

**Entanglement generation by adiabatic navigation in the space of symmetric multiparticle states**Razmik G. Unanyan,<sup>1,\*</sup> Michael Fleischhauer,<sup>1</sup> Nikolay V. Vitanov,<sup>2,†</sup> and Klaas Bergmann<sup>1</sup><sup>1</sup>*Fachbereich Physik der Universität, 67653 Kaiserslautern, Germany*<sup>2</sup>*Theoretical Physics Division, Department of Physics, Sofia University, James Boucher 5 Boulevard, 1126 Sofia, Bulgaria*

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We propose a technique for robust and efficient navigation in the Hilbert space of entangled symmetric states of a multiparticle system with externally controllable linear and nonlinear collective interactions. A linearly changing external field applied along the quantization axis creates a network of well separated level crossings in the energy diagram of the collective states. One or more transverse pulsed fields applied at the times of specific level crossings induce adiabatic passage between these states. By choosing the timing of the pulsed field appropriately, one can transfer an initial product state of all  $N$  spins into (i) any symmetric state with  $n$  spin excitations and (ii) the  $N$ -particle analog of the Greenberger-Horne-Zeilinger state. This technique, unlike techniques using pulses of specific area, does not require precise knowledge of the number of particles and is robust against variations in the interaction parameters. We discuss potential applications in two-component Bose condensates and ion-trap systems.

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**I. INTRODUCTION**

Entanglement is a unique quantum feature which has enjoyed considerable attention in the last few years. It plays a crucial role in many rapidly developing areas of contemporary quantum physics, such as quantum information [1] and fundamental tests of quantum mechanics [2]. Various quantum systems have been suggested for controlled creation of entanglement, e.g., trapped ions [3], spins in magnetic field [4], quantum dots [5], cavity-quantum-electrodynamics systems [6], crystal lattices [7], Josephson junctions [8], and others.

In order to entangle  $N$  spin- $\frac{1}{2}$  particles interaction between the spins is required and external control of this interaction is necessary to generate specific many-particle states. For the latter purpose one can use sequences of resonant external pulses of precise area, e.g.,  $\pi$  pulses. While this technique is conceptually simple, it is very sensitive to variations in the pulse area and resonance mismatch, which can be caused by temporal and spatial fluctuations of the external field and may lead to significant errors. Hence an important practical challenge is to design robust and efficient methods for a controlled navigation in the multiparticle Hilbert space. This is of particular importance in mesoscopic systems, where only limited control over the interaction parameters and the number of particles is possible.

The simplest interaction that can lead to entanglement in a collection of spins involves either pairwise nearest-neighbor interactions or a collective coupling between all particles. An example for the first case is the Ising model [9], while the collective coupling of ions to a phonon mode in an ion trap [3] or the self-interaction in a Bose-Einstein condensate (BEC) [10] is an example for the second.

We here analyze the second type of systems and propose a method for controlled, efficient, and robust navigation in the space of symmetric multiparticle entangled states. No precise knowledge of interaction parameters or particle numbers is needed and only certain adiabaticity criteria have to be fulfilled. The proposed technique is a multiparticle generalization of our earlier proposal [11] for the creation of entanglement in a pair of two-state systems by using adiabatic passage induced by a suitably crafted external field. The multiparticle problem adds some new challenges as it involves in general multistep as well as direct transitions; this opens a variety of paths between any pair of multiparticle states. We use this to advantage and demonstrate that certain paths are insensitive to the number of particles  $N$ , a property that is particularly significant for systems with large  $N$ , for which  $N$  is not usually known exactly.

The scheme proposed in the present paper allows us to create robustly and efficiently maximum entanglement starting from a product state. In particular, it can be used to create various special entangled states, such as of the Greenberger-Horne-Zeilinger (GHZ) type or the so-called  $W$  states. Moreover, arbitrary transitions among the collective states can be realized. After introducing the concept of the method we discuss two specific implementations: an ion-trap scheme similar to that of Mølmer and Sørensen [12] and a coupled two-component BEC in the two-mode approximation [13].

**II. NAVIGATION IN THE HILBERT SPACE OF THE COLLECTIVE STATES****A. Controlled collective spin interaction**

We consider a collection of  $N$  identical spin- $\frac{1}{2}$  particles with a total angular momentum operator  $\hat{\mathbf{J}}$ . The simplest collective interaction that allows to entangle individual spins is quadratic in one of the components of  $\hat{\mathbf{J}}$ , viz. (in units  $\hbar = 1$ )

$$\hat{H}(t) = \xi \hat{J}_z^2 + \mathbf{B}(t) \cdot \hat{\mathbf{J}}, \quad (1)$$

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where  $\xi$  is the spin-spin interaction constant and  $\mathbf{B}(t)$  is some time-dependent external field. Special cases of this type of Hamiltonian have been discussed by several authors (see, e.g., [14] and references therein).

Since the Hamiltonian (1) contains only collective spin operators, only multiparticle states with the same total spin are coupled. In particular, if the system is initially in the product state of all spins in the spin-down state  $|\downarrow\downarrow\dots\downarrow\rangle$ , only symmetric collective states with the same (maximum) angular momentum  $J = \frac{1}{2}N$  will interact. Thus the  $2^N$ -dimensional Hilbert space reduces to the  $(N+1)$ -dimensional subspace of symmetric multiparticle states. Because each of these states is characterized by a definite number of excitations  $n=0,1,\dots,N$  (which corresponds to an angular momentum projection  $m = -J+n \in \{-J, J\}$ ), we shall use  $n$  to label the states. They are given by

$$|0\rangle = |\downarrow\downarrow\dots\downarrow\rangle, \quad (2a)$$

$$|1\rangle = \binom{N}{1}^{-1/2} \sum_{i=1}^N \hat{\sigma}_i^+ |0\rangle, \quad (2b)$$

$$|2\rangle = \binom{N}{2}^{-1/2} \sum_{i=1}^{N-1} \sum_{j=i+1}^N \hat{\sigma}_i^+ \hat{\sigma}_j^+ |0\rangle, \quad (2c)$$

⋮

$$|N\rangle = |\uparrow\uparrow\dots\uparrow\rangle, \quad (2d)$$

where  $\hat{\sigma}_i^+$  is the Pauli spin-flip operator, which inverts the spin of the  $i$ th particle. In particular, the single-excitation symmetric state  $|1\rangle$  is an  $N$ -particle analog of the  $W$  state [15],

$$|1\rangle = \frac{1}{\sqrt{N}} \{ |\uparrow\downarrow\downarrow\dots\downarrow\rangle + |\downarrow\uparrow\downarrow\dots\downarrow\rangle + \dots + |\downarrow\downarrow\dots\uparrow\rangle \}.$$

This state is maximally robust against disposal of any of its qubits [15]. Obviously, state  $|N-1\rangle$ , which has  $N-1$  spins up and one spin down, is also of the  $W$  type.

We assume, similarly to [11], that the particle-field coupling consists of a linearly changing component along the  $z$  direction (chosen as the quantization axis) and a pulsed component in the  $x$  direction. Then the Hamiltonian (1) takes the form

$$\hat{H}(t) = \xi \hat{J}_z^2 - A t \hat{J}_z + \Omega(t) \hat{J}_x, \quad (3)$$

where  $A$  is assumed real and positive. Because the operator  $\hat{J}_x$  connects only adjacent states, the linkage pattern is chain-wise. The direct coupling between each pair of adjacent states  $|n\rangle$  and  $|n+1\rangle$  is given by

$$\Omega_{n,n+1}(t) = \sqrt{J(J+1) - m(m+1)} \Omega(t), \quad (4)$$

i.e., all couplings have the time dependence of the external pulse, but different magnitudes.

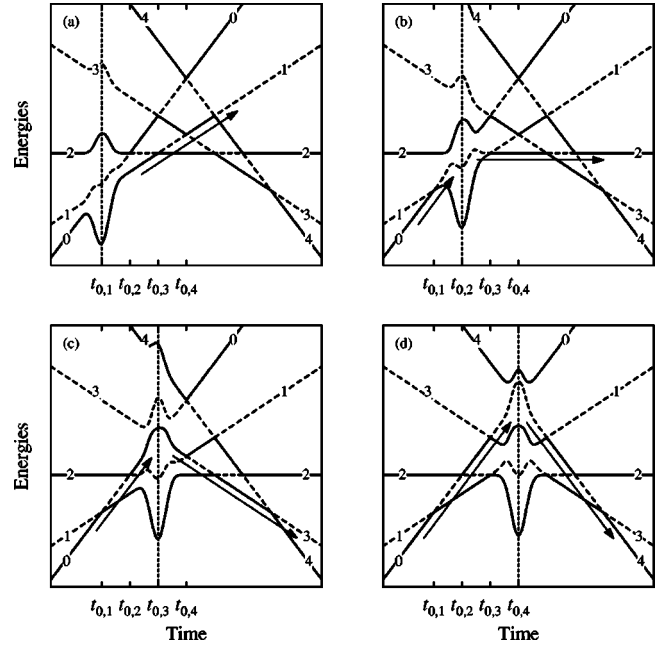


FIG. 1. Energies of the five eigenstates of the Hamiltonian (3) for  $N=4$  particles. At equidistant times  $t_{nk}$  there are diabatic level crossings between the energies (5) of the collective states. States  $|0\rangle$  and  $|4\rangle$  are product states, all others are entangled symmetric states. Application of an external coupling pulse at time  $t_{nk}$  leads to an avoided crossing and adiabatic population transfer between states  $|n\rangle$  and  $|k\rangle$ . The four frames apply to the cases when an external coupling pulse is applied at the crossings (a)  $t_{01}$ , (b)  $t_{02}$ , (c)  $t_{03}$ , and (d)  $t_{04}$ . As a result, four navigation routes (indicated by arrows) are created connecting the initial state  $|0\rangle$  to states  $|1\rangle$ ,  $|2\rangle$ ,  $|3\rangle$ , and  $|4\rangle$ , respectively.

The energy of each state  $|n\rangle$  changes linearly in time with a slope proportional to  $m = -J+n$ ,

$$E_n(t) = m^2 \xi - m A t. \quad (5)$$

This creates a web of level crossings in the energy diagram of the collective states as shown in Fig. 1 for the case of  $N=4$  particles. These level crossings can be used to design various navigation routes between the collective states.

As follows from Eq. (5), states  $|n\rangle$  and  $|k\rangle$  cross at time

$$t_{nk} = (n+k-N) \frac{\xi}{A}. \quad (6)$$

These crossings are equidistant and separated by a time interval  $\tau = \xi/A$ , which does not depend on the number of particles  $N$ , but only on the interaction parameters. Pairs of states with the same total number of excitations  $n+k$  cross at the same time. However, due to the presence of the nonlinear interaction term  $m^2 \xi$ , all level crossings are well separated in energy.

The positions of the crossings may or may not depend on  $N$ . For example, the crossing between states  $|0\rangle$  and  $|1\rangle$  is situated at  $t_{01} = -(N-1)\xi/A$ . Hence this time will be known only if  $N$  is known exactly; we will see below that this restricts the possible scenarios of adiabatic transfer. In

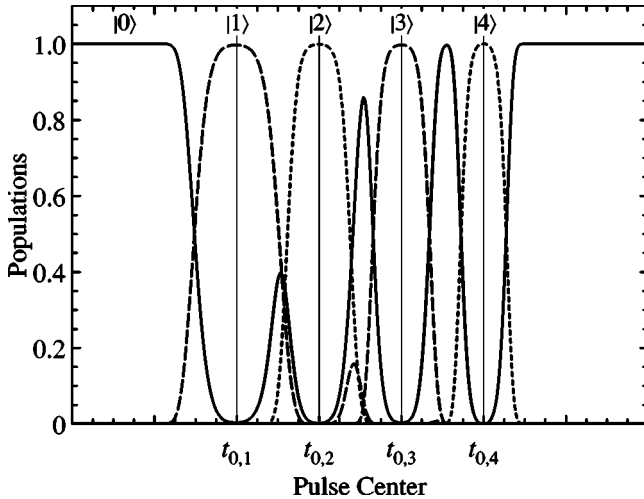


FIG. 2. Numerically calculated populations of the symmetric collective states  $|n\rangle$  in a four-particle system, initially in the product state  $|0\rangle$ , plotted against the center  $T_0$  of a Gaussian coupling pulse,  $\Omega(t) = \Omega_0 \exp[-(t-T_0)^2/T^2]$ , for  $\xi = 20T^{-1}$ ,  $\Omega_0 = 50T^{-1}$ , and  $A = 5T^{-2}$ .

contrast, the crossing between the two product states  $|0\rangle$  and  $|N\rangle$  does not depend on  $N$ , because it is situated at  $t_{0N} = 0$ , and the time  $t = 0$  is determined by the zero value of the longitudinal field,  $B_z(0) = 0$ . The crossing between states  $|N\rangle$  and  $|1\rangle$ , which is situated at  $t_{1N} = \xi/A$ , does not depend on  $N$  either.

## B. Principles of navigation in Hilbert space

### 1. Navigation routes

Once a network of level crossings is created, one can design in principle any navigation route in the Hilbert space by choosing properly the timing of the pulsed external field. It is most natural to assume that the multiparticle system is prepared initially in one of the product states, for example in  $|0\rangle$ . If the system evolves along this state and the designed route requires that it must make a transition to the entangled state  $|n\rangle$ , one should apply a sufficiently strong (adiabatic) external pulse at the time of diabatic crossing  $t_{0n}$  between  $|0\rangle$  and  $|n\rangle$ . The interaction will open an avoided crossing between the energies of the corresponding adiabatic states and will force the system to make an adiabatic transition from  $|0\rangle$  to  $|n\rangle$  [16]. On the contrary, if the route requires the system to remain in the same collective state, one should ensure that there is negligible interaction between this state and the other collective states. With the leeway in choosing the time dependence and the intensity of the pulsed external field, one can link any initial state to any final state by using one or more suitably timed pulses.

In Fig. 2 we show the numerically calculated populations of the five symmetric collective states  $|n\rangle$  in a four-particle system plotted against the center  $T_0$  of a Gaussian coupling pulse. Depending on the timing of this pulse, the population is transferred from the initial state  $|0\rangle$  to different collective states. In agreement with the above discussion, the maximum transfer efficiency for each collective state  $|n\rangle$  is achieved

when  $T_0$  is near the crossing  $t_{0n}$  between  $|0\rangle$  and  $|n\rangle$ . One also recognizes that the range of times over which complete transfer occurs decreases as the number of excitations associated with the transfer increases.

The present technique allows us also to transfer entanglement. Indeed, if the many-particle system is initially in the entangled state  $|n\rangle$ , it can be transferred adiabatically to another entangled state  $|k\rangle$  by applying an adiabatic pulse at the crossing time  $t_{nk}$  of these states. An entangled state  $|n\rangle$  can also be transferred into one of the unentangled states  $|0\rangle$  or  $|N\rangle$ ; this can be used for measurement of entanglement.

### 2. Conditions

For such a state engineering to be successful, the level crossings must be well separated, i.e., the width  $T$  of the external pulsed field must be small in comparison with the time separation between the crossings:  $T \ll \xi/A$ .

On the other hand, the condition for Landau-Zener population transfer around the crossing at  $t_{nk}$  can be shown to lead to the following conditions:

$$\Omega_{nk}(t_{nk})T \gg \sqrt{AT} \gg 1, \quad (7)$$

where  $\Omega_{nk}(t_{nk})$  is the effective coupling between states  $|n\rangle$  and  $|k\rangle$ , estimated at the crossing time  $t_{nk}$ . For adjacent states, which are connected directly, this coupling is given by Eq. (4). For example, the above conditions suffice to estimate the feasibility of the direct transition  $|0\rangle \rightarrow |1\rangle$ .

For states that are coupled via one or more intermediate states,  $\Omega_{nk}(t)$  is an effective multi-quantum coupling. This coupling can be estimated perturbatively when  $\Omega \ll N\xi$  [17] by eliminating adiabatically the off-resonant intermediate states, which yields

$$\Omega_{nk} \propto \left( \frac{\Omega}{N\xi} \right)^{|n-k|} \ll 1.$$

Hence for  $\Omega \ll N\xi$  the effective coupling between  $|n\rangle$  and  $|k\rangle$  is very small and cannot induce adiabatic evolution. Thus we have to consider the case  $\Omega \gtrsim N\xi$ , which is, however, accessible only numerically.

### C. Choice of navigation path

As the energy diagram in Fig. 1 suggests, there are multiple paths linking each pair of collective states. Each of these paths has certain advantages and disadvantages, depending on the particular experimental situation.

#### 1. Transition between the product states

The two product states  $|0\rangle$  and  $|N\rangle$  can be linked in several different ways. First, one can apply a single adiabatic pulse at their level crossing at time  $t_{0N} = 0$ , as shown in Fig. 3 (upper frame). This approach is independent of the number of particles  $N$  because the crossing time  $t_{0N} = 0$  is well defined by the zero of the linearly increasing  $z$  field. In other words, even if we do not know the exact  $N$ , we can find the exact location of the crossing between the completely unexcited state  $|0\rangle$  and the completely excited state  $|N\rangle$ . How-

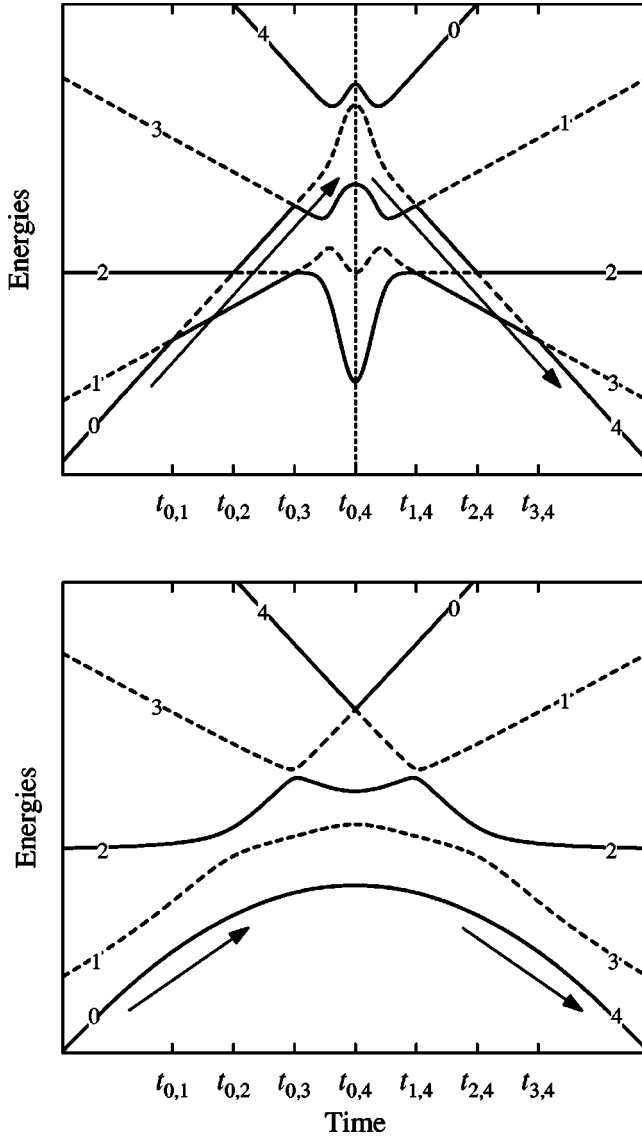


FIG. 3. Alternative routes for the transition between the product states  $|0\rangle \rightarrow |4\rangle$  in a four-particle system: a single narrow pulse applied at the crossing  $t_{0,4}$  between  $|0\rangle$  and  $|4\rangle$  (top), and a wide pulse covering all crossings (bottom).

ever, using this crossing between  $|0\rangle$  and  $|N\rangle$  requires much stronger field because these states are not coupled directly, but only via an  $N$ -quanta transition.

Alternatively, one can use a train of pulses centered at each crossing, so that the population will flow from  $|0\rangle$  through all intermediate states to reach  $|N\rangle$  at the end:  $|0\rangle \rightarrow |1\rangle \rightarrow |2\rangle \rightarrow \dots \rightarrow |N\rangle$ . Because this navigation route (the lowest solid curve in Fig. 3) passes only through crossings of directly coupled adjacent levels, much less field intensity is needed to satisfy the adiabatic condition. However, the first crossing at  $t_{01} = -(N-1)\xi/A$  depends on the number of particles  $N$ , i.e., this approach is only applicable if  $N$  is known exactly.

A third possibility, applicable only to the transition  $|0\rangle \rightarrow |N\rangle$ , is to apply a sufficiently wide single pulse, covering all crossings, as shown in Fig. 3 (lower frame). In this case,

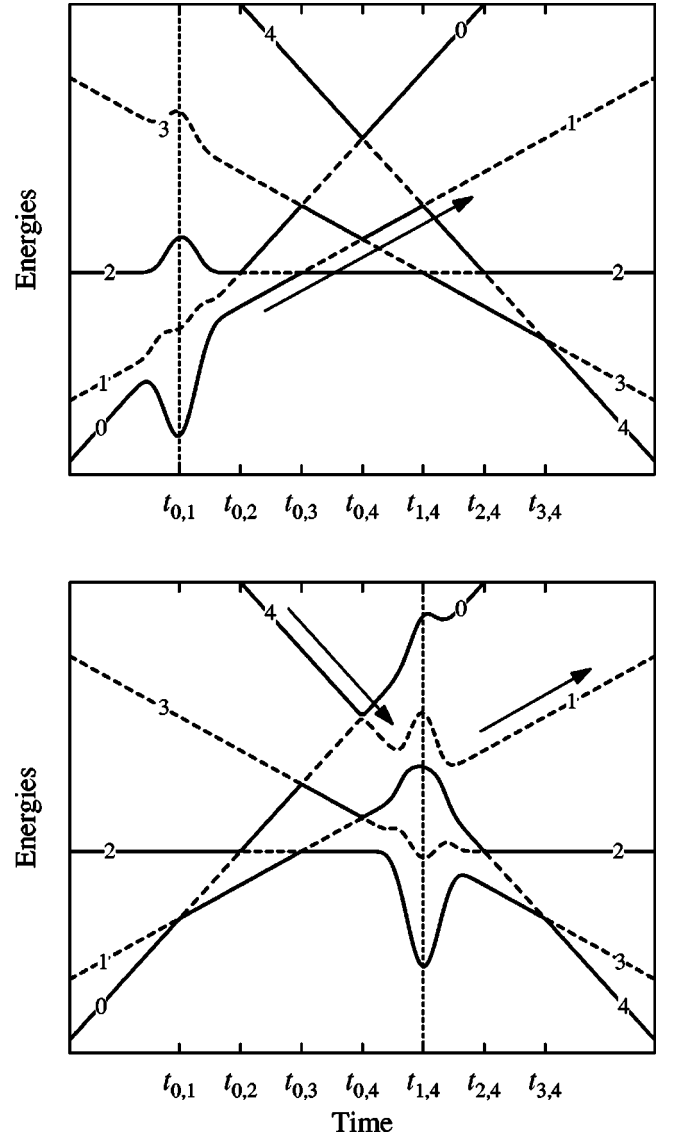


FIG. 4. Alternative routes for the creation of the  $W$  state  $|1\rangle$  in a four-particle system:  $|0\rangle \rightarrow |1\rangle$  (upper plot) and  $|4\rangle \rightarrow |1\rangle$  (lower plot).

only an approximate knowledge of  $N$  is required, because there are no stringent restrictions on the pulse width and timing.

## 2. Creation of $W$ states

One faces similar choices for the transitions from one of the product states to any entangled state  $|n\rangle$ . For example, the  $N$ -particle  $W$  state can be created using the transition  $|0\rangle \rightarrow |1\rangle$  by applying a single adiabatic pulse at time  $t_{01} = -(N-1)\xi/A$ , as shown in Fig. 4 (upper plot). Because this is a transition between adjacent collective states, only a moderately strong field is required. However, this approach is only applicable if the number of particles is known precisely because the crossing time  $t_{01}$  depends on  $N$ : an error in  $N$ , even as small as  $\Delta N = 1$ , is inadmissible since then the pulse will be applied at a wrong crossing.



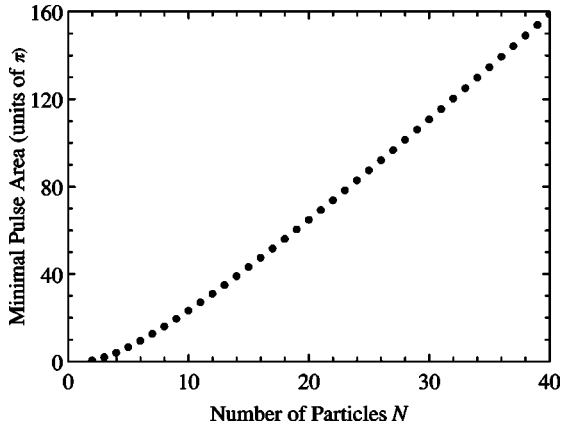


FIG. 5. Numerically calculated minimal pulse area  $\mathcal{A}_{\min}$  (defined as the pulse area for which 90% transfer efficiency is achieved) for the transition from the product state  $|N\rangle$  to the  $W$  state  $|1\rangle$  as a function of the particle number  $N$ . We have used a Gaussian pulse,  $\Omega(t) = \Omega_0 \exp[-(t-T_0)^2/T^2]$ , with  $\xi = 20T^{-1}$ ,  $A = 10T^{-2}$ , and  $T_0 = \xi/A$ .

Alternatively, state  $|1\rangle$  can be populated using the transition  $|N\rangle \rightarrow |1\rangle$ , as shown in Fig. 4 (lower plot). For this path, the crossing time  $t_{N1} = \xi/A$  does not depend on  $N$ . However, because this is an  $(N-1)$ -quanta transition, the coupled states differ by a large number of spin excitations. Then a much stronger field may be needed to widen the much narrower avoided crossing and induce adiabatic evolution. Similar conclusions apply to the other  $W$ -state  $|N-1\rangle$  and to any other entangled state  $|n\rangle$ .

### 3. Feasibility of the $N$ -invariant scenario

The  $N$ -invariance of the latter approach, which uses the (multi-quanta) crossings near  $t=0$  rather than the (single-quanta) outside crossings, is a very attractive feature because it allows us to use this technique without knowing the precise number of particles  $N$ . The only problem is that for multi-quanta transitions a stronger external field is needed to induce adiabatic passage. Therefore we have performed numerical simulations to estimate the minimal pulse area  $\mathcal{A}_{\min}$  needed to achieve 90% transfer efficiency for the transition  $|N\rangle \rightarrow |1\rangle$  from the product state  $|N\rangle$  to the  $W$  state  $|1\rangle$ . This area is plotted in Fig. 5 as a function of the particle number  $N$ .

As the figure demonstrates,  $\mathcal{A}_{\min}$  increases nearly quadratically for small  $N$  and approaches a nearly linear dependence for large  $N$ . This (slow) linear increase implies that the conditions on the required resources for application of this technique to many-particle systems are not very strong. The linear behavior can be understood qualitatively by noting that, as the numerical simulations show, the coupling term  $\hat{J}_x$  in Eq. (3) (which is  $\propto \Omega_0 \propto \mathcal{A}_{\min}$ ) is of the same order of magnitude as the vertical energy splitting (which is  $\propto N\xi$ ). Hence, the transition probability must scale with the parameter  $\Omega_0/(N\xi)$ .

### D. Generation GHZ states

Another interesting application of the present scheme, after a slight modification, is the creation of the so-called GHZ state [18],

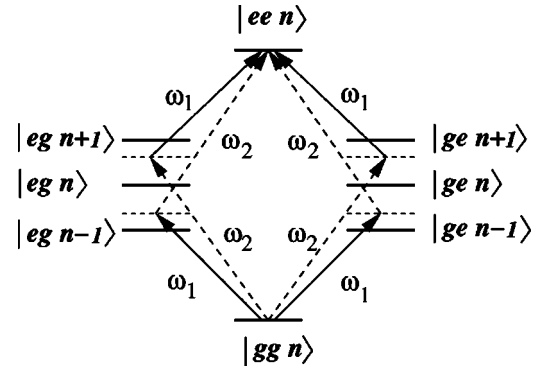


FIG. 6. Ion-trap system for realization of an effective nonlinear spin Hamiltonian. The application of a symmetrically detuned bichromatic field to ions with ground state  $|g\rangle$  and excited state  $|e\rangle$  leads to two-photon resonance between the collective states  $|gg\rangle$  and  $|ee\rangle$  independent of the trap oscillation quantum number  $n$ .

$$|\text{GHZ}\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |N\rangle). \quad (8)$$

Starting from the product state  $|0\rangle$  one has to create first an equal superposition of this state and the  $W$  state  $|1\rangle$ , for example, by a single  $\pi/2$  pulse. Then the present technique can be used to transfer the population of  $|1\rangle$  to the other product state  $|N\rangle$  by applying a single adiabatic pulse at the crossing  $t_{1N}$ ,

$$|0\rangle \xrightarrow{\pi/2} \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \xrightarrow{\text{adiab}} \frac{1}{\sqrt{2}}(|0\rangle + |N\rangle).$$

## III. IMPLEMENTATIONS

In the following we briefly discuss two implementations of the technique discussed above. The first one involves two-photon transitions in trapped ions, the second a coherently coupled two-component Bose-Einstein condensate.

### A. Ion-trap system

Recently Sørensen and Mølmer suggested a realization of a nonlinear Hamiltonian in a two-level ion trap system [12], displayed in Fig. 6. In their scheme, two laser fields (in the Lamb-Dicke limit of light coupling) are applied with frequencies  $\omega_1$  and  $\omega_2$  that are symmetrically detuned from the single-photon resonance by a detuning  $\delta$ . Then there is a two-photon resonance between states with two ions in the ground level  $|gg\rangle$  and two ions in the excited level  $|ee\rangle$ . If the detuning is larger than the Rabi frequencies of the two lasers ( $\delta \gg \Omega_1 = \Omega_2 \equiv \Omega$ ) there is no single-photon excitation.

In a coarse-grained time-averaged picture an effective Hamiltonian proportional to  $\xi \hat{J}_z^2$  emerges [12], with  $\xi = 2\eta^2 \Omega^2 \nu / (\nu^2 - \delta^2)$ ,  $\eta$  being the Lamb-Dicke parameter, and  $\nu$  the phonon frequency. If, in addition to the detuned bichromatic fields, a resonant laser coupling the  $z$  component and one coupling the  $x$  component are applied, the Hamil-

tonian (3) is obtained. Since  $\xi$  is proportional to the Lamb-Dicke parameter  $\eta \sim 1/\sqrt{N}$ , the crossing times (6) depend on the number of particles. However, this is not a significant problem because  $N$  is usually precisely known for trapped ions.

### B. Two-component BEC

Another example for our model Hamiltonian (3) is the trapped atomic BEC in two different internal states  $a$  and  $b$  [10,13], demonstrated in recent experiments with the hyperfine states  $|F=1, M=\pm 1\rangle$  in sodium [19]. The external coupling field  $\mathbf{B}(t)$  can be provided by a chirped radio-frequency pulse.

The Hamiltonian of a two-component BEC with  $s$ -wave scattering and coherent coupling between the components in the two-mode approximation is given by

$$\hat{H}(t) = \hat{H}_0 + \hat{H}_{int}(t), \quad (9a)$$

$$\hat{H}_0 = E_a a^\dagger a + E_b b^\dagger b + \frac{U_{aa}}{2} a^{\dagger 2} a^2 + \frac{U_{bb}}{2} b^{\dagger 2} b^2, \quad (9b)$$

$$\hat{H}_{int}(t) = \frac{U_{ab}}{2} a^\dagger a b^\dagger b + \Omega(t) [a b^\dagger e^{-i\phi(t)} + b a^\dagger e^{i\phi(t)}]. \quad (9c)$$

Here  $a$  and  $b$  are the annihilation operators for the bosons in the two states  $|a\rangle$  and  $|b\rangle$ , and  $E_a$  and  $E_b$  are their energies.  $U_{aa}$ ,  $U_{bb}$ , and  $U_{ab}$  characterize the particle-particle interaction when the particles are in states  $|a\rangle$  and  $|b\rangle$ . The time-dependent functions  $\Omega(t)$  and  $\phi(t)$  are the amplitude and the phase of the coherent coupling between the two components. If the elastic  $s$ -wave scattering lengths of the two components are equal ( $U_{aa} = U_{bb} = U$ ), as in the sodium experiment [19], the nonlinear spin coupling  $\xi$  will be independent of the number of particles  $N$ .

In terms of the Schwinger bosonic representation of the angular momentum,

$$\hat{J}_x = \frac{1}{2}(ab^\dagger + ba^\dagger), \quad \hat{J}_y = -\frac{1}{2}i(ab^\dagger - ba^\dagger),$$

$$\hat{J}_z = \frac{1}{2}(a^\dagger a - b^\dagger b),$$

the Hamiltonian (9) takes the form

$$\hat{H}(t) = \alpha \hat{J}_z + \xi \hat{J}_z^2 + \Omega(t) [\hat{J}_x \cos \phi(t) + \hat{J}_y \sin \phi(t)],$$

where  $\alpha = E_a - E_b$  and  $\xi = U - \frac{1}{2}U_{ab}$ . After a unitary transformation  $|\Psi(t)\rangle = \exp[-i\phi(t)\hat{J}_x]|\Phi(t)\rangle$  we obtain a Hamiltonian equivalent to Eq. (9),

$$\hat{H}(t) = \xi \hat{J}_z^2 + [\alpha - \dot{\phi}(t)] \hat{J}_z + 2\Omega(t) \hat{J}_x.$$

### IV. SUMMARY

In the present paper we have proposed and analyzed a robust adiabatic scheme for generating symmetric entangled states of a many-particle system. A constant nonlinear interaction in the collective spin projection  $\hat{J}_z^2$  combined with a time-dependent linear interaction in  $\hat{J}_z$  creates a web of level crossings between the collective multiparticle states. Application of pulsed fields along the  $x$  component of the collective spin at the times of appropriate level crossings induces single or multistep transitions between the states. In this way a controlled adiabatic navigation in the  $(N+1)$ -dimensional subspace of the symmetric collective states is possible allowing, for example, generation of  $N$ -particle  $W$  and GHZ states. The suggested method is robust against parameter variations and does not request an exact knowledge of the particle number, but only demands proper timing of the pulsed field. It should be noted that the nonlinear interaction  $\hat{J}_z^2$  guarantees the separation of the level crossings and is hence necessary for the navigation in the Hilbert space.

An initial product state is connected to symmetric entangled states via various pathways involving avoided crossings induced by single and/or multiparticle interactions. If the exact number of particles is known, it is in general possible to find a pathway that runs only through single-excitation crossings; then only a moderately strong field is needed to induce adiabatic passage. If  $N$  is not known, a pathway can be chosen that involves multiphoton avoided crossings. Our numerical studies indicate that in this case the required field scales nearly linearly with  $N$ . We have given two explicit examples for the application of our scheme, ion traps and coherently coupled two-component Bose condensates.

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