies,[8] and recently by Ueda, and co-workers.[9] The resulting distribution reads as

$$P(m;t,T)=\sum_{n=m}^{\infty}\begin{bmatrix} n \\ m\end{bmatrix}[1-\exp(-\lambda T)]^m \times[\exp(-\lambda T)]^{n-m}\rho_{nn} \ , \tag{1.4}$$

and the number of counts becomes

$$\langle m\rangle=[1-\exp(-\lambda T)]\langle n\rangle \ . \tag{1.5}$$

For small values of λT ($\lambda T \ll 1$), Eqs. (1.4) and (1.5) agree with (1.2) and (1.3).

The common situation of photodetection is that the light emitted from some source propagates in free space and falls on the photodetector spatially well separated from the source. Any photon not absorbed by the detector at a given time propagates away. From this point of view it is not understandable that according to Eq. (1.5) all photons would be absorbed if T becomes sufficiently large. The reason for this discrepancy is that Eq. (1.5) is based on a single-mode theory which neglects propagation effects as well as the size and the location of the detector. Hence this model corresponds to the case of the field and the detector being in some cavity rather than to the practically relevant case of light source and detector being separated from each other. To describe the propagation of the radiation from the source to the detector a multimode theory is required, even if the light is quasimonochromatic. An attempt to solve this problem has been made by Chmara;[10] however, explicit formulas in terms of the input field have not been given.

In the present article a nonperturbative multimode approach to the detector-field interaction is presented. The application of Green's-function[11] techniques renders it possible to derive an explicit expression for the photocounting distribution. Moreover, we take into account that, in general, the field is not a free field but is attributed to some atomic sources. In contrast to the nonperturbative single-mode investigations mentioned above, in our theory the effect of field damping simply consists in a renormalization of the detector efficiency in the Mandel formula. This renormalized detector efficiency is always less than unity even if the number of detector atoms becomes large, so that now the first objection to Mandel's formula becomes irrelevant. The answer to the second objection made requires a very careful definition of the photons to be detected.

In the present approach the detector is described by an ensemble of harmonic oscillators. The details and the physical meaning of this model are explained in Sec. II. In Sec. III the counting statistics is calculated. In Sec. IV our results are related to Mandel's illustrative concept of configuration-space photon numbers.[12] The question whether or not an unphysical behavior of the number of counts might be expected in the case when the detection time becomes large is discussed in Sec. V. Section VI is devoted to a summary and some conclusions.

Throughout this article the rotating-wave approximation is used. Hence the results derived cannot be applied to ultrashort detection. This problem will be the subject of a future article.

II. MODEL OF THE DETECTION PROCESS

With regard to the photon field, the photodetection process is a second kind (destructive) of measurement because the photons are absorbed by the detector atoms.[8,13] In this picture the detection process cannot be described by von Neumann's projection postulate[14] but, for instance, by the so-called open-system approach.[8] In practice, however, the number of photoelectrons emitted or the number of excited detector atoms is observed rather than the number of photons. This measurement may be regarded as being a first kind (nondestructive) of measurement, in other words, von Neumann's projection postulate can be applied. Clearly the two pictures should lead to equivalent results, as can be seen from the calculations of Mollow[5] and Srinivas and Davies[8] or from the discussion recently given by Imoto, Ueda, and Ogawa.[15] In what follows we will regard the photodetection as a measurement of the number of detector excitations (photoelectrons).

Let us study a situation that is representative for many practical cases: A radiation field generated by some radiation sources is focused on a detector with nearly normal incidence, so that we may use the one-dimensional model, illustrated in Fig. 1. The vector potential of the radiation field can be divided into a positive- and a negative-frequency part, viz.

$$\hat{A}(x,t)=\hat{A}^{(+)}(x,t)+\hat{A}^{(-)}(x,t) \ . \qquad (2.1)$$

In what follows we want to consider only modes propagating in the $+x$ direction. This simplification may be justified by the relative position of the sources and the detector and by the fact that highly efficient detectors reflect only a very small fraction of the incident radiation. Hence $\hat{A}^{(+)}(x,t)$ is given by

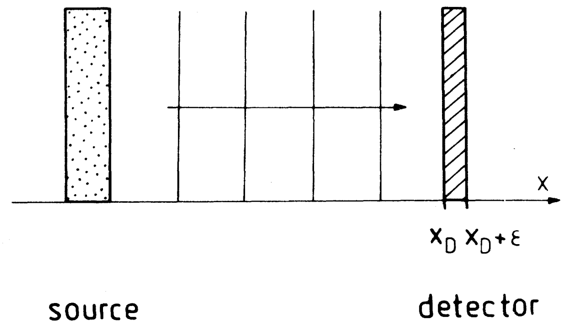

FIG. 1. One-dimensional model of the photodetection process.

$$\hat{A}^{(+)}(x,t)=\int_0^{\infty}dk\left[\frac{\hbar}{4\pi\epsilon_0 Fck}\right]^{1/2}\hat{a}(k,t)e^{ikx} \ , \qquad (2.2)$$

where F is the effective cross section of the beam, which is assumed to coincide with the detector area. The radiation field is assumed to be generated by some sources via the interaction

$$\hat{H}_{sf}(t)=-\int_{-\infty}^{\infty}dx\,\hat{j}(x,t)\hat{A}(x,t) \ , \qquad (2.3)$$

where $\hat{j}(x,t)$ is the current density of the source. An ideal detector should have a linear response, so that saturation effects may be disregarded. Realistic detectors show such a linear behavior if the number of incident photons is small compared to the number of detector atoms. Under these conditions, the probability of a certain detector atom getting excited during the detection time interval is small and the atomic transitions may well be approximated by (harmonic) oscillator transitions. In this sense we model the detector by an ensemble of harmonic oscillators. Moreover, a broadband detector is considered, which means that the bandwidth of the spectrum of the oscillator frequencies is large compared to the bandwidth of the light to be detected. In this case the detector acts as a bath which leads to irreversible photoabsorption. The operators $\hat{C}^{\dagger}_{\mu j},\hat{C}_{\mu j}$ are the creation and destruction operators of the jth oscillator with the frequency ω_μ. The interaction of the field with the oscillators is described in the rotating-wave approximation

$$\hat{H}_{df}(t)=-\frac{e}{m}\int dx\,[\hat{P}^{(-)}(x,t)\hat{A}^{(+)}(x,t)+\text{H.c.}]\Xi(t) \ . \qquad (2.4)$$

Here

$$\hat{P}^{(-)}(x,t)=i\sum_{\mu,j}\left[\frac{\hbar m\omega_\mu}{2}\right]^{1/2}\hat{C}^\dagger_{\mu j}(t)\delta(x-x_j) \qquad (2.5)$$

is the negative-frequency part of the momentum operator of the oscillator transition. In Eq. (2.4) we have introduced the function $\Xi(t)$ in order to describe the switching of the detector

$$\Xi(t)=\Theta(t-t_1)\Theta(t_2-t)\ . \qquad (2.6)$$

Here (t_1,t_2) is the detection time interval. In what follows we will work in the interaction picture in order to apply the S-matrix formalism. In this picture, the dynamics of a state vector is formally described with the help of the unitary time evolution operator

$$S(t,t')=T_c\exp\left[-\frac{i}{\hbar}\int_{t'}^{t}d\tau[\hat{H}_{df}(\tau)+\hat{H}_{sf}(\tau)]\right], \qquad (2.7)$$

where T_c is the causal time-ordering operator. Since in our model the presence of radiation sources is explicitly taken into account, the radiation field and the detector, respectively, can be assumed to be in the ground states $|0\rangle_f$ and $|0\rangle_d$ for $t\to-\infty$,

$$|\psi(t=-\infty|\rangle=|\psi_0\rangle\equiv|0\rangle_f|0\rangle_d|\psi\rangle_s\ . \qquad (2.8)$$

At the end of the measuring time interval (t_1,t_2) the state vector can formally be expressed by

$$|\psi(t_2)\rangle=S(t_2,-\infty)|\psi_0\rangle\ . \qquad (2.9)$$

The counting distribution is given by the distribution of the number of atomic excitations $\hat{m}$ at the time t_2, which reads in our harmonic oscillator model as

$$\hat{m}=\sum_{\mu,j}\hat{C}^\dagger_{\mu j}\hat{C}_{\mu j}\ . \qquad (2.10)$$

III. FACTORIAL MOMENTS AND COUNTING STATISTICS

From Eqs. (2.9) and (2.10) the $k$the moment of the number of counts reads as

$$\langle:\hat{m}(t_2,t_1)^k:\rangle=\sum_{\mu_1,j_1}\cdots\sum_{\mu_k,j_k}\langle\hat{C}^\dagger_{\mu_1 j_1}(t_2)\cdots\hat{C}^\dagger_{\mu_k j_k}(t_2)\times\hat{C}_{\mu_k j_k}(t_2)\cdots\hat{C}_{\mu_1 j_1}(t_2)\rangle\ , \qquad (3.1)$$

where the two time arguments t_1 and t_2 indicate that, according to Eqs. (2.4) and (2.6), the detector operates during the time interval (t_1,t_2), and : : denotes normal ordering. Now the procedure is the following: The factorial

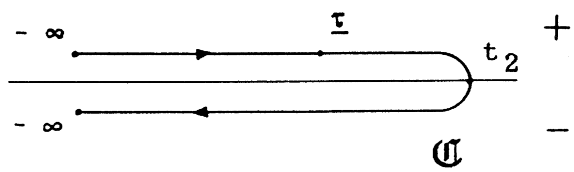

FIG. 2. Time contour $\mathfrak{C}$. A correlation function of Heisenberg operators can be expressed by operators in the interaction picture via two time evolution operators, one being time ordered, the other one antitime ordered. To obtain a single exponential operator we introduce a time contour $\mathfrak{C}$ and a corresponding time-ordering operator $T_{\mathfrak{C}}$.

moments (3.1) will be expressed in terms of source-current correlation functions via integral transformations. The corresponding integral kernels, which are Green's functions of the radiation field interacting with the detector, will be calculated. They turn out to be proportional to the free Green's functions. Since correlation functions of the input field are related to source correlation functions just by these free Green's functions, we can express the factorial moments of the counting statistics by correlation functions of the input field and a renormalized detector response function.

A. The mean number of counts and its relation to source quantities

The method of calculation of the factorial moments will be illustrated for the first moment (mean number of counts). The generalization to higher-order moments is straightforward. By combining Eqs. (2.8)–(2.10), in the interaction picture the number of counts may be expressed in the form

$$\langle\hat{m}(t_2;t_1)\rangle=\sum_{\mu,j}\langle\psi_0|S(-\infty,t_2)\hat{C}^\dagger_{\mu,j}(t_2)\hat{C}_{\mu j}(t_2)\times S(t_2-\infty)|\psi_0\rangle\ . \qquad (3.2)$$

In order to deal with only one time evolution operator we introduce a time contour $\mathfrak{C}$ depicted in Fig. 2 and a time-ordering operator $T_{\mathfrak{C}}$ on $\mathfrak{C}$, which coincides with the causal time-ordering operator T_c on the positive branch and the anticausal time-ordering operator T_a on the negative branch. To distinguish the time arguments on $\mathfrak{C}$ from ordinary time arguments we underline the former ones. We therefore may rewrite Eq. (3.2) as

$$\langle\hat{m}(t_2;t_1)\rangle=\sum_{\mu,j}\langle\psi_0|T_{\mathfrak{C}}S_{\mathfrak{C}}\hat{C}^\dagger_{\mu j}(t_2)\hat{C}_{\mu j}(t_2)|\psi_0\rangle\ . \qquad (3.3)$$

Expanding $S_{\mathfrak{C}}$ into a power series and eliminating $\hat{C}^\dagger_{\mu j}$ and $\hat{C}_{\mu j}$ with the help of Wick's theorem[11] yields

$$\langle \hat{m}(t_2;t_1)\rangle = \sum_{\mu,j}\left[\frac{i}{\hbar}\right]^2 \frac{\hbar\omega_\mu e^2}{2m}\int_{t_1}^{t_2} d\tau \int_{t_1}^{t_2} d\tau' \langle 0|\hat{C}_{\mu j}(\tau)\hat{C}^\dagger_{\mu j}(t_2)|0\rangle\langle 0|\hat{C}_{\mu j}(t_2)\hat{C}^\dagger_{\mu j}(\tau')|0\rangle$$

$$\times\Bigg\langle \psi_0 \Bigg| T_{\mathfrak{C}}\Bigg\{ \hat{A}^{(-)}(x_j,\tau_{(-)})\hat{A}^{(+)}(x_j,\tau'_{(+)})$$

$$\times\left[\sum_{n=1}^{\infty}\frac{1}{[2(n-1)]!}\left[\frac{i}{\hbar}\right]^{2(n-1)}\int d\underline{1}\int d\underline{1}'\cdots\int d(\underline{n-1})\int d(\underline{n-1})'\left[\frac{e}{m}\right]^{2(n-1)}\right.$$

$$\left.\times[\hat{P}^{(-)}(\underline{1})\hat{A}^{(+)}(\underline{1})+\mathrm{H.c.}]\cdots[\hat{P}^{(-)}(\underline{n-1})'\hat{A}^{(+)}(\underline{n-1})'+\mathrm{H.c.}]\right]$$

$$\times\exp\left[-\frac{i}{\hbar}\int d\underline{\tau}\,\hat{H}_{sf}(\underline{\tau})\right]\Bigg\}\Bigg|\psi_0\Bigg\rangle , \tag{3.4a}$$

where the variables $\underline{1},\underline{1}',\ldots$ are used as abbreviations for $(x_1,\underline{\tau}_1)$, $(x'_1,\tau'_1),\ldots$, and $\tau_{(-)}$ $(\tau_{(+)})$ on the negative (positive) branch. This equation can be rewritten in the compact form

$$\langle \hat{m}(t_2;t_1)\rangle = \sum_{\mu,j}\frac{\omega_\mu e^2}{2\hbar m}\int_{t_1}^{t_2} d\tau\int_{t_1}^{t_2} d\tau' e^{-i\omega_\mu(\tau-\tau')}\langle \hat{A}^{(-)}_{sd}(x_j,\tau)\hat{A}^{(+)}_{sd}(x_j,\tau')\rangle , \tag{3.4b}$$

where $\hat{A}^{(\pm)}_{sd}$ are the field operators the time dependence of which is governed by the interaction with the source (s) as well as with the detector (d). Analogously, one would obtain the following for higher-order moments:

$$\langle :\hat{m}^k(t_2;t_1):\rangle = \sum_{\mu_1,j_1}\cdots\sum_{\mu_k,j_k}\frac{\omega_{\mu_1}e^2}{2\hbar m}\cdots\frac{\omega_{\mu_k}e^2}{2\hbar m}$$

$$\times\int_{t_1}^{t_2} d\tau_1\int_{t_1}^{t_2} d\tau'_1\cdots\int_{t_1}^{t_2} d\tau_k\int_{t_1}^{t_2} d\tau'_k e^{-i\omega_{\mu_1}(\tau_1-\tau'_1)}\cdots e^{-i\omega_{\mu_k}(\tau_k-\tau'_k)}$$

$$\times\langle T_a[\hat{A}^{(-)}_{sd}(x_{j_1},\tau_1)\cdots\hat{A}^{(-)}_{sd}(x_{j_k},\tau_k)]$$

$$\times T_c[\hat{A}^{(+)}_{sd}(x_{j_1},\tau'_1\cdots\hat{A}^{(+)}_{sd}(x_{j_k},\tau'_k)]\rangle . \tag{3.5}$$

On first glance this result looks like the perturbative one, but the difference to the perturbative calculations is that $\hat{A}_{sd}$ describes the field damped due to the interaction with the detector.

For the following it is convenient to express in Eq. (3.4b) the field correlation function in terms of source-quantity correlation function. Fleischhauer and Schubert[16] have shown that

$$\langle \hat{A}^{(-)}_{sd}(x_j,t)\hat{A}^{(+)}_{sd}(x_j,t')\rangle = \frac{1}{\hbar^2}\int dx\int d\tau\int dx'\int d\tau' D^-(x_j,t;x,\tau)D^+(x_j,t';x',\tau')\langle\hat{j}_f(x,\tau)\hat{j}_f(x',\tau')\rangle , \tag{3.6}$$

where D^- and D^+ are the following Green's functions:

$$D^-(1;2) = {}_d\langle 0|_f\langle 0|T_a\hat{A}^{(-)}_d(1)\hat{A}^{(+)}_d(2)|0\rangle_f|0\rangle_d ,$$
$$D^+(1;2) = {}_d\langle 0|_f\langle 0|T_c\hat{A}^{(+)}_d(1)\hat{A}^{(-)}_d(2)|0\rangle_f|0\rangle_d . \tag{3.7}$$

Here the time dependence of the operators $\hat{A}^{(\pm)}_d(t)$ is only governed by the detector-field interaction. Note that the time dependence of $\hat{j}_f(t)$ is determined by the source-field interaction. In order to obtain expressions in terms of the input field, the time dependence of which is only governed by the source-field interaction, we make use of the fact that the intensity of this field can be expressed in terms of source quantities as well:

$$\langle \hat{A}^{(-)}_s(x_j,t)\hat{A}^{(+)}_s(x_j,t')\rangle = \frac{1}{\hbar^2}\int dx\int d\tau\int dx'\int d\tau' d^-(x_j,t;x,\tau)d^+(x_j,t';x',\tau')\langle\hat{j}_f(x,\tau)\hat{j}_f(x',\tau')\rangle , \tag{3.8}$$

where

$$d^-(1;2) = {}_f\langle 0|T_a\hat{A}^{(-)}(1)\hat{A}^{(+)}(2)|0\rangle_f ,$$
$$d^+(1;2) = {}_f\langle 0|T_c\hat{A}^{(+)}(1)\hat{A}^{(-)}(2)|0\rangle_f \tag{3.9}$$

are the well-known Green's function of the free field $\hat{A}$.

We now turn to the problem of expressing D^- (D^+) in terms of d^- (d^+), which allows us to derive formulas in terms of the input field.

B. Renormalization of the detector response due to the field damping

Because of the linear character of the detector-field interaction there holds a simple closed Dyson equation for the Green's functions defined in Eq. (3.7):

$$D^+(1;2)=d^+(1;2)-\frac{1}{\hbar^2}\int_{t_1}^{t_2}d3\int_{t_1}^{t_2}d4\,d^+(1;3)C^+(3;4)D^+(4;2)\ , \tag{3.10}$$

where C^+ is the Green's function of the detector oscillators

$$\begin{aligned}C^+(1;2)&=\sum_{\mu,j}\frac{e^2\hbar\omega_\mu}{2m}\delta(x_1-x_j)\delta(x_2-x_j)\times{}_d\langle 0|T_c\hat{C}_{\mu j}(\tau_1)\hat{C}^+_{\mu j}(\tau_2)|0\rangle_d\\&=\Theta(\tau_1-\tau_2)\sum_{\mu,j}\frac{e^2\hbar\omega_\mu}{2m}\delta(x_1-x_j)\delta(x_2-x_j)\times e^{-i\omega_\mu(\tau_1-\tau_2)}\ .\end{aligned} \tag{3.11}$$

Making use of Eqs. (3.11) and (3.9) we can simplify the Dyson equation (3.10) and obtain

$$D^+(1;2)=d^+(1;2)-\frac{e^2}{4(2\pi)m\epsilon_0 Fc}\sum_{\mu,j}\omega_\mu\int_{t_1}^{\tau_1}d\tau\int_{t_1}^{\tau}d\tau' e^{-i\omega_\mu(\tau-\tau')}\int_0^\infty dk\frac{1}{k}e^{ik(x_1-x_j)-ikc(\tau_1-\tau')}D^+(x_j,\tau';2)\ . \tag{3.12}$$

According to our broadband detector assumption, in Eq. (3.12) we can replace the sum over μ by a frequency integral with constant frequency density ρ_ω:

$$\sum_\mu \omega_\mu e^{-i\omega_\mu(\tau-\tau')}=\rho_\omega\int_0^\infty d\omega\,\omega\, e^{-i\omega(\tau-\tau')}\ . \tag{3.13}$$

Since we consider fields with optical frequencies, in Eq. (3.13) we may approximate the integral by extending the lower limit of integration to $-\infty$ and obtain

$$\sum_\mu \omega_\mu e^{-i\omega_\mu(\tau-\tau')}\approx 2\pi i\rho_\omega\frac{\partial}{\partial\tau}\delta(\tau-\tau')\ . \tag{3.14}$$

Inserting this equation into Eq. (3.12), extending in Eq. (3.12) the lower limit of the k integration to minus infinity, and replacing the sum over the position index by an integral (spatial density ρ_x) yields, for $x_D\le x_1\le x_D+\epsilon$,

$$D^+(1;2)=d^+(1;2)-\frac{e^2\rho_\omega\rho_x\pi}{4\epsilon_0 Fmc}\int_{x_D}^{x_1}dx\,D^+\left[x_1,\tau_1-\frac{x_1-x}{c};2\right]\ , \tag{3.15}$$

where x_D is the position of the entrance plane and ϵ is the thickness of the sensitive detection layer (cf. Fig. 1). Because of

$$d^+(x_1,\tau_1;x_2,\tau_2)=d^+(x_1-c\tau_1,0;x_2,\tau_2)\ , \tag{3.16}$$

the solution of Eq. (3.15) reads as

$$D^+(1;2)=d^+(1;2)e^{-\kappa(x_1-x_D)}\ , \tag{3.17a}$$

$$\kappa=\frac{e^2\rho_x\rho_\omega\pi}{4\epsilon_0 Fmc} \tag{3.17b}$$

for $x_D\le x_1\le x_D+\epsilon$. Making use of this result the field correlation function in (3.4b) and (3.5) can be expressed in terms of the input field

$$\langle\hat{A}^{(-)}_{sd}(x,\tau)\hat{A}^{(+)}_{sd}(x,\tau)\rangle=e^{-2\kappa(x-x_D)}\langle\hat{A}^{(-)}_{s}(x,\tau)\hat{A}^{(+)}_{s}(x,\tau)\rangle\ , \tag{3.18}$$

$$\begin{aligned}&\langle T_a[\hat{A}^{(-)}_{sd}(x_1,\tau_1)\cdots\hat{A}^{(-)}_{sd}(x_k,\tau_k)]T_c[\hat{A}^{(+)}_{sd}(x_1,\tau_1')\cdots\hat{A}^{(+)}_{sd}(x_k,\tau_k')]\rangle\\&\quad=e^{-2\kappa(x_1-x_D)}\cdots e^{-2\kappa(x_k-x_D)}\langle T_a[\hat{A}^{(-)}_{s}(x_1,\tau_1)\cdots\hat{A}^{(-)}_{s}(x_k,\tau_k)]T_c[\hat{A}^{(+)}_{s}(x_1,\tau_1')\cdots\hat{A}^{(+)}_{s}(x_k,\tau_k')]\rangle\ .\end{aligned} \tag{3.19}$$

Equation (3.8) is obviously the quantum version of the classical Lambert-Beer law and is, of course, an expected result. But note that a corresponding relationship for higher-order moments is not obvious. In fact, due to the damping of the field, vacuum components are coupled in such a way that they do not disappear automatically; see Ref. 16. The fact that these vacuum terms are not present in the final expression (3.19) is a result of the quantum-mechanically consistent ordering prescriptions used. From Eqs. (3.4b), (3.5), (3.18), and (3.19) follows for the moments of the counting statistics that

$$\langle\hat{m}(t_2;t_1)\rangle=\int_{x_D}^{x_D+\epsilon}dx\frac{e^2\rho_\omega\rho_x\pi}{\hbar m}e^{-2\kappa(x-x_D)}\int_{t_1}^{t_2}d\tau\, i\langle\hat{E}^{(-)}_s(x,\tau)\hat{A}^{(+)}_s(x,\tau)\rangle\ , \tag{3.20}$$

$$\langle :\hat{m}^k(t_2;t_1): \rangle = \left[\frac{ie^2\rho_\omega\rho_x\pi}{\hbar m} \right]^k \times \int_{x_D}^{x_D+\epsilon} dx_1 \cdots \int_{x_D}^{x_D+\epsilon} dx_k e^{-2\kappa(x_1-x_D)} \cdots e^{-2\kappa(x_k-x_D)} \times \int_{t_1}^{t_2} d\tau_1 \cdots \int_{t_1}^{t_2} d\tau_k \langle T_a[\hat{E}_s^{(-)}(x_1,\tau_1)\cdots \hat{E}_s^{(-)}(x_k,\tau_k)] \times T_c[\hat{A}_s^{(+)}(x_k,\tau_k)\cdots \hat{A}_s^{(+)}(x_1,\tau_1)]\rangle \ . \tag{3.21}$$

The set of factorial moments Eq. (3.21) determines the photocounting distribution

$$P(m;t,T) = \left\langle \Omega \frac{\hat{\Phi}^m}{m!} \exp(-\hat{\Phi}) \right\rangle , \tag{3.22a}$$

$$\hat{\Phi} = \int_{x_D}^{x_D+\epsilon} dx \int_t^{t+T} d\tau \frac{ie^2\rho_\omega\rho_x\pi}{\hbar m} \times e^{-2\kappa(x-x_D)} \hat{E}_s^{(-)}(x,\tau)\hat{A}_s^{(+)}(x,\tau) \ . \tag{3.22b}$$

It is worth noting that in our multimode, nonperturbative approach, in contrast to the nonperturbative single-mode result given in Eq. (1.4), the photocounting distribution obtained formally looks like the quantum Mandel formula Eq. (1.1). It differs from Eq. (1.1) in the expression for the detector response.

Let us apply these results to the particular case when the variation of the field amplitude over the detector volume can be disregarded (pointlike detector). In this case we may perform the space integration in Eq. (3.22b) and obtain

$$\hat{\Phi} = \alpha \frac{2\epsilon_0 Fc}{\hbar} \int_t^{t+T} d\tau \, i\hat{E}_s^{(-)}(x_D,\tau)\hat{A}_s^{(+)}(x_D,\tau) \ , \tag{3.23a}$$

where

$$\alpha \equiv 1 - e^{-\eta} \ , \tag{3.23b}$$

$$\eta \equiv \kappa\epsilon = \frac{e^2\rho_\omega\pi}{4\epsilon_0 Fmc}\rho_x\epsilon = \frac{e^2\rho_\omega\pi}{4\epsilon_0 Fmc} N \ , \tag{3.23c}$$

N being the number of detector atoms. The effect of the field damping is obviously a renormalization of the perturbative detector efficiency in such a way that the renormalized efficiency tends to unity as the number of detector atoms goes to infinity. In the limit of perturbation theory ($\eta \ll 1$) we simply have $\alpha \approx \eta$.

IV. RELATION TO MANDEL'S CONFIGURATION-SPACE PHOTON NUMBER CONCEPT

From Eq. (3.23a) we see that in the case of a pointlike detector the field relevant for photodetection is the field within the time interval (t_1,t_2) at the position of the entrance plane of the detector x_D. This field may be represented by Fourier decomposition as follows:

$$\hat{A}_s^{(+)}(x_D,t) = \frac{1}{\sqrt{T}} \sum_l \left[\frac{\hbar}{2\epsilon_0 cF\omega_l} \right]^{1/2} \hat{a}_l e^{-i\omega_l(t-x_D/c)} \ , \quad t_1 < t < t_1+T = t_2 \tag{4.1}$$

$$\hat{a}_l = \left[\frac{2\epsilon_0 cF\omega_l}{\hbar} \right]^{1/2} \frac{1}{\sqrt{T}} \int_{t_1}^{t_2} d\tau \, e^{i\omega_l(\tau-x_D/c)} \times \hat{A}_s^{(+)}(x_D,\tau) \ , \tag{4.2}$$

where $\omega_l = 2\pi l/T$, l being non-negative integers. Apart from the values at the boundary times $t=t_1$ and t_2, the field defined in Eq. (4.1) of course agrees with the actual incoming field [Eq. (2.2)] in the relevant time interval (t_1,t_2). Note that the two boundary values are meaningless, because we are interested in quantities integrated over the time interval. Using Eq. (4.1) is clearly not allowed in the case when the time is outside the measuring interval, because the incoming field is, in general, not periodic with period $T=t_2-t_1$. From Eq. (4.1) it is easily seen that the operator of the electric field strength $\hat{E}_s^{(+)}(x_D,t)$ may be represented as

$$\hat{E}_s^{(+)}(x_D,t) = \frac{1}{\sqrt{T}} \sum_l i \left[\frac{\hbar\omega_l}{2\epsilon_0 cF} \right]^{1/2} \hat{a}_l e^{-i\omega_l(t-x_D/c)} \ , \quad t_1 < t < t_1+T = t_2 \tag{4.3}$$

so that the operators $\hat{a}_l$ may also be determined from $\hat{E}_s^{(+)}(x_D,t)$, viz.

$$\hat{a}_l = -i \left[\frac{2\epsilon_0 cF}{\hbar\omega_l} \right]^{1/2} \frac{1}{\sqrt{T}} \int_{t_1}^{t_2} d\tau \, e^{i\omega_l(\tau-x_D/c)} \times \hat{E}_s^{(+)}(x_D,\tau) \ . \tag{4.4}$$

Combining Eqs. (4.1), (4.3), and (3.20) (for a pointlike detector) yields

$$\langle \hat{m}(t_2;t_1) \rangle = \alpha \sum_l \langle \hat{a}_l^\dagger \hat{a}_l \rangle \ . \tag{4.5}$$

It is near at hand that the operators $\hat{a}_l^\dagger$ and $\hat{a}_l$, respectively, may be interpreted as photon creation and destruction operators. To satisfy ourselves that this is the case, let us consider the commutator $[\hat{a}_l, \hat{a}_{l'}^\dagger]$, which, according to Eqs. (4.2) and (4.4), may be written as

$$[\hat{a}_l,\hat{a}_{l'}^\dagger] = -i\frac{2\epsilon_0 Fc}{\hbar} \left[\frac{\omega_{l'}}{\omega_l} \right]^{1/2} \frac{1}{T} \int_{t_1}^{t_2} d\tau \int_{t_1}^{t_2} d\tau' e^{i\omega_l(\tau-x_D/c)} e^{-i\omega_{l'}(\tau'-x_D/c)} [\hat{E}_s^{(+)}(x_D,\tau), \hat{A}_s^{(-)}(x_D,\tau')] \ . \tag{4.6}$$

To evaluate the (time-dependent) commutator on the right-hand side of Eq. (4.6) we note that in the case of sources being present the commutation relation $[\hat{E}_s^{(+)}(x_1,\tau_1)\hat{A}_s^{(-)}(x_2,\tau_2)]$ may differ from the corresponding free-field commutation relation in an additional, so-called time-delayed term.[17] However, the condition for nonzero time-delayed commutator contributions is that the field can propagate from x_1 (at time t_1) to x_2 (at time t_2) via the sources. Clearly, this might be the case if the light propagating in the positive-x direction and falling on the detector could be reflected into the sources by the detector. Since in our model blooming of the detector is assumed to suppress such reflections, the time-delayed terms mentioned above may be disregarded. We remind the reader that our starting point of considering only modes propagating in the positive-x direction is closely related to this assumption about the action of the photodetector. From these arguments, in Eq. (4.6) the (time-dependent) field commutator may be viewed as simply being the corresponding free-field commutator. Applying Eq. (2.2) we may therefore write

$$[\hat{E}_s^{(+)}(x_D,\tau),\hat{A}_s^{(-)}(x_D,\tau')] = i\frac{\hbar}{2c\epsilon_0 F}\frac{1}{2\pi}\int_0^\infty d\omega\, e^{-i\omega(\tau-\tau')} . \tag{4.7}$$

We now evaluate the integral in this equation by extending the integration to $-\infty$ and obtain

$$[\hat{E}_s^{(+)}(x_D,\tau),\hat{A}_s^{(-)}(x_D,\tau')] = i\frac{\hbar}{2c\epsilon_0 F}\delta(\tau-\tau') . \tag{4.8}$$

Combining Eqs. (4.8) and (4.7) then yields

$$[\hat{a}_l,\hat{a}_{l'}^\dagger]=\delta_{ll'} , \tag{4.9}$$

so that the operators $\hat{a}_l^\dagger \hat{a}_l$ may indeed be viewed as photon creation and destruction operators. Defining the operator of the total number of photons

$$\hat{n}(t_2;t_1)\equiv\sum_l \hat{a}_l^\dagger\hat{a}_l , \tag{4.10}$$

we may rewrite Eq. (4.3) as

$$\langle \hat{m}(t_2;t_1)\rangle=\alpha\langle\hat{n}(t_2;t_1)\rangle . \tag{4.11}$$

Here, the arguments t_1 and t_2 of $\hat{n}$ indicate that, according to Eq. (4.2), the operator $\hat{n}$ depends on the actual counting interval. Clearly, the operator $\hat{n}(t_2;t_1)$ may be interpreted as the operator of the total number of photons falling on the photodetector (situated at the position x_D) during the time interval (t_1,t_2).

The procedure outlined above is closely related to the configuration-space photon number concept introduced by Mandel[12] via the so-called detection operators, which may be defined by the relation

$$\hat{\mathcal{A}}(x_D,t)=\left[\frac{c}{2\pi}\right]^{1/2}\int_0^\infty dk\,\hat{a}(k,t)e^{ikx_D} . \tag{4.12}$$

Combining Eqs. (3.20) and (2.2), taking into account that the bandwidth of the light is small compared with the center frequency, and making use of Eq. (4.12), we may rewrite Eq. (3.20) as

$$\langle\hat{m}(t_2;t_1)\rangle=\alpha\int_{t_1}^{t_2}d\tau\langle\hat{\mathcal{A}}^\dagger(x_D,\tau)\hat{\mathcal{A}}(x_D,\tau)\rangle . \tag{4.13}$$

Comparing this equation with the result given in Eq. (4.4), we see that the relation

$$\int_{t_1}^{t_2}d\tau\langle\hat{\mathcal{A}}^\dagger(x_D,\tau)\hat{\mathcal{A}}(x_D,\tau)\rangle=\langle\hat{n}(t_2,t_1)\rangle \tag{4.14}$$

is valid. That is, $\hat{\mathcal{A}}(x_D,t)$ $[\mathcal{A}^\dagger(x_D,t)]$ may be regarded as being the photon/s$^{1/2}$ units destruction (creation) operator of the field incident on the photodetector (located at x_D). Note that in Eq. (4.12) the time dependence of the operators $\hat{a}(k,t)$ must be determined from the solution of the light-source interaction problem. We remind the reader that in the rotating-wave approximation any time interval must be large compared with λ/c, λ being the characteristic wavelength of the light field under study. In particular, the detection time interval T must be large enough so that $cT\gg\lambda$. If we let in the time integral in Eq. (4.4) $\tau-x_D/c=t_1-x/c$, $d\tau=-dx/c$, we see that it may be rewritten in the form of a length integral. Similar to Mandel's concept, the photon number operator Eq. (4.10) may therefore be regarded as representing the number of photons in a volume $V=Fl$ at time t_1, the effective length being $l=cT$ $(l\gg\lambda)$. The advantage of our approach is that taking into account sources does not raise complications in the case, when the effective detection volume V and the region of space which the sources are located in overlap.

V. DOES THE NUMBER OF COUNTS EXCEED THAT OF PHOTONS IF THE MEASURING TIME TENDS TO INFINITY?

The example commonly studied in the discussion of this problem is a single-mode free-field Fock state of photon number n. In this context, Davies and Srinivas argued that, on the basis of a relation of the type given in Eq. (3.22a), in the case of a sufficiently long duration of detection, the number of counts recorded can become larger than n.[8] If both the detector and the light field are inside a resonatorlike setup, they are right. In this case Eq. (1.4) must be used, which accounts for the effect that a photon not absorbed by the detector at one time is reflected by the cavity mirrors and is available for detection for later times too. However, to describe standard photodetection experiments, where the light is detected outside the region of space in which it is generated and/or amplified by means of resonatorlike devices, a continuous quantization is necessary. Here the argument of Davies and Srinivas fails, because light having passed through the detector cannot return to it. To clarify this point let us consider the (somewhat academic) case of the radiation field to be detected in a single-mode free-field Fock state. We note that the detection of the photons of a free-field single-mode Fock state requires, in principle, an infinite observation time, because the photons are homogeneously distributed over the whole universe. The number of counts recorded during any finite time interval vanishes. To show this, we start from Eq. (2.2) in the form

$$\hat{A}^{(+)}(x,t)=\lim_{L\to\infty}\sum_{l=0}^{\infty}\left[\frac{\hbar}{2\epsilon_0 cFlk_l}\right]^{1/2}\hat{a}_l(t)e^{ik_lx}, \qquad (5.1)$$

where $k_l=(2\pi/L)l$. Since for a single-mode field the relation $\langle\hat{a}_l^\dagger\hat{a}_l\rangle=\delta_{ll'}\delta_{ll_0}\langle\hat{a}_{l_0}^\dagger\hat{a}_{l_0}\rangle$ is valid, where $n=\langle\hat{a}_{l_0}^\dagger\hat{a}_{l_0}\rangle$ is the number of photons of the mode, from Eq. (3.20) the mean number of counts is given by

$$\langle m\rangle=\alpha\lim_{L\to\infty}\frac{cT}{L}n=0 \quad \text{if } T \text{ is finite}. \qquad (5.2)$$

Hence the detector should operate during the whole time scale (from $-\infty$ to $+\infty$ in order to have the chance to absorb the photons of the Fock state. In this case, from Eq. (4.14) together with Eq. (4.12) [and $\hat{a}(k,t)=\hat{a}(k)\exp(-ikct)$], we easily obtain

$$\langle\hat{n}(\infty,-\infty)\rangle=\int_0^\infty dk\,\langle\hat{a}^\dagger(k)\hat{a}(k)\rangle. \qquad (5.3)$$

In particular, in the case of a single-mode Fock state we have

$$\langle\hat{a}^\dagger(k)\hat{a}(k)\rangle=n\delta(k-k_0), \qquad (5.4)$$

so that

$$\langle\hat{n}(\infty,-\infty)\rangle=n, \qquad (5.5)$$

and, according to Eq. (4.11),

$$\langle\hat{m}(\infty,-\infty)=\alpha n. \qquad (5.6)$$

As expected, the number of photons incident on the photodetector exactly corresponds to the number of the photons of the single-mode Fock state, the fraction αn of which are being detected. Clearly since $\alpha\le 1$ the number of counts recorded does not exceed the number of initially present photons.

A much more instructive and important example from the physical point of view is that of a leaky cavity with a single excited cavity quasimode shown in Fig. 3. For the case of a quantization in free space the vector potential of the field traveling outside the cavity in $+x$ direction can be related to the cavity quasimode operators as shown by Knöll, Vogel and Welsch[18] by

$$\hat{A}^{(+)}(x_D,t)=\sum_m\Gamma^{1/2}e^{i\phi_m}\left[\frac{\hbar}{2\epsilon_0 Fc\omega_m}\right]^{1/2}\hat{a}_m\left[t-\frac{x_D}{c}\right]+\hat{A}^{(+)}_{\text{free}}, \qquad (5.7)$$

where $\hat{A}^{(+)}_{\text{free}}$ is the part of the field resulting from the reflection of incident vacuum fields at the cavity. Γ is the damping factor of the cavity

$$\Gamma=\frac{c}{2l}|\tilde{t}|^2, \qquad (5.8)$$

where $|\tilde{t}|^2$ represents the transmission coefficient of the outcoupling mirror and l is the cavity length. Because of the damping, the mean number of cavity photons decreases with increasing time according to

$$\langle\hat{a}_m^\dagger(t)\hat{a}_m(t)\rangle=\langle\hat{a}_m^\dagger(0)\hat{a}_m(0)\rangle e^{-\Gamma t}. \qquad (5.9)$$

If the quasimode is in a Fock state $|n\rangle$ at $t=t_1-x_D/c$ we obtain, according to Eq. (3.20) for $\langle\hat{m}\rangle$,

$$\langle\hat{m}(t_2;t_1)\rangle=\alpha c\int_{t_1}^{t_2}d\tau\frac{\Gamma}{c}n\,e^{-\Gamma(\tau-t_1)}=\alpha(1-e^{-\Gamma T})n. \qquad (5.10)$$

If $T=t_2-t_1$ is large compared to the cavity lifetime Γ^{-1}, the exponential in Eq. (5.10) tends to zero, which means that all photons have left the cavity and have been counted with efficiency α.

The arguments given here show that, in full agreement with Eq. (4.11), the number of counts cannot exceed the number of initial photons, even if the measuring time tends to infinity.

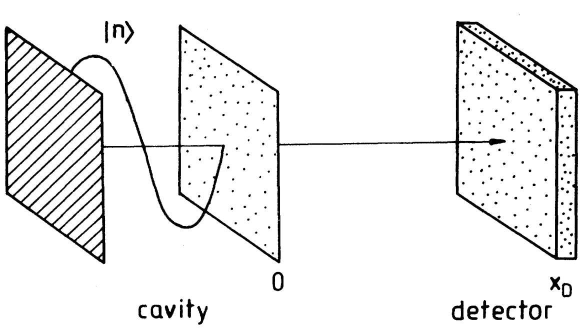

FIG. 3. A photodetector (efficiency α) measures the number of photons leaving a leaky cavity. If a single-cavity quasimode is in a Fock state $|n\rangle$ at $t=0$, the number of cavity photons at time t is given by $n\exp(-\Gamma t)$, where Γ is the cavity damping rate. The mean number of counts after a detection time T is then given by $\alpha n[1-\exp(-\Gamma T)]$ and is hence less than n.

VI. SUMMARY AND CONCLUSIONS

In this article we have studied the photodetection process for the case that a multimode light field generated by some kind of radiation sources propagates through free space and falls on a photodetector. The field damping due to its interaction with the detector atoms has been taken into account. Making use of Green's-function techniques and the rotating-wave approximation, we are able to calculate the statistics of the counts recorded in terms of input field quantities. The structure of the nonperturbative formulas obtained is more similar to those known from perturbation theory[1–3] than to the result of the nonperturbative single-mode treatment.[4–9]

Whereas in the single-mode description the field-damping modifies the structure of the perturbative formulas with respect to the temporal evolution, in our model the effect of damping consists in a spatial modification of the field over the detector volume. In the particular case of a pointlike detector this effect simply leads to a renormalization of the detector efficiency. The reason for the difference between the two descriptions may be seen in the completely different physical situations studied. The single-mode treatment corresponds to the case of the detector and the field being inside a cavity.

In practice, however, the field detected is usually the field outgoing from a cavity or other optical arrangements and propagating through free space into the detector. This is just the situation studied in our model.

Since in the former case any photon not giving rise to a count at one time is available for detection at later times, the long-time behavior of the perturbative formulas is corrected. In our case any photon not absorbed propagates away, so that there is no reason for a correction of the long-time behavior of the perturbative formulas. However, a detector atom at position x interacts with the field attenuated due to the absorption of photons by the atoms at positions $x'<x$ (Lambert-Beer law). In this context we note that, in general, the field is attenuated not only by the detection process but also by additional loss mechanisms, which in our model may be described by the interaction of the light with additional harmonic oscillators. In this case the product of the densities $\rho_\omega \rho_x$ in Eq. (3.22b) is smaller than the corresponding quantity in Eq. (3.17b), so that in Eqs. (3.23) the detector efficiency is modified according to $\xi[1-\exp(-\eta)]$, with $\xi<1$.

It is worth noting that the renormalization of the detector efficiency guarantees that the number of counts recorded during a certain time interval cannot exceed the number of photons falling on the detector during this time interval.

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