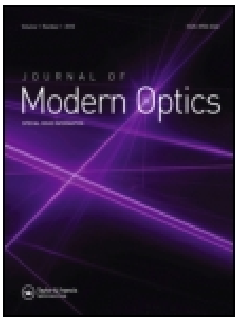

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# The influence of optical processing through linear passive systems on the quantum properties of light

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**Abstract.** A microscopic quantum description of the interaction between a radiation field coupled to sources and linear passive optical systems is presented. The quantum input–output relations for correlation functions that determine the quantum statistical properties are established under general conditions. In particular the influence of the vacuum field is taken into account. The treatment has advantageously been performed with the help of Green function techniques, which allow one to include loss mechanisms and dispersion of the linear medium. To illustrate the general results two interesting examples, the diminishing of squeezing due to a lossy Fabry–Perot interferometer and the action of a black screen, are given.

## 1. Quantum against classical propagation

Linear passive systems, which are the basic elements in the generation and the transmission of light, can strongly change the statistical properties of the electromagnetic field [1–4, 6]. In particular the application of non-classical light [5] requires an exact quantum analysis since its relatively low noise can disadvantageously be affected by the action of these elements. In the present article we describe this action in a general way, that means we take into account the presence of radiation sources and include linear loss mechanisms and dispersion.

In the classical linear propagation problem the output field $E_{\text{out}}(t)$ is related to the signal input field $E_s(t)$ or the current $j_s(t)$ of the radiation source by the linear integral relations

$$\begin{aligned} E_{\text{out}}(t) &= \int_0^\infty \mathrm{d}\tau\, G(\tau) E_s(t-\tau) \\ &= \int_0^\infty \mathrm{d}\tau\, D(\tau) j_s(t-\tau), \end{aligned} \tag{1}$$

(see Born and Wolf, Principles of Optics) where $G(\tau)$, $D(\tau)$ represent the time dependent instrumental functions. In a quantum mechanical description as indicated in figure 1, all vacuum field components must be taken into account as well. Hence the quantum input–output relations read:

$$\begin{aligned} \hat{E}_{\text{out}}(t) &= \int_0^\infty \mathrm{d}\tau\, G(\tau)\hat{E}_s(t-\tau) + \int_0^\infty \mathrm{d}\tau\, \tilde{G}(\tau)\hat{E}_{\text{vac}}(t-\tau) \\ &= \int_0^\infty \mathrm{d}\tau\, D(\tau)\hat{j}_s(t-\tau) + \int_0^\infty \mathrm{d}\tau\, \tilde{G}(\tau)\hat{E}_{\text{vac}}(t-\tau), \end{aligned} \tag{2}$$

where $\tilde{G}(\tau)$ is the instrumental function corresponding to the vacuum input port. We can investigate the alteration of quantum statistical properties of the field from input

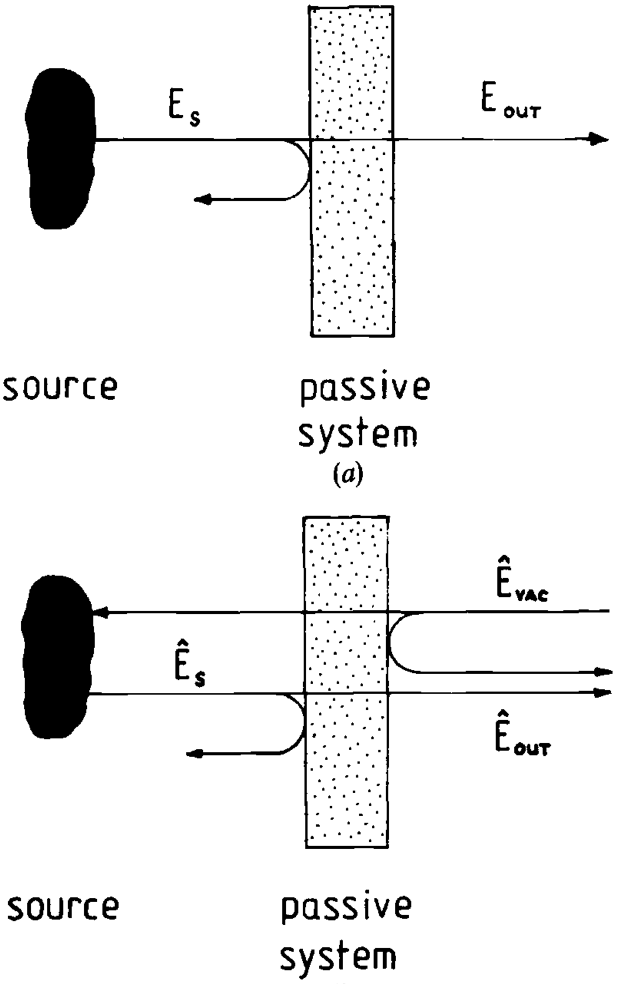


Figure 1. Classical (*a*) against quantum (*b*) propagation problem. While in the classical propagation problem the output field is determined only by the signal input field in the quantum case the vacuum part of the field has to be taken into account. This vacuum field prevents a straight forward calculation of an input–output relation, because it interacts *via* the sources with the signal field.

to output by evaluating the relation between correlation functions of the input signal field $\hat{E}_s(t)$ or source variables $\hat{j}_s(t)$ and the output field $\hat{E}_{out}(t)$. Unfortunately the vacuum term in equation (2) prevents a straightforward calculation of such relations from equation (2). Note that in normally ordered correlation functions of the output field (all quantum statistical measures can be expressed by a combination of them) the vacuum terms may be disregarded only if the vacuum field $\hat{E}^{(\pm)}_{vac}(t)$ commutes with the signal input field $\hat{E}^{(\pm)}_s(t)$. In the presence of sources, however, the vacuum input field interacts *via* the sources with the signal field, as indicated in figure 1 (*b*). Hence the commutator $[\hat{E}^{(-)}_{vac}(t), \hat{E}^{(-)}_s(t')]$ is in general non-zero. Only if $t$ is larger than $t'$ it vanishes, because the signal field is independent of the input vacuum field at a later instance of time. That means the vacuum terms in equation (2) are meaningless in correlation functions which are normally and time-ordered with respect to the *input* fields. In some special cases as investigated for instance in the input–output theory of Gardiner and Collett [7, 8] the system's response is $\delta(\tau)$-like and hence the *input* fields are time-ordered if the *output* fields are. So the vacuum contributions may be disregarded for these systems if normally and time-ordered correlation functions of the output field are calculated. However, the handling of the vacuum field is non-trivial if more general linear devices are considered. The case of a lossless and non-dispersive linear medium has been investigated by Knöll, Vogel and Welsch [9]. Making use of a quantization procedure which includes the influence of the linear passive medium they were able to calculate the commutator $[\hat{E}^{(-)}_{vac}(t), \hat{E}^{(-)}_{s}(t')]$ in terms of source quantities and could so eliminate the vacuum terms. However, this treatment can not be generalized to dispersive and lossy media. For such media a completely different approach is necessary. As an appropriate tool we have used the method of Green functions, which has been successfully applied in the field of solid state physics. For simplicity we investigate an arrangement with special geometry, but nevertheless general conclusions can be drawn. We show, that for arbitrary linear passive systems the vacuum input fields can be eliminated in an input–output relation of normally and time-ordered correlation functions. We obtain linear integral relations between such correlation functions of the output and the signal input, where the integral kernels are the well-known classical instrumental functions of the passive system. Since all quantum statistical measures, such as the Poisson-excess or the squeezing parameter can be expressed by a combination of normally and time-ordered correlation functions of different order the correspondence between the classical and quantum propagation problem allows a straightforward evaluation of the influence of optical processing on the quantum statistical properties of the radiation.

Our paper is organized as follows: The model is described in section 2. The passive system is assumed to be an ensemble of damped harmonic oscillators with different frequencies, which can be considered as a linear approach to a system of damped two-level atoms. For the sake of simplicity we consider only a quasi-one-dimensional arrangement. We derive in section 3 an integral relation between the relevant field correlation functions and the source variable correlation functions. The corresponding integral kernel or instrumental function is calculated in section 4 and proves to be identical to the classical instrumental function. In addition to the results presented in section 3, we derive in section 6 relations of the output field correlation functions to the corresponding signal field functions. A general property of realistic linear optical processing namely the diminishing of non-classical properties is illustrated for the case of squeezed light transmitted through a lossy Fabry–Perot interferometer in section 7. Moreover our approach allows to describe the action of a 'black' screen a device often used in experimental arrangements for instance as a shutter but not well described up to now from the quantum theory point of view.

## 2. Model

A general passive system can be well described by an ensemble of damped two-level atoms in a thermodynamic equilibrium. If the intensities of the incoming light fields are appropriately small, the action of the system is linear. In this case the effective population of the upper atomic levels is negligible, and therefore the Pauli operators of the two level systems can be replaced by Bose variables. This corresponds to the lowest order of the Holstein–Primakoff expansion of Pauli operators [12]. Hence we may consider as a model of wide generality a layer of damped harmonic oscillators, which are homogeneously distributed over the region $-l/2 \leqslant x < l/2$. This model can be regarded as the natural extension of the classical dispersion theory. This quasi one-dimensional arrangement is depicted in figure 1.

The sources irradiate the passive system from the left and a detector or other optical instruments are placed behind the passive system. $\hat{b}_{i\lambda}$ and $\hat{b}_{i\lambda}^+$ are the Bose annihilation and creation operators of the oscillators with the frequency $\omega_\lambda$ at a place $x_i$. The 'vector' potential $\hat{A}(x, t)$ of the radiation is decomposed into the annihilation and creation operators $\hat{a}_k$ and $\hat{a}_k^+$ in the usual way [13]:

$$\hat{A}(x, t) = \sum_k \left(\frac{\hbar}{2\varepsilon_0 \omega_k L}\right)^{1/2} [\hat{a}_k(t) \exp(ikx) + \text{h.c.}]. \tag{3}$$

The Hamiltonian of the system includes the free electromagnetic field, the oscillators of the passive system, the bath with zero temperature, the field-oscillator, and the oscillator-bath coupling.

$$\hat{H} = \hat{H}_1 + \hat{H}_s^0 + \hat{H}_{sf}, \tag{4}$$

$$\hat{H}_1 = \hat{H}_{ps}^0 + \hat{H}_f^0 + \hat{H}_{psf}. \tag{5}$$

$\hat{H}_1$ is the Hamiltonian of the passive system (ps) interacting with the electromagnetic field (f). The Hamiltonian $\hat{H}_{ps}^0$ of the passive system describes the harmonic oscillators and their damping by an additional Bosonic bath. The interaction operator between the passive system and the electromagnetic field $\hat{H}_{psf}$ reads in pA-approximation

$$\hat{H}_{psf}(t) = -\sum_{i\lambda} \frac{e}{m} \hat{p}_{i\lambda}(t) \hat{A}(x_i, t) = -\int_{-l/2}^{l/2} dx\, \hat{B}(x, t) \hat{A}(x, t), \tag{6}$$

where

$$\hat{p}_{i\lambda} = i\left(\frac{\hbar m \omega_\lambda}{2}\right)^{1/2} (\hat{b}_{i\lambda}^+(t) - \hat{b}_{i\lambda}(t))$$

is the momentum operator of the $\lambda$-th oscillator and

$$\hat{B}(x, t) = -\sum_{i\lambda} \frac{e}{m} \delta(x - x_i) \hat{p}_{i\lambda}(t) \hat{H}_s^0$$

is the source Hamiltonian and $\hat{H}_{sf}$ describes the interaction between the sources and the field

$$\hat{H}_{sf}(t) = -\int_{-\infty}^{\infty} dx\, \hat{j}(x, t) \hat{A}(x, t), \tag{7}$$

where $\hat{j}(x, t)$ is the current density operator of the source atoms.

Under the above conditions we may start from an initial vacuum state $|0\rangle$ of the light field. Moreover, because $T = 0$, the passive system is initially in its ground state $|g\rangle$, which makes it possible to use the Green function technique with respect to these variables. The source atoms are initially in an arbitrary state. In the remainder of this paper we will use the following notation; dynamical variables without subscript are to be thought in the Heisenberg picture, operators with the subscript I correspond to the interaction picture, that means they evolve according to their free Hamiltonians $\hat{H}^0$. In addition operators of the subsystem (ps+f) with a subscript W evolve according to $\hat{H}_1$. If $S$ and $S_1$ are the time evolution operators according to

$$S(t_2, t_1) = T_- \exp\left[\frac{i}{\hbar} \int_{t_1}^{t_2} d\tau \int_{-\infty}^{\infty} dx\, \hat{j}_I(x, \tau) \hat{A}_W(x, \tau)\right], \tag{8}$$

and

$$S_1(t_2, t_1) = T_- \exp\left[\frac{i}{\hbar}\int_{t_1}^{t_2} d\tau \int_{-\infty}^{\infty} dx\, \hat{B}_I(x, \tau)\hat{A}_I(x, \tau)\right], \tag{9}$$

where $T_-$ ($T_+$) is the time (anti-time) ordering operator, the following relations between these three types hold:

$$\hat{A}(x, t) = S(-\infty, t)\hat{A}_W(x, t)S(t, -\infty), \tag{10}$$

$$\hat{A}_W(x, t) = S_1(-\infty, t)\hat{A}_I(x, t)S_1(t, -\infty). \tag{11}$$

## 3. Relation between field and source correlation functions

The measurable quantities of the electromagnetic field in the case of detectors basing on the external photoeffect are the following correlation functions of operators in the Heisenberg picture

$$\langle T_+[\hat{A}^{(+)}(1)\dots\hat{A}^{(+)}(N)]T_-[\hat{A}^{(-)}(1')\dots\hat{A}^{(-)}(M')]\rangle. \tag{12}$$

The symbols (+) and (−) denote here and in the remainder the positive or negative frequency part of the vector potential. The numbers $1, \dots, N$ are used as an abbreviation for the time and space coordinates $(x_1, t_1), \dots, (x_N, t_N)$. The appearance of the frequency components in (12) is a result of a rotating wave approximation in the detector–field interaction (DFRWA) [13]. Originally the measurable quantities contain the whole potential operators

$$\langle T_+[\hat{A}(1)\dots\hat{A}(N)]T_-[\hat{A}(1')\dots\hat{A}(M')]\rangle. \tag{13}$$

For our purposes it is more convenient to use the latter expression and to apply the DFRWA at the end of the calculations.

Now we want to apply the method of (non-equilibrium) Green functions in particular the Keldysh technique [11]. In doing so we introduce a time contour $\mathscr{C}$ consisting of two branches (figure 2). A physical time may be located on the positive or negative branch. In addition one defines a time ordering operator $T_{\mathscr{C}}$ that orders operators of different time arguments in the following sense: on the upper branch $T_{\mathscr{C}}$ corresponds to the ordinary time ordering operator $T_-$, on the lower branch to $T_+$, respectively. In addition a time of the lower branch is later as any time on the upper branch. Now the correlation function (13) can be expressed by

$$\langle T_+[\hat{A}(1)\dots\hat{A}(N)]T_-[\hat{A}(1')\dots\hat{A}(M')]\rangle$$
$$= \langle T_{\mathscr{C}}\hat{A}(1_-)\dots\hat{A}(N_-)\hat{A}(1'_+)\dots\hat{A}(M'_+)\rangle, \tag{14}$$

the subscript $\pm$ at the time arguments denotes the branch of the contour. With the help of the time evolution operator (8) on $\mathscr{C}$ we can rewrite (14) into

$$\langle T_{\mathscr{C}}\hat{A}(1_-)\dots\hat{A}(M'_+)\rangle = \langle T_{\mathscr{C}}S_{\mathscr{C}}\hat{A}_W(1_-)\dots\hat{A}_W(M'_+)\rangle. \tag{15}$$

Following the usual procedures of the Green's function formalism we expand $S_{\mathscr{C}}$ into a power series and eliminate the field variables $\hat{A}_W$ with the help of Wick's theorem [10]. As a result the correlation function (15) can be expressed by a source operator correlation function via an linear integral transformation. The integral kernel

$$D_W(1, 2) \equiv \langle T_{\mathscr{C}}\hat{A}_W(1)\hat{A}_W(2)\rangle, \tag{16}$$

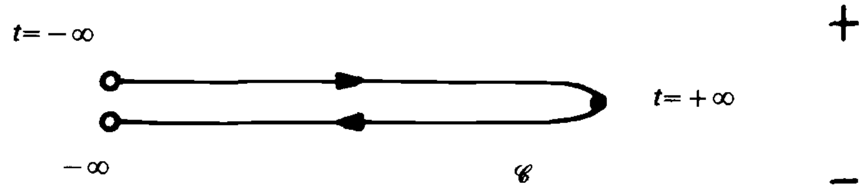


Figure 2. The Keldysh time contour.

is the Green function of the light interacting with the passive system. For a second-order correlation function one would obtain for instance:

$$\langle T_{\mathscr{C}}\hat{A}(1)\hat{A}(2)\rangle = D_{\mathrm{W}}(1,2) - \frac{1}{\hbar^2}\int_{\mathscr{C}} d1' \int_{\mathscr{C}} \mathrm{d}2'\, D_{\mathrm{W}}(1,1')D_{\mathrm{W}}(2,2')\langle T_{\mathscr{C}}\hat{j}(1')\hat{j}(2')\rangle, \quad (17)$$

where the time integration goes over the whole contour $\mathscr{C}$. Introducing a matrix notation

$$\mathbf{D}(1,2) \equiv \begin{bmatrix} D^{++}(1,2) & D^{+-}(1,2) \\ D^{-+}(1,2) & D^{--}(1,2) \end{bmatrix} \equiv \begin{bmatrix} \langle T_{-}\hat{A}(1)\hat{A}(2)\rangle & \langle \hat{A}(2)\hat{A}(1)\rangle \\ \langle \hat{A}(1)\hat{A}(2)\rangle & \langle T_{+}\hat{A}(1)\hat{A}(2)\rangle \end{bmatrix}, \quad (18)$$

analogously for $\mathbf{D}_{\mathrm{W}}$, and

$$\mathbf{j}(1,2) \equiv \begin{bmatrix} j^{++}(1,2) & j^{+-}(1,2) \\ j^{-+}(1,2) & j^{--}(1,2) \end{bmatrix} \equiv \begin{bmatrix} \langle T_{-}\hat{j}(1)\hat{j}(2)\rangle & -\langle \hat{j}(2)\hat{j}(1)\rangle \\ -\langle \hat{j}(1)\hat{j}(2)\rangle & \langle T_{+}\hat{j}(1)\hat{j}(2)\rangle \end{bmatrix}, \quad (19)$$

these contour integrals can be transformed into ordinary integrals:

$$\mathbf{D}(1,2) = \mathbf{D}_{\mathrm{W}}(1,2)$$
$$-\frac{1}{\hbar^2}\int_{-\infty}^{\infty} \mathrm{d}t_1' \int_{-\infty}^{\infty} \mathrm{d}t_2' \int_{-\infty}^{\infty} \mathrm{d}x_1' \int_{-\infty}^{\infty} \mathrm{d}x_2'\, \mathbf{D}_{\mathrm{W}}(1,1')\mathbf{j}(1',2')\mathbf{D}_{\mathrm{W}}^{\mathrm{T}}(2',2). \quad (20)$$

The matrix element $D^{+-}(1,1)$ yields for instance after the DFRWA an expression for the field intensity behind the passive system. As we will show in section 4 the matrix $\mathbf{D}_{\mathrm{W}}$ diagonalizes for this approximation, and so we obtain for $D^{+-}(1,1)$ after the DFRWA:

$$\langle \hat{A}^{(+)}(1)\hat{A}^{(-)}(1)\rangle = \frac{1}{\hbar^2}\int_{-\infty}^{\infty} \mathrm{d}1' \int_{-\infty}^{\infty} \mathrm{d}1''\, D_{\mathrm{W}}^{--}(1,1')D_{\mathrm{W}}^{++}(1,1'')\langle \hat{j}(1')\hat{j}(1'')\rangle. \quad (21)$$

Similar relations can be derived for higher correlation functions. For instance, there holds:

$$\langle T_{+}[\hat{A}^{(+)}(1)\hat{A}^{(+)}(2)]T_{-}[\hat{A}^{(-)}(2)\hat{A}^{(-)}(1)]\rangle$$
$$= \frac{1}{\hbar^4}\int_{-\infty}^{\infty} \mathrm{d}1' \int_{-\infty}^{\infty} \mathrm{d}2' \int_{-\infty}^{\infty} \mathrm{d}1'' \int_{-\infty}^{\infty} \mathrm{d}2''$$
$$\times D_{\mathrm{W}}^{--}(1,1')D_{\mathrm{W}}^{--}(2,2')D_{\mathrm{W}}^{++}(2,2'')D_{\mathrm{W}}^{++}(1,1'')$$
$$\times \langle T_{+}[\hat{j}(1')\hat{j}(2')]T_{-}[\hat{j}(2'')\hat{j}(1'')]\rangle. \quad (22)$$

In classical linear optics the output light field is determined by the source current via a linear integral relation according to equation (1). The integral kernel $D$ in this relation is in the case of the vacuum simply the free space propagator of the field. In the presence of passive systems it moreover describes the response of the passive system on the field. We noted in section 1 that there is no similar relation for the field operators, because of the additional vacuum terms in equation (2). However, we see from equations (21) and (22), that for the measurable quantities (which are normally and time-ordered correlation functions) as the field intensity (equation (21)) or intensity correlation (equation (22)) such a relation exists. The integral kernel $D_W^{++}$ in equations (21, 22) is according to equation (16) the Green function of the radiation field interacting with the passive system. Hence the response of the passive system in the quantum case is completely described by it. It will be shown in the next section that it equals the well known classical propagator, as expected. It is not surprising that a normal ordering of the output operators is necessary to avoid the appearance of vacuum terms in equations (21) and (22). However, it is just the combination of the normal with the time-ordering that actually leads to the disappearance of vacuum contributions and to input–output relations which are apart from the ordering of the operators equivalent to the classical input–output relations. An explanation for this will be given in section 6.

## 4. Calculation of the instrumental function $D_W(1,2)$

In this section we derive a Dyson equation for $D_W(1,2)$. The self-energy part of this equation is a Green function of the non-interacting passive system, which is well known, and therefore allows an almost trivial calculation of $D_W$. With the help of the time evolution operator

$$S_{1\mathscr{C}} = T_{\mathscr{C}} \exp\left[\frac{\mathrm{i}}{\hbar}\int_{\mathscr{C}} \mathrm{d}t \int_{-l/2}^{l/2} \mathrm{d}x\, \hat{B}_{\mathrm{I}}(x,t)\hat{A}_{\mathrm{I}}(x,t)\right], \tag{23}$$

the required Green function $D_W(1,2)$ can be expressed by

$$D_W(1,2) = \langle T_{\mathscr{C}} S_{1\mathscr{C}} \hat{A}_{\mathrm{I}}(1)\hat{A}_{\mathrm{I}}(2)\rangle. \tag{24}$$

Following the Green function formalism again, we finally obtain the Dyson equation

$$D_W(1,2) = D_0(1,2) - \frac{1}{\hbar^2}\int_{\mathscr{C}} \mathrm{d}1' \int_{\mathscr{C}} \mathrm{d}2'\, D_0(1,1')B_0(1',2')D_W(2',2), \tag{25}$$

where $B_0(1,2) = \langle T_{\mathscr{C}} \hat{B}_{\mathrm{I}}(1)\hat{B}_{\mathrm{I}}(2)\rangle$ is the Green function of the non-interacting passive system, which is calculated in the microscopic oscillator model in the Appendix. Using Feynman graphs the Dyson equation can simply be expressed as shown in figure 3. Introducing $2\times 2$ matrices analogously to section 3, and noting from the Appendix, that $B_0(1,2) = B_0(\tau = t_1 - t_2)\ \delta(x_1 - x_2)\ \varepsilon(x_1)$ with $\varepsilon(x_1)$ being 1 for $-l/2 \leqslant x_1 \leqslant l/2$ and 0 elsewhere, we obtain the matrix form of the Dyson equation. A Fourier transformation with respect to $\tau = t_1 - t_2$ according to

$$F(\omega) \equiv \int_{-\infty}^{\infty} \mathrm{d}\tau \exp(\mathrm{i}\omega\tau)\, F(\tau), \tag{26}$$

yields

$$\mathbf{D}_W(x_1, x_2; \omega) = \mathbf{D}_0(x_1, x_2; \omega) - \frac{1}{\hbar^2}\int_{-l/2}^{l/2} \mathrm{d}x\, \mathbf{D}_0(x_1, x; \omega)\mathbf{B}_0(\omega)\mathbf{D}_W(x_1, x; \omega). \tag{27}$$

In the DFRWA $D_W^{-+}$ and $D_W^{--}$ are relevant only for negative $\omega$-values, and $D_W^{+-}$ and $D_W^{++}$ for positive $\omega$ values. Because $D_0^{+-}(x_1, x_2; \omega)$ and $B_0^{+-}(\omega)$ vanish for positive $\omega$ values and $D_0^{-+}$ and $B_0^{-+}$ for negative values respectively (see Appendix), the matrix

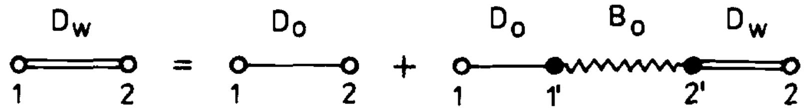


Figure 3. Graphical expression of the Dyson equation of the Green's function $D_W(1,2)$. The solid line represents the non-interacting Green function of the electromagnetic field, the wavy line that of the non-interacting passive system, respectively.

Dyson equation decouples into the following equations in the corresponding $\omega$-regions:

$$D_W^{+-}(x_1,x_2;\omega)=D_W^{-+}(x_1,x_2;\omega)=0, \tag{28 a}$$

$$D_W^{\pm\pm}(x_1,x_2;\omega)=D_0^{\pm\pm}(x_1,x_2;\omega) -\frac{1}{\hbar^2}\int_{-l/2}^{l/2} \mathrm{d}x\, D_0^{\pm\pm}(x_1,x_2;\omega)B_0^{\pm\pm}(\omega)D_W^{\pm\pm}(x_1,x_2;\omega). \tag{28 b}$$

The integral equation (28 *b*) can be solved by a transformation into an ordinary differential equation.

We are interested in the field properties behind the passive system. Hence we consider the case $x_1>l/2$. On the other hand the position of the sources is normally in front of or inside the dielectricum and therefore the cases $x_2<-l/2$ and $-l/2\leqslant x_2\leqslant l/2$ are of interest. The solution of (28 *b*) are for these cases:

$$D_W^{++}(x_1,x_2;\omega)=\frac{\hbar}{2\varepsilon_0\omega c}\frac{1-r^2}{1-r^2\exp\left(2\mathrm{i}\frac{\omega}{c}nl\right)}\exp\left[\mathrm{i}\frac{\omega}{c}(n-1)l\right]\exp\left[\mathrm{i}\frac{\omega}{c}(x_1-x_2)\right],$$
$$x_1>\frac{l}{2}\ \text{and}\ x_2<-\frac{l}{2}, \tag{29}$$

and

$$D_W^{++}(x_1,x_2;\omega)=\frac{\hbar}{2\varepsilon_0\omega c}\frac{1-r^2}{1-r^2\exp\left(2\mathrm{i}\frac{\omega}{c}nl\right)}$$
$$\left\{\frac{n+1}{2n}\exp\left[\mathrm{i}\frac{\omega}{c}(n-1)\frac{l}{2}\right]\exp\left[\mathrm{i}\frac{\omega}{c}(x_1-nx_2)\right]+\frac{n-1}{2n}\exp\left[\mathrm{i}\frac{\omega}{c}(3n-1)\frac{l}{2}\right]\right.$$
$$\left.\times\exp\left[\mathrm{i}\frac{\omega}{c}(x_1+nx_2)\right]\right\},\ x_1>\frac{l}{2}\ \text{and}\ -\frac{l}{2}\leqslant x_2\leqslant\frac{l}{2}, \tag{30}$$

with

$$r\equiv\frac{n-1}{n+1}, \tag{31}$$

and

$$n(\omega)\equiv\left(1+\frac{\mathrm{i}B_0^{++}(\omega)}{\varepsilon_0\hbar\omega^2}\right)^{1/2}. \tag{32}$$

These Green functions are the same as in the classical propagation problem, provided $n(\omega)$ in equation (32) is regarded as the (possibly complex) refractive index of the medium.

In the following section we will show that this interpretation of $n(\omega)$ is valid.

**5. Physical interpretation of $B_0^{++}(\omega)$**

The dynamical behaviour of an excited free electromagnetic field in the presence of a passive system is determined by the evolution of the density matrix $\hat{\rho}$ in the interaction picture by

$$i\hbar\frac{d}{dt}\hat{\rho}(t)=[\hat{H}_1(t),\hat{\rho}(t)], \tag{33}$$

or equivalently by the time evolution operator $S_1(t,-\infty)$

$$\begin{aligned}\hat{\rho}(t)&=S_1(t,-\infty)\hat{\rho}_0S_1(-\infty,t)\\&=S_1(t,-\infty)|0\rangle_{ps}|\psi\rangle_{ff}\langle\psi|_{ps}\langle 0|S_1(-\infty,t),\end{aligned} \tag{34}$$

where $\hat{\rho}_0$ is the density matrix at $t=-\infty$ which is assumed to be the product of the ground state of the passive system $|0\rangle_{ps}$ and an arbitrary state $|\psi\rangle_f$ of the field. Since we are interested in field correlation functions only we can trace out the degrees of freedom of the passive system. If we assume that the passive system remains approximately in its initial state, the tracing procedure reduces to

$$\hat{\rho}_f(t)=\frac{{}_{ps}\langle 0|\hat{\rho}(t)|0\rangle_{ps}}{\mathrm{Tr}_f\{\langle 0|\hat{\rho}(t)|0\rangle\}}. \tag{35}$$

The time evolution of $\tilde{\rho}_f$ is then governed by

$$\begin{aligned}{}_{ps}\langle 0|S_1(t,-\infty)|0\rangle_{ps}=T_-\exp\Bigg[\frac{i}{\hbar}\int_{-\infty}^{\infty}dt'\int_{-\infty}^{t'}dt''\int_{-l/2}^{l/2}dx\frac{i}{\hbar}B_0^{++}(t'-t'')\\ \times\hat{A}_I(x,t')\hat{A}_I(x,t'')\Bigg].\end{aligned} \tag{36}$$

The original time evolution operator $S_1$ had been the formal solution of the Heisenberg equation of motion

$$i\hbar\frac{d}{dt}S_1(t,t_0)=\hat{H}_{psfI}(t)S_1(t,t_0), \tag{37}$$

with the condition $S_1(t,t)=1$. Hence a derivation of the averaged evolution operator should lead to an 'effective Hamiltonian'. Carrying out the derivation we obtain

$$i\hbar_{ps}\langle 0|S_1(t_1t_0)|0\rangle_{ps}=\hat{H}_{effI}(t)_{ps}\langle 0|S_1(t,t_0)|0\rangle_{ps}, \tag{38}$$

with

$$\hat{H}_{effI}(t)=-\int_{-\infty}^{\infty}d\tau\int_{-l/2}^{l/2}dx\frac{i}{\hbar}B_0^{++}(\tau)\hat{A}_I(x,t)\hat{A}_I(x,t-\tau). \tag{39}$$

Now we want to compare the microscopic 'effective Hamiltonian' with a macroscopic one. We start with the classical Maxwell equations in the presence of a dielectric without external charges or currents

$$\mathrm{curl}\,\mathbf{B}=\mu_0(\varepsilon_0\dot{\mathbf{E}}+\dot{\mathbf{P}}),\quad \mathrm{curl}\,\mathbf{E}=-\dot{\mathbf{B}}, \tag{40}$$

$$\mathrm{div}\,\mathbf{E}=-\frac{1}{\varepsilon_0}\mathrm{div}\,\mathbf{P},\quad \mathrm{div}\,\mathbf{B}=0. \tag{41}$$

We assume a linear relation between the polarization $\mathbf{P}$ and the electric field $\mathbf{E}$

$$\mathbf{P}(\mathbf{r},t)=\int_{-\infty}^{\infty} \mathrm{d}\tau\, \kappa(\tau)\mathbf{E}(\mathbf{r},t)\varepsilon(\mathbf{r}), \tag{42}$$

with $\varepsilon(\mathbf{r})$ being 1 inside the dielectric and 0 outside. As can be seen from equation (37) we may define a current density $\mathbf{j}_\mathrm{M}$ of the medium by

$$\mathbf{j}_\mathrm{M}=\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{P}.$$

Passing over to a quantum description, we replace the dynamical variables by operators. The interaction between the electromagnetic field and the passive system is described by the Hamiltonian

$$\hat{H}'_{\mathrm{effI}}(t)=-\int_{-l/2}^{l/2} \mathrm{d}x\, \hat{P}_\mathrm{I}(x,t)\hat{A}_\mathrm{I}(x,t), \tag{43}$$

in the $jA$ approximation. A comparison of equations (42) and (39) leads after a Fourier transformation with respect to $t$ to the identification:

$$\frac{\mathrm{i}}{\hbar}B_0^{++}(\omega)=\omega^2\kappa(\omega)=\omega^2\varepsilon_0[\varepsilon(\omega)-1]=\omega^2\varepsilon_0[n^2(\omega)-1]. \tag{44}$$

From this equation one can see, that the abbreviation $n(\omega)$, used in equation (32), can really be interpreted as the refractive index of the medium. This refractive index can be determined by experimental methods, and hence the expression for $D_\mathrm{W}^{++}(x_1,x_2,\omega)$ can be calculated explicitly for a given medium.

## 6. Relation between field correlation functions in the presence of passive systems and input field correlation functions

For a lot of practical applications a relation between input and output field correlation functions is of greater interest than a relation between source and output variables we have studied up to now.

Because of the linear structure of the Dyson equation (28 *b*) one can make the following ansatz for $D_\mathrm{W}^{++}$:

$$D_\mathrm{W}^{++}(1,2)=\int \mathrm{d}3\, G^{++}(1,3)D_0^{++}(3,2). \tag{45}$$

If we insert this expression for instance into equation (21) we obtain

$$\langle\hat{A}^{(+)}(1)\hat{A}^{(-)}(2)\rangle=\frac{1}{\hbar^2}\int \mathrm{d}1'\int \mathrm{d}1''\int \mathrm{d}2'\int \mathrm{d}2''\, G^{--}(1,1')G^{++}(2,2') \times D_0^{--}(1,1'')D_0^{++}(2',2'')\langle\hat{j}(1'')\hat{j}(2'')\rangle. \tag{46}$$

If no passive system is present, denoted by the subscript '0', one would obtain

$$\langle\hat{A}^{(+)}(1')\hat{A}^{(-)}(2')\rangle_0=\frac{1}{\hbar^2}\int \mathrm{d}1''\int \mathrm{d}2''\, D_0^{--}(1',1'')D_0^{++}(2',2'')\langle\hat{j}(1'')\hat{j}(2'')\rangle. \tag{47}$$

In most practical cases the action of the passive system on the dynamical behaviour of the sources is negligible. Then $\langle\hat{j}(1)\hat{j}(2)\rangle_0$ equals $\langle\hat{j}(1)\hat{j}(2)\rangle$, and equation (47) can be inserted into equation (46)

$$\langle\hat{A}^{(+)}(1)\hat{A}^{(-)}(2)\rangle=\int \mathrm{d}1'\int \mathrm{d}2'\, G^{--}(1,1')G^{++}(2,2)\langle\hat{A}^{(+)}(1')\hat{A}^{(-)}(2')\rangle_0. \tag{48}$$

Similar relations can be derived for higher-order correlation functions.

$$\langle T_+[\hat{A}^{(+)}(1)\ldots\hat{A}^{(+)}(N)]T_-[\hat{A}^{(-)}(1')\ldots\hat{A}^{(-)}(M')]\rangle$$
$$=\int \mathrm{d}\tilde{1}\ldots\int \mathrm{d}\tilde{N}\int \mathrm{d}\tilde{1}'\ldots\int \mathrm{d}\tilde{M}'\, G^{--}(1,\tilde{1})\ldots G^{--}(N,\tilde{N})G^{++}(1',\tilde{1}')\ldots G^{++}(M',\tilde{M}')$$
$$\times\langle T_+[\hat{A}^{(+)}(\tilde{1})\ldots\hat{A}^{(+)}(\tilde{N})]T_-[\hat{A}^{(-)}(\tilde{1}')\ldots\hat{A}^{(-)}(\tilde{M}')]\rangle_0. \quad (49)$$

It can be seen from equation (49), that in analogy to the field–source relation investigated in section 3, time- and normally ordered correlation functions of the output can be expressed by input signal field correlation functions of the same ordering and, that, as far as this kind of ordering is considered, no additional terms from the vacuum occur.

The importance of the time ordering of the output correlation function comes to light if we recall an argument first given by Collet and Gardiner [8]. The negative (positive) frequency component of the incoming vacuum field $\hat{A}^{(-)}_{\mathrm{vac}}(t)$ commutes with the corresponding component of the signal field $\hat{A}^{(-)}_{\mathrm{s}}(t')$ if $t>t'$, because the signal field cannot be affected by the vacuum field at a later instance of time. This leads to a disappearence of vacuum terms if the *input* operators are time (anti-time) ordered. We had shown now, that the time-ordering of the output operators is transfered to the input operators (compare (equations (49) or (21)). Hence in the presence of radiation sources the time-ordering of the output operators is necessary to eliminate vacuum terms.

The instrumental function $G^{++}$ can simply be derived from $D_{\mathrm{W}}^{++}$. For the Fourier transformed quantity $G^{++}(x_1,x_2;\omega)$ follows from (45) and (28 *b*)

$$G^{++}(x_1,x_2;\omega)=\delta(x_1-x_2)-\frac{1}{\hbar^2}D_{\mathrm{W}}^{++}(x_1,x_2;\omega)B_0^{++}(\omega)\Xi(x_2), \quad (50)$$

where

$$\Xi(x)=\begin{cases}1, & x>0,\\ 0, & x=0,\\ -1, & x<0.\end{cases}$$

For $x_1<l/2$, the following holds:

$$G^{++}(x_1,x_2;\omega)=\delta(x_1-x_2)+\frac{\mathrm{i}\omega}{c}\frac{r}{1-r^2\exp[2\mathrm{i}n(\omega/c)l]}$$
$$\times\{(n+1)\exp[\mathrm{i}(\omega/c)(n-1)l/2]\exp[\mathrm{i}(\omega/c)(x_1-nx_2)]$$
$$+(n-1)\exp[\mathrm{i}(\omega/c)(3n-1)l/2]\exp[\mathrm{i}(\omega/c)(x_1+nx_2]\}. \quad (51)$$

This expression equals the classical instrumental function. Hence we may conclude that in normally and time-ordered correlation functions the output field operators can be replaced by input field operator expressions according to the *classical* relation

$$A^{(-)}_{\mathrm{out}}(x,t)=\int \mathrm{d}x'\int \mathrm{d}t'\, G^{++}(x,t;x',t')A^{(-)}_{\mathrm{in}}(x',t'), \quad (52)$$

and the order of the operators must be transferred from the output to the input variables. In order to estimate the changes of the field statistics one has simply to rewrite the corresponding statistical measures into normally and time-ordered correlation functions and to apply equation (52) making use of the above outlined correspondence between the classical and quantum propagation problem.

At this point it should be noted, that the disappearance of the vacuum contributions in the special correlation functions under consideration does not mean, that the statistical properties of the radiation are unaffected by the vacuum incoupling. Most of the interesting statistical measures like the intensity or field fluctuation variances are not normally ordered, although they are expressible in terms of various normally ordered quantities of different order.

## 7. Consideration of two physically instructive examples

The significance of the foregoing general statement regarding the relation between the input and the output correlation functions of an optical arrangement will be illustrated by the following two examples.

### 7.1. *The influence of the linear passive system on squeezed light*

The utilization of the advantageous properties of squeezed light requires the knowledge of its spatial and temporal behaviour. Therefore we aim at the squeezing parameters of the radiation that is assumed to traverse the linear passive medium as modelled in section 2.

Squeezing can advantageously be described with the help of the field fluctuation excess [5]

$$F \equiv \langle \Delta\hat{E}^2 \rangle - \langle \Delta\hat{E}^2 \rangle_{\text{vac}}, \tag{53}$$

that is the difference of the field-strength variances of the radiation state under consideration and the vacuum state. In the case of a nearly monochromatic wave the electric field-strength operator can be expressed by

$$\hat{E}(\Phi) = \mathrm{i}g\hat{a}\exp \mathrm{i}\Phi + \text{h.c.} = \hat{E}^{(-)}(\Phi) + \hat{E}^{(+)}(\Phi), \tag{54}$$

where $g = (\hbar\omega/2\varepsilon_0 V)^{1/2}$ is the amplitude. The fluctuation excess depends, *via* $\hat{E}$, on the phase of the field strength $\Phi = kx - \omega t$. The vacuum fluctuations are given by $\langle \Delta\hat{E}^2 \rangle_{\text{vac}} = g^2$. We can express $\hat{E}$ and $F$ with the help of the positive and negative part of the vector potential by inserting $\hat{E}^{(\pm)} = \mp \mathrm{i}\hat{A}^{(\pm)}\omega$. Now we want to consider the field-fluctuation excess at an arbitrary space-time point behind the linear optical arrangement with the instrumental function $G^{\pm\pm}$. This point shall be labelled by the coordinate 1. Denoting the input correlation functions by $\langle \ldots \rangle_0$, the equations (49, 53 and 54) yield the result

$$\begin{aligned} F(1) = \omega^2 \int \mathrm{d}1' \int \mathrm{d}2' \\ \{ & G^{--}(1,1')G^{--}(1,2')[\langle \hat{A}^{(+)}(1') \rangle_0 \langle \hat{A}^{(+)}(2') \rangle_0 - \langle \hat{A}^{(+)}(1')\hat{A}^{(+)}(2') \rangle_0] \\ & + G^{++}(1,1')G^{++}(1,2')[\langle \hat{A}^{(-)}(1') \rangle_0 \langle \hat{A}^{(-)}(2') \rangle_0 - \langle \hat{A}^{(-)}(1')\hat{A}^{(-)}(2') \rangle_0] \\ & - 2G^{--}(1,1')G^{++}(1,2')[\langle \hat{A}^{(+)}(1') \rangle_0 \langle \hat{A}^{(-)}(2') \rangle_0 - \langle \hat{A}^{(+)}(1')\hat{A}^{(-)}(2') \rangle_0] \}. \end{aligned} \tag{55}$$

Thus, the normally ordered input correlation functions up to the second order and the instrumental function uniquely determine the field fluctuation excess at the output.

To get a deeper insight, we will study equation (55) in the case of input radiation with well-defined squeezing properties: Yuen [14] has introduced squeezed radiation states $|\beta_i\mu, \nu\rangle$, that are right hand eigenstates of an operator $\hat{b}$ according to

$$\hat{b}|\beta;\mu,\nu\rangle = \beta|\beta;\mu,\nu\rangle, \qquad \hat{b} \equiv \mu\hat{a} + \nu\hat{a}^+. \tag{56}$$

The complex numbers $\mu, \nu$ are assumed to obey the condition $|\mu|^2 = 1 + |\nu|^2$ there holds $[\hat{b}, \hat{b}^+] = \hat{I}$. $\nu = 0$ characterizes a coherent state, while $|\nu| > 0$ characterizes squeezed states with $F < 0$ in a certain phase region. $|\nu|$ is the so called squeezing parameter, since the squeezing character increases with growing $|\nu|$.

A straightforward calculation yields for the Yuen states.

$$\left.\begin{aligned} \langle \hat{a} \rangle &= \mu^* \beta - \nu \beta^*, \\ \langle \hat{a}^2 \rangle &= \langle \hat{a} \rangle^2 - \mu^* \nu, \\ \langle \hat{a}^+ \hat{a} \rangle &= \langle \hat{a}^+ \rangle \langle \hat{a} \rangle + |\nu|^2, \end{aligned}\right\} \tag{57}$$

and therefore we have finally at the output the field-fluctuation excess

$$\begin{aligned} F(1) = 2g^2 \int d1' \int d2'\, G^{--}(1, 1') G^{++}(2, 2') \exp[i(2' - 1')] |\nu|^2 \\ \times g^2 \left\{ \int d1' \int d2' G^{++}(1, 1') G^{++}(2, 2') \exp[i(2' + 1')] \mu^* \nu + \text{c.c.} \right\}. \end{aligned} \tag{58}$$

Carrying out the time and space integration we obtain

$$\begin{aligned} F(x, t) = 2g^2 \Bigg\{ G^{--}(\omega) G^{++}(\omega) |\nu|^2 + \frac{1}{2} \Bigg( G^{++}(\omega) G^{++}(\omega) \\ \times \exp\left[ -2i\omega\left(t - \frac{x}{c}\right)\right] \mu^* \nu + \text{c.c.} \Bigg) \Bigg\}, \end{aligned} \tag{59}$$

where $G^{++}(\omega)$ is given by

$$\begin{aligned} G^{++}(\omega) &\equiv \int_{-\infty}^{\infty} dx'\, G^{++}(x, x'; \omega) \exp\left[ i\frac{\omega}{c}(x' - x)\right] \\ &= \frac{(1 - r^2) \exp\left[ i\frac{\omega}{c}(n - 1) l\right]}{1 - r^2 \exp\left( 2i\frac{\omega}{c} nl \right)}. \end{aligned} \tag{60}$$

The absolute value $G$ and the phase $\Psi$ of the instrumental function $G^{++}(\omega)$ depend on the frequency and the characteristic instrumental parameters of the optical arrangement. Equation (59) leads to the fluctuation excess

$$F(x, t) = G^2(\omega) g^2 [2|\nu|^2 + 2|\mu||\nu| \cos(2\Phi + \psi_\nu - \psi_\mu - 2\Psi)], \tag{61}$$

at the output, where $\psi_\nu$, $\psi_\mu$ are the phases of $\nu$, $\mu$. A clear comparison between the input excess $F$ and the output excess can be given if we rewrite equation (61) into

$$F(\Phi) = F_0(\Phi - \Psi) G^2(\omega). \tag{62}$$

This reveals the following facts: if squeezed light at the input exists (i.e. $F_0(\Phi) < 0$ for a certain phase region $\Delta\Phi$), also squeezed light at the output can be found; however, the phase region is shifted by the space-time independent phase $\Psi$. The 'squeezing strength' (the maximum of $-F/g^2$) can be strongly affected by the transmission $G^2(\omega)$. Equation (61) exhibits that the minimum value of the relative field-strength variance at the output is

$$\left( \frac{\langle \Delta \hat{E}^2 \rangle}{g^2} \right)_{\min} = 1 + G^2(\omega) 2|\nu| [|\nu| - (|\nu|^2 + 1)^{1/2}], \tag{63}$$

which is attained at $2(\Phi-\Psi)+\psi_\nu-\psi_\mu=\pi$. For large $|\nu|$ the minimum value of the relative variance at the output decreases asymptotically with

$$\left(\frac{\langle\Delta\hat{E}^2\rangle}{g^2}\right)_{\min}\simeq 1-G^2(\omega)+\frac{G^2(\omega)}{4|\nu|^2}, \tag{64}$$

while the corresponding input expression is

$$\left(\frac{\langle\Delta\hat{E}^2\rangle_0}{g^2}\right)_{\min}\simeq\frac{1}{4|\nu|^2}. \tag{65}$$

The comparison between equations (65 and 64) reveals that the squeezing strength of the input radiation can exactly be found in the output too, if the transmission $G^2(\omega)$ of the optical arrangement takes on the value unity. This holds for instance, in the case of a lossless Fabry–Perot arrangement, if

$$\omega\approx\omega_m=\frac{m\pi c}{2nl}$$

(m—positive integer). Transmission values $G^2(\omega)<1$ occur for $\omega\neq\omega_m$; (60) and (64) show the strong decrease of the squeezing strength, if the difference $|\omega-\omega_m|$ becomes greater than the half-width of the transmission region

$$\Delta\omega=\frac{c}{2\pi nl}\frac{1-r^2}{r}$$

about $\omega_m$. The diminishing of the transmission, leads to the diminishing of the squeezing. In the case of a lossy Fabry–Perot arrangement a small but non-vanishing imaginary part of the refractive index diminishes the maximum obtainable transmission to

$$G^2(\omega)=\frac{(1-R^2)\exp\left(-n_2\frac{\omega}{c}l\right)}{1-R^2\exp\left(-2n_2\frac{\omega}{c}l\right)}, \tag{66}$$

where $n=n_1+in_2$ and $R=(n_1-1)/(n_1+1)$.

### 7.2. *The action of a black screen*

In a lot of experimental arrangements shutters or apertures are used to modulate the time or space structure of the radiation coming from a source. In practice these devices are partially reflecting and absorbing screens. The two limit cases are the totally reflecting and the totally absorbing, i.e. 'black' screen. The definition of both aspects in classical electrodynamics leads to serious difficulties. However in a theory that treats the electromagnetic field as a scalar quantity such a definition seems to be possible. The description of a totally reflecting screen is rather trivial and well known in such a scalar treatment, but the description of a black screen has not been given before. We will do that now as another example of the application of our investigation. According to classical optics a black screen should have a refractive index with real part 1 and imaginary part inifinity. Hence the parameter $r$ of equation (31) equals unity. The Green function $D_W^{++}(x_1, x_2; \omega)$ describes the propagation of signals from a source at a point $x_1$ to a point $x_2$ (in the sense of normally and time-ordered correlation functions). From (29) we see, that there is no signal transmission from left to right of the screen and *vice versa*.

$$D_W^{++}(x_1, x_2; \omega) = 0, \qquad x_1 > \frac{l}{2} \text{ and } x_2 < -\frac{l}{2}. \tag{67}$$

Furthermore there is no connection between points inside and outside the passive system, as can be seen from equation (30)

$$D_W^{++}(x_1, x_2; \omega) = 0, \qquad x_1 > \frac{l}{2} \text{ and } -\frac{l}{2} \leqslant x_2 \leqslant \frac{l}{2}. \tag{68}$$

If both points lie on the same side of the screen, for instance if $x_1 > l/2$ and $x_2 > l/2$, the corresponding function can be calculated directly from the Dyson equation (28). The function $D_W^{++}$ in the integral vanishes, because $x$ lies inside and $x_2$ outside the passive system. Hence it follows:

$$D_W^{++}(x_1, x_2; \omega) = D_0^{++}(x_1, x_2; \omega), \qquad x_1, x_2 > \frac{l}{2}. \tag{69}$$

From this simple investigation one can conclude, that the 'black' screen subdivides the space into two nonconnected subspaces. In each of these subspaces the Green function is the same as in the whole free space. That means, for the 'black' screen the mode structure remains unchanged in contrast to the case of a mirror.

## 8. Summary and conclusions

The purpose of our paper was to investigate the changes of statistical measures from input to output of an radiation field interacting with a general linear passive system in the presence of sources. This change can be deduced from the alteration of field correlation functions. The problem in such an investigation, the handling of the vacuum field, has been treated in a general way.

With the help of the Green function method we were able to express time and normally ordered output correlation functions in terms of corresponding signal-input correlation functions. By the combination of both ordering procedures the vacuum terms appearing in the operator input–output relation could be eliminated in a general way. The instrumental response functions calculated by quantum theoretical means coincide with the corresponding classical response functions, as expected. Hence there is a strong correspondence to the classical linear propagation problem. For an arbitrary combination of linear passive systems the total output is given by the signal-input *via* classical relations, if normally and time-ordered output correlation functions are considered. In order to determine the alteration of statistical properties the corresponding measures as for instance the Poisson-excess or the squeezing parameter have to be expressed by appropriate combinations of such correlation functions. Making use of the above mentioned correspondence between the quantum and classical propagation problem we can immediately estimate the influence of linear optical processing on the statistical properties of the radiation.

To illustrate the application of our theory two physically relevant examples have been given.

## Appendix

*Non-interacting Green functions*

(*a*) *The electromagnetic field*

$$D_0^{++}(1,2)\equiv\langle 0|T_-\hat{A}_I(x_1,t_1)\hat{A}_I(x_2,t_2)|0\rangle$$
$$=\sum_k \frac{\hbar}{2\varepsilon_0 L\omega_k}\{\theta(t_1-t_2)\exp[ik(x_1-x_2)]\exp[-i\omega_k(t_1-t_2)]$$
$$+\theta(t_2-t_1)\exp[-ik(x_1-x_2)]\exp[i\omega_k(t_1-t_2)]\}. \quad (A\,1)$$

$$D_0^{++}(x_1,x_2;\omega)=\frac{\hbar}{2\varepsilon_0\omega c}\left\{\Xi(\omega)\cos\left[\frac{\omega}{c}(x_1-x_2)\right]+i\Xi(x_1-x_2)\sin\left[\frac{\omega}{c}(x_1-x_2)\right]\right\}. \quad (A\,2)$$

with

$$\Xi(x)=\begin{cases}1, & x>0,\\ 0, & x=0,\\ -1, & x<0.\end{cases}$$

For $x_1>x_2$ and $\omega>0$ follows

$$D_0^{++}(x_1,x_2;\omega)=\frac{\hbar}{2\varepsilon_0\omega c}\exp\left[i\frac{\omega}{c}(x_1-x_2)\right]. \quad (A\,3)$$

Because of

$$D_0^{--}(1,2)=D_0^{++}(1,2)^*, \quad (A\,4)$$

there follows

$$D_0^{--}(x_1,x_2;\omega)=D_0^{++}(x_1,x_2;\omega)=-\frac{\hbar}{2\varepsilon_0\omega c}\exp\left[-i\frac{\omega}{c}(x_1-x_2)\right]. \quad (A\,5)$$

From

$$D_0^{-+}(1,2)=\sum_k \frac{\hbar}{2\varepsilon_0 L\omega_k}\exp[ik(x_1-x_2)]\exp[-i\omega_k(t_1-t_2)], \quad (A\,6)$$

follows

$$D_0^{-+}(x_1,x_2;\omega)=\frac{\hbar}{\varepsilon_0\omega c}\Theta(\omega)\cos\left[\frac{\omega}{c}(x_1-x_2)\right], \quad (A\,7)$$

and analogously

$$D_0^{+-}(x_1,x_2;\omega)=-\frac{\hbar}{\varepsilon_0\omega c}\Theta(-\omega)\cos\left[\frac{\omega}{c}(x_1-x_2)\right], \quad (A\,8)$$

with $\Theta(x)$ being the unit step function. From (A 7) and (A 8) one can see that $D_0^{-+}$ vanishes for negative values of $\omega$ and $D_0^{+-}$ for positive values respectively.

(*b*) *The passive system*

A harmonic oscillator coupled to a Bosonic bath evolves according to the Heisenberg–Langevin equation

$$\dot{\hat{b}}_{i\lambda}(t)=-i\omega_\lambda\hat{b}_{i\lambda}(t)-\gamma_\lambda\hat{b}_{i\lambda}(t)+\hat{F}_{i\lambda}(t). \quad (A\,9)$$

$\hat{F}_{i\lambda}(t)$ being the $\delta$-correlated fluctuation operator.

$$\langle \hat{F}_{i\lambda}(t)\rangle = \langle \hat{F}_{i\lambda}(t)\hat{F}_{j\mu}(t')\rangle q = 0, \quad \text{(A 10)}$$

$$\langle \hat{F}^{+}_{i\lambda}(t)\hat{F}_{j\mu}(t')\rangle = 0, \quad \text{(A 11)}$$

$$\langle \hat{F}_{i\lambda}(t)\hat{F}^{+}_{j\mu}(t')\rangle = \delta_{\lambda\mu}\delta_{ij}2\gamma_{\lambda}\delta(t-t'), \quad \text{(A 12)}$$

where the temperature of the bath is assumed to be zero. The formal solution of (A 9) reads:

$$\hat{b}_{i\lambda}(t) = \exp\left[-(\mathrm{i}\omega_{\lambda}+\gamma_{\lambda})t\right]\left\{\hat{b}_{i\lambda}(0) + \int_{0}^{t} \mathrm{d}\tau\, \hat{F}_{i\lambda}(\tau)\exp\left[(\mathrm{i}\omega_{\lambda}+\gamma_{\lambda})\tau\right]\right\}. \quad \text{(A 13)}$$

In the equilibrium case one obtains for the correlation functions of $\hat{b}_{i\lambda}$:

$$\langle \hat{b}_{i\lambda}(t)\rangle = \langle \hat{b}_{i\lambda}(t)\hat{b}_{j\mu}(t')\rangle = 0, \quad \text{(A 14)}$$

$$\langle \hat{b}^{+}_{i\lambda}(t)\hat{b}_{j\mu}(t')\rangle = 0, \quad \text{(A 15)}$$

$$\langle \hat{b}_{i\lambda}(t)\hat{b}^{+}_{j\mu}(t')\rangle = \delta_{ij}\delta_{\lambda\mu}2\gamma_{\lambda}\exp\left[-\mathrm{i}\omega_{\lambda}(t-t')\right]\exp\left(-\gamma_{\lambda}|t-t'|\right). \quad \text{(A 16)}$$

Hence for the non-interacting Green function follows:

$$B_0^{+\,+}(1,2) = \delta(x_1-x_2)\varepsilon(x_1)\Big\{\Theta(t_1-t_2)\sum_{\lambda}\xi_{\lambda}\exp\left[-\mathrm{i}\omega_{\lambda}(t_1-t_2)\right]\exp\left[-\gamma_{\lambda}(t_1-t_2)\right]$$
$$+\Theta(t_2-t_1)\sum_{\lambda}\xi_{\lambda}\exp\left[\mathrm{i}\omega_{\lambda}(t_1-t_2)\right]\exp\left[-\gamma_{\lambda}(t_2-t_1)\right]\Big\}. \quad \text{(A 17)}$$

A Fourier transformation yields

$$B_0^{+\,+}(x_1,x_2;\omega) = \delta(x_1-x_2)\varepsilon(x_1)\sum_{\lambda}\xi_{\lambda}\left(\frac{\gamma_{\lambda}+\mathrm{i}(\omega-\omega_{\lambda})}{\gamma_{\lambda}^2+(\omega-\omega_{\lambda})^2} + \frac{\gamma_{\lambda}-\mathrm{i}(\omega+\omega_{\lambda})}{\gamma_{\lambda}^2+(\omega+\omega_{\lambda})^2}\right). \quad \text{(A 18)}$$

Analogously it follows for $B_0^{-\,+}(x_1,x_2;\omega)$ that

$$B_0^{-\,+}(x_1,x_2;\omega) = \delta(x_1-x_2)\varepsilon(x_1)\sum_{\lambda}\xi_{\lambda}\frac{\gamma_{\lambda}}{\gamma_{\lambda}^2+(\omega-\omega_{\lambda})^2}. \quad \text{(A 19)}$$

In the above expressions we used the abbreviations

$$\varepsilon(x) \equiv \begin{cases} 1 & -l/2 \leqslant x \leqslant l/2 \\ 0 & \text{elsewhere,} \end{cases} \quad \text{(A 20)}$$

$$\xi_{\lambda} \equiv \frac{\hbar\omega_{\lambda}e^2}{2m}. \quad \text{(A 21)}$$

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[14] YUEN, H. P., 1976, *Phys. Rev.* A, **13,** 2226.