Topological Growing of Laughlin States in Synthetic Gauge Fields

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We suggest a scheme for the preparation of highly correlated Laughlin states in the presence of synthetic gauge fields, realizing an analogue of the fractional quantum Hall effect in photonic or atomic systems of interacting bosons. It is based on the idea of growing such states by adding weakly interacting composite fermions along with magnetic flux quanta one by one. The topologically protected Thouless pump ("Laughlin's argument") is used to create two localized flux quanta and the resulting hole excitation is subsequently filled by a single boson, which, together with one of the flux quanta, forms a composite fermion. Using our protocol, filling $1/2$ Laughlin states can be grown with particle number N increasing linearly in time and strongly suppressed number fluctuations. To demonstrate the feasibility of our scheme, we consider two-dimensional lattices subject to effective magnetic fields and strong on-site interactions. We present numerical simulations of small lattice systems and also discuss the influence of losses.

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Introduction.—In recent years, topological states of matter [1–8] have attracted a great deal of interest, partly due to their astonishing physical properties (like fractional charge and statistics), but also because of their potential practical relevance for quantum computation [9,10]. While these exotic phases of matter were first explored in the context of the quantum Hall effect of electrons subject to strong magnetic fields [11,12], there has been considerable progress, recently, towards their realization in cold-atom [13–16], as well as photonic [17–23], systems. Particularly attractive features of such quantum Hall simulators are the comparatively large intrinsic length scales which allow coherent preparation, manipulation, and spatially resolved detection of exotic many-body phases and their excitations.

In electronic systems, the preparation of topological states of matter relies on quick thermalization and cooling below the many-body gap. While this is already hard to achieve in cold-atom systems (partly due to the small required temperatures), cooling is even less of an option in photonic systems due to the absence of effective thermalization mechanisms. On the other hand, lasers with narrow linewidths allow for a completely different avenue towards preparation of extremely pure quantum states. For instance, it was suggested to use the coherence properties of lasers to directly excite two (and more) photon Laughlin (LN) states in nonlinear cavity arrays [24], where the laser plays the role of a coherent pump. However, this approach has the inherent problem of an extremely small multiphoton transition amplitude. While this might be acceptable for small systems of $N = 2$, 3 photons, it makes the preparation of true many-body states with $N \gg 2$ practically impossible. Moreover, the prepared states in this case contain superpositions of different photon-numbers rather than being Fock states.

In this Letter, we suggest an alternative scheme for the preparation of topologically ordered states of strongly interacting bosons, specifically for the $1/2$ LN state, and we discuss systems allowing for an implementation of the scheme with state-of-the-art technology. It consists of growing such states and makes direct use of the Thouless pump [25] connected to the many-body topological invariant. In the case of quantum Hall physics, the latter is realized by local flux insertion in the spirit of Laughlin's argument for the quantization of the Hall conductivity σ_H [26]: Introducing magnetic flux $\phi/2\pi = 2$ (in units of the flux quantum) in the center of the system produces a quantized outwards Hall current $\sim\sigma_H \partial_t \phi$, leaving behind a hole, see Fig. 1(a).

In the next step, the so-created hole can be replenished by a single boson. In view of the composite fermion (CF) picture [27,28] of the fractional quantum Hall effect, this refilling step can be interpreted as the addition of a single CF (composed of a bare boson and one flux quantum) into a free orbital of the CF Landau level (LL), using up the remaining flux quantum. To refill the hole deterministically by a single boson, we consider a coherent pump in the center of the system. Excitations by more than one particle are prohibited by the many-body gap, and the coherent coupling can not decrease the total particle number because the central cavity is empty initially. Thus, our final state has sub-Poissonian boson number statistics. A complementary scheme, where holes resulting from boson losses are dynamically refilled in the entire system using single photon pumps, has recently been suggested for photonic

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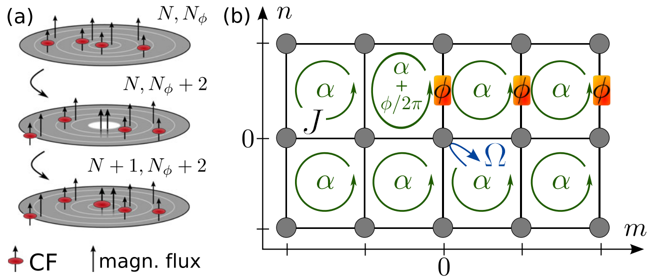

FIG. 1 (color online). (a) The key idea of our scheme is to grow LN states by introducing weakly interacting CFs into the system. This is achieved by adding magnetic flux (arrows) in the center and replenishing the arising hole by a new boson (red bullet). (b) We consider the Hofstadter-Hubbard model (flux α per plaquette). Additional flux ϕ can be introduced in the center by adiabatically changing the complex phase of the hoppings marked with a box. Furthermore, the central site is assumed to be externally accessible for a coherent drive (Rabi frequency Ω).

systems [29]. Our protocol, in contrast, does not rely on an explicit single photon source.

A key advantage of our scheme, compared to [24,30,31], is the ability to grow LN states with a size increasing linearly in time. To reach N particles with given fidelity $1-\varepsilon$, the protocol has to be carried out sufficiently slowly to avoid errors in the repumping protocol. For $\varepsilon \ll 1$, the total required time scales like

$$T \sim \frac{N^{3/2}}{\Delta_{\mathrm{LN}}\varepsilon^{1/2}}, \tag{1}$$

where Δ_{LN} is the bulk many-body gap. In contrast to previously proposed schemes [24,30,31], T only grows algebraically with N.

Model.—We consider a 2D lattice with complex hopping elements (amplitude J) realizing an effective magnetic field, supplemented by Hubbard-type on-site interactions (strength U). This model is illustrated in Fig. 1 and can be described by the following Hamiltonian:

$$\begin{aligned}\hat{\mathcal{H}}_{\mathrm{int}}+\hat{\mathcal{H}}_0 =& \frac{U}{2}\sum_{m,n}\hat{a}^\dagger_{m,n}\hat{a}_{m,n}(\hat{a}^\dagger_{m,n}\hat{a}_{m,n}-1)\\ &-J\sum_{m,n}[e^{-i2\pi\alpha n}\hat{a}^\dagger_{m+1,n}\hat{a}_{m,n}+\hat{a}^\dagger_{m,n+1}\hat{a}_{m,n}+\mathrm{H.c.}],\end{aligned}$$

where we used the Landau gauge and set $\hbar = 1$. Following Jaksch and Zoller's proposal for the creation of synthetic gauge fields [32], there have been numerous suggestions regarding how this Hamiltonian can be implemented in photonic [18,23,24,33,34], circuit-QED [35–37], or atomic [31,38] systems, and in the last case, this goal has already been achieved [15,16].

Local flux insertion can most easily be realized by changing the hopping elements from site $(m \geq 0, n = 0)$ to $(m, 1)$ by a factor $e^{i\phi}$, see Fig. 1(b). These links are, thus, described by

$$\hat{\mathcal{H}}_\phi = -J\sum_{m\geq 0}[e^{-i\phi}\hat{a}^\dagger_{m,1}\hat{a}_{m,0}+\mathrm{H.c.}], \tag{2}$$

modifying the total magnetic flux through the central plaquette to $\alpha - \phi/2\pi$. $\hat{\mathcal{H}}_\phi$ is motivated by recent experiments with photons [18,23], where the hopping phases can locally and temporally be manipulated [39]. Finally to replenish the system with bosons, we place a weak coherent pump ($\Omega \ll 4\pi\alpha J$) in the center,

$$\hat{\mathcal{H}}_\Omega = \Omega e^{-i\omega t}\hat{a}^\dagger_{0,0} + \mathrm{H.c.} \tag{3}$$

In the following, we present the details of our scheme, neglecting local boson losses (rate γ) for the moment. We include losses again, afterwards, in the discussion of the performance of our scheme.

Protocol—continuum.—We begin by discussing the continuum case when the magnetic flux per plaquette $\alpha \ll 1$ is small, allowing us to make use of angular momentum L_z as a conserved quantum number. The continuum can be described by LLs, which are eigenstates of $\hat{\mathcal{H}}_0$ in the limit $\alpha \to 0$ with energies $E_n = (n+1/2)\omega_c$ $(n = 0, 1, 2, \ldots)$ and $\omega_c = 4\pi\alpha J$ denoting the cyclotron frequency, see, e.g., [28]. The magnetic length is defined as $\ell_B = a/\sqrt{2\pi\alpha}$, where a denotes the lattice constant. In symmetric gauge, the single particle states of the lowest LL (LLL) are labeled by their angular momentum quantum number $l = 0, 1, 2, \ldots$ [28], and we define boson creation operators of these orbitals as $\hat{b}^\dagger_l$. Now, we discuss the preparation of filling $\nu = N/N_\phi = 1/2$ LN states, but the generalization to other fillings is straightforward.

To create the first excitation from vacuum $|0\rangle$, we switch on the coherent pump (3) with frequency $\omega = \omega_c/2$, which, due the blockade [40] (caused by strong boson-boson interactions), only allows a single particle to enter the system. Since we drive locally in the center, no angular momentum is transferred, and we, thus, arrive at the state $|\Psi_1\rangle = \hat{b}^\dagger_0|0\rangle$. This argument is true when excitations of higher LLs can be neglected, allowing us to project the coherent pump (3) into the LLL, $\hat{\mathcal{H}}_\Omega \approx [\hat{b}^\dagger_0 e^{-i\omega t}\Omega^{(1)}_{\mathrm{eff}} + \mathrm{H.c.}]$ with $\Omega^{(1)}_{\mathrm{eff}} = \Omega\sqrt{\alpha}$. To this end, we require a weak pump, $\Omega \ll \omega_c$, also sufficiently feeble for the blockade to work, i.e., $\Omega^{(1)}_{\mathrm{eff}} \ll \Delta_{\mathrm{LN}}$. In the continuum, the gap can be estimated from $\Delta_{\mathrm{LN}} \approx \min(V_0, \omega_c)$, where $V_0 = U\alpha/2$ is given by Haldane's zeroth-order pseudopotential [41]. To prepare $|\Psi_1\rangle$ from $|0\rangle$ as described, the coherent pump has to be switched on for a time $T_\pi = \pi/2\Omega^{(1)}_{\mathrm{eff}}$ (corresponding to a π pulse in the effective two-level system defined by $|0\rangle$ and $|\Psi_1\rangle$), which works when losses are negligible, $\gamma \ll \Omega^{(1)}_{\mathrm{eff}}$ [42].

Next, we adiabatically introduce two units of magnetic flux into the center of the system. Thereby, the initial state $|\Psi_1\rangle = \hat{b}^\dagger_0|0\rangle$ attains two units of angular momentum, and we end up in $|\Psi_2\rangle = \hat{b}^\dagger_2|0\rangle$ [43]. This state has a ring structure with a hole in its center, which—repeating the first step of our protocol—can be replenished by an additional particle using the coherent pump. Because the latter only couples to the center of the system, it can not reduce the total particle number. The combined insertion of magnetic flux and a boson can be understood as addition of a single CF, with one flux-quantum binding to the boson to form a CF in the reduced magnetic field corresponding to the remaining flux quantum. Crucially, in contrast to the first step, the new state is not the simple product state $\hat{b}_0^\dagger|\Psi_2\rangle$. Instead, the blockade mechanism allows us to pump only into the $N=2$ LN state $|\mathrm{LN},2\rangle$, which is the only zero-energy state with the correct total angular momentum $L_z=2$, while all other states are detuned from the pumping frequency by the gap Δ_{LN}. As a consequence, the corresponding Rabi frequency is reduced by a Franck-Condon factor (FCF), $\Omega_{\mathrm{eff}}^{(2)}/\Omega = \langle \mathrm{LN},2|\hat{b}_0^\dagger|\Psi_2\rangle\sqrt{\alpha}$.

Having established our protocol for two bosons, the extension to N-particle LN states $|\mathrm{LN},N\rangle$ is straightforward. In this case, local flux insertion is used to create the state $|2\mathrm{qh},N-1\rangle$ with two quasiholes (qh), which are subsequently refilled by the coherent pump to prepare $|\mathrm{LN},N\rangle$. The corresponding transition amplitude $\Omega_{\mathrm{eff}}^{(N)}$ is reduced by a many-body FCF,

$$\Omega_{\mathrm{eff}}^{(N)}/\Omega = \sqrt{\alpha}\langle \mathrm{LN},N|\hat{b}_0^\dagger|2\mathrm{qh},N-1\rangle. \tag{4}$$

Using exact diagonalization (ED) of small systems ($N=1,\ldots,9$), we find that $\Omega_{\mathrm{eff}}^{(N)}$ is nearly constant as a function of N, and we extrapolate $\Omega_{\mathrm{eff}}^{(\infty)} \approx 0.70\ \Omega\sqrt{\alpha}$. Thus, our pump works equally for large and small boson numbers.

A natural explanation regarding why highly correlated many-body states can be grown in the relatively simple fashion described above is provided by the composite fermion picture: LN states are separable (Slater determinant) states of noninteracting CFs filling the CF-LLL [27]. Thus, introducing CFs one-by-one into the orbitals of this LLL, LN states can easily be grown.

Protocol—lattice.—To ensure a sizable cyclotron gap ω_c, a not too small flux per plaquette α is desirable, where lattice effects become important. We will now study this regime, which is also of great experimental relevance [15,16,23]. The spectrum of the Hamiltonian $\hat{\mathcal{H}}_0(\alpha)$ is the famous Hofstadter butterfly [44], consisting of a self-similar structure of magnetic sub-bands. When interactions are taken into account, LN-type states can still be identified at filling $\nu=1/2$ [31,38].

The basic ideas directly carry over from the continuum to the lattice case. Because the many-body Chern number is strictly quantized, Laughlin's argument shows that a hole excitation can still be created by local flux insertion. However, due to the formation of magnetic sub-bands, such a quasihole becomes dispersive and will propagate away from the center. This leaves us only a restricted time to refill the defect, and leads to a reduced efficiency of repumping. To circumvent this problem, we introduce a trap for quasiholes. A static, repulsive potential of the form

$$\hat{\mathcal{H}}_{\mathrm{pot}} = \sum_{m,n} \frac{g}{\sqrt{2\pi}\ell_B/a} e^{-(m^2+n^2)a^2/2\ell_B^2}\hat{a}_{m,n}^\dagger\hat{a}_{m,n}, \tag{5}$$

is sufficient for a gapped ground state at every point in the protocol. An alternative would be to include carefully chosen long-range hoppings leading to a completely flat band [45].

In the following, we use ED to simulate our protocol for small systems. To get rid of boundary effects, which can be pronounced in small systems, we consider a spherical geometry [46] and take into account lattice effects by using a buckyball-type lattice. The hopping elements on all links have amplitude J, and their phases were chosen such that the flux per plaquette is α. Because the total flux N_ϕ is integer quantized, it holds $\alpha = N_\phi/N_p$ with $N_p = 32$ being the number of plaquettes. We checked numerically (using ED) that, for the values of $\alpha \le 0.2$ used in this Letter, there are gapped LN-type ground states, provided that the condition $N_\phi = 2(N-1)$ for $\nu=1/2$ LN states on a sphere is fulfilled. We find gaps of the order $\Delta_{\mathrm{LN}} \approx 0.1J$, as predicted for a square lattice [31,38]. To describe the effect of local flux insertion $N_\phi \to N_\phi + \phi/2\pi$, we slightly increase $\alpha \to \alpha + \phi/(2\pi N_p)$ everywhere, except on the central plaquette where $\alpha \to \alpha - (1-1/N_p)\phi/2\pi$ changes by $-\phi/2\pi$ in thermodynamic limit (i.e., for $N_p \to \infty$). Starting from an incompressible LN-type ground state, we checked numerically that the correct number of low-lying quasihole states is obtained and that they can be gapped out by the potential Eq. (5) [47].

In Fig. 2, we present a numerical simulation of our full protocol on the C_{60} buckyball lattice. We start from vacuum and $N_\phi = 0$ flux quanta. Then, the coherent pump Eq. (3) is switched on for a time $T_\Omega = 6\pi/\Omega$ (with $\Omega = 0.05J$) and one boson is inserted with an overlap close to one to the target $N=1$ ground state. The driving frequency ω is

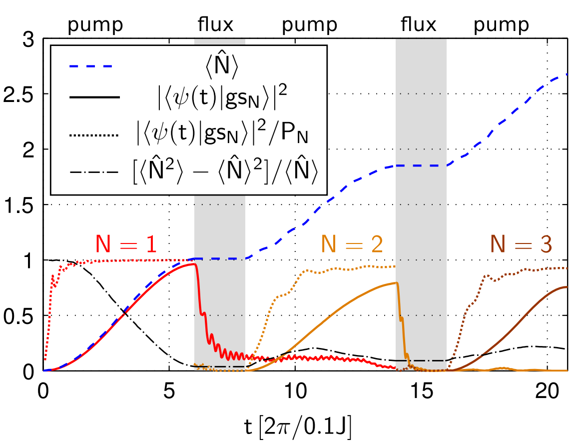

FIG. 2 (color online). Simulation of the full protocol on a C_{60} buckyball as described in the text, for $U = 10J$ and including the static potential (5) with $g = J$. The overlaps (solid line, conditioned on the targeted particle number N—dotted line) together with particle-number fluctuations (dashed-dotted line) indicate the accuracy of our protocol.

chosen to be resonant on the transition from the $N = 0$ to the $N = 1$ ground state. After introducing two more flux quanta in a time $2 \times 20\pi/J$, of the order $2\pi/\Delta_{\mathrm{LN}} \approx 60/J$, the whole protocol is repeated, and we, finally, arrive close to a three particle LN-type ground state. We find that the overlaps of the prepared states to the targeted N particle ground states $|\mathrm{gs}_N\rangle$ are close to one after all steps, and the overlaps conditioned on having the correct particle number N (occurring with probability P_N) are even larger. At the end of the protocol, the $N = 3$ boson ground state at $N_\phi = 4$ is prepared with high fidelity, which carries the signatures of a LN-type state. Importantly, the particle number fluctuations after a completed cycle are strongly suppressed $[\langle \hat{N}^2\rangle - \langle \hat{N}\rangle^2]/\langle \hat{N}\rangle \ll 1$.

In our simulations, we neglected edge effects and bulk losses. The latter result in a finite boson lifetime, such that, in the growing scheme, the mean density $\rho(r)$ decays with the distance r from the center. In continuum, we find $\rho(r) \approx (1/4\pi\ell_B^2)\exp[-\gamma T_0(r^2/4\ell_B^2)]$, with T_0 being the duration of a single step of the protocol. In a forthcoming publication [48], we study larger systems using a simplified model of noninteracting CFs on a lattice and show that our protocol still works when edge-effects are taken into account.

In Fig. 2, we observe that the fidelity $\mathcal{F}_N = |\langle \psi(t)|\mathrm{gs}_N\rangle|$ for preparation of the N-particle LN-type ground state (gs) is limited, mostly by the inefficiency of the pump. High fidelity, however, is a prerequisite for measuring, e.g., braiding phases of elementary excitations, which play a central role for topological quantum computation [10]. Taking into account couplings between low-energy states of the N and $N + 1$ boson sectors, induced by the coherent pump (3), we find the following expression for the fidelity:

$$\mathcal{F}_N \sim \exp\left[-\left(\frac{\Lambda^2}{\Delta_{\mathrm{LN}}^2 T_0^2} + \gamma T_0 \frac{N}{2}\right)\frac{N}{2}\right]. \tag{6}$$

The second term in the exponent describes boson loss, whereas the first term takes into account imperfections of the blockade in the repumping process with rates scaling like $(\Omega_{\mathrm{eff}}\Lambda/\Delta_{\mathrm{LN}})^2$. Here Λ is a parameter depending on nonuniversal FCFs, which, in the continuum case $\alpha \to 0$, is found to be $\Lambda = 1.4$ from finite-size extrapolations of ED results. In a lattice, Λ takes larger values and, from Fig. 2, we estimate $\Lambda \approx 10$. In Eq. (6), we neglected fidelity losses from flux insertion, which only leads to small corrections of Λ however, even when using the approximation $T_0 \approx T_\pi = \pi/2\Omega_{\mathrm{eff}}$. We observe a competition between losses $\sim T_0$ and errors of the pump $\sim 1/T_0^2$. Thus, for a target fidelity $\mathcal{F}_N = 1 - \varepsilon$, only LN states of a restricted number of bosons $N \leq N_{\max}$ can be grown,

$$N_{\max} = 1.365\varepsilon^{3/5}\left(\frac{\Delta_{\mathrm{LN}}}{\Lambda\gamma}\right)^{2/5}. \tag{7}$$

To do so, a time $T = N_{\max} T_0 = 1.22 N_{\max}^{3/2}\varepsilon^{-1/2}\Lambda/\Delta_{\mathrm{LN}}$ is required, which yields Eq. (1).

Experimental realization.—Our protocol can be implemented in photonic cavity arrays [18,23,24,33,34], where the main experimental challenges are the required large interactions $U \gtrsim J$ and small losses $\gamma \ll \Delta_{\mathrm{LN}}/N^{5/2}$. Strong nonlinearities can be realized, e.g., by placing single atoms into the cavities [33] or coupling them to quantum dots [23] or Rydberg gases [23,49,50]. Most promising are circuit-QED systems, where loss rates $\gamma = (0.1\ \mathrm{ms})^{-1}$ have been achieved [51] (and $\gamma = 1\ \mathrm{ms}^{-1}$ seems feasible). The strong coupling regime can be reached and single-photon nonlinearities $U = 100$ MHz are realistic [37]. For the case when $U \approx J$ and for $\alpha \approx 0.1$, the LN gap can be estimated to $\Delta_{\mathrm{LN}} \approx 0.05U = 5$ MHz [38], which corresponds to $\Delta_{\mathrm{LN}}/\gamma \approx 3 \times 10^3$. For an infidelity of $\epsilon = 0.1$, this yields $N_{\max} = 7.4$ in a continuum system ($N_{\max} = 3.4$ for $\Lambda \approx 10$ as in our simulation). To observe interesting many-body physics on a qualitative level, $\epsilon = 0.5$ should be sufficient, which results in $N_{\max} \approx 20$ in continuum. To reach even larger photon numbers, an array of multiple flux and photon pumps could be envisioned.

Alternatively, our scheme could be realized in ultracold atomic systems [15,16], where large interactions U and negligible decay γ are readily available [52]. In this case, an idea for realizing local flux insertion would be to use optical Raman beams with nonzero angular momentum [53], or as an alternative, quasiholes could be introduced by placing a focused laser-beam close to the edge of the system and increasing its intensity adiabatically [54]. Independent of the system, means for detecting LN-type ground states are required, and several approaches were discussed regarding how this can potentially be achieved [24,31,55–59].

Summary and outlook.—We proposed a scheme for the preparation of highly correlated LN states of bosons in artificial gauge fields. LN states can be understood in terms of weakly interacting CFs, and our protocol is based on the idea of growing noncorrelated states of the latter. We demonstrated that this can be achieved by first creating LN quasihole excitations which are subsequently refilled with bosons. Importantly, our protocol only requires a preparation time scaling slightly faster than linear with system size.

Our scheme is not restricted to the preparation of LN states of bosons. For example, we expect that the $\nu = 1$ bosonic Moore-Read Pfaffian [5,60,61] supporting non-Abelian topological order, can also be grown using our technique. Moreover, preparing bosons in higher LLs opens the possibility to simulate exotic Haldane pseudopotentials, mimicking the effect of long-range interactions without the need to implement these in the first place. We also expect that our scheme can be adapted for the preparation of fractional quantum Hall states of fermions.

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