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Influence of pump-field phase diffusion on laser gain in a double-Λ non-inversion laser

Michael Fleischhauer [1], Mikhail D. Lukin, Dmitri E. Nikonov, Marlan O. Scully [2]

Department of Physics, Texas A&M University, College Station, TX 77843, USA

Texas Laser Laboratory, Houston Advanced Research Center, 4800 Research Forest Drive, The Woodlands, TX 77381, USA

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Abstract

The influence of pump-field phase diffusion on the gain in a non-degenerate double-Λ non-inversion laser is studied. The phase diffusion of the lower-level coherence leads to loss contributions which are not directly proportional to the number density of atoms and occur in addition to the well known effect of reduced coherent population trapping. Lasing without inversion is however still possible even if the phases of the pump fields are not locked.

In recent years the possibility of coherent light amplification without the need of population inversion (LWI) became a subject of intensive theoretical [1] and experimental [2] activity. One particular atomic system capable of amplifying without inversion is the double-Λ scheme shown in Fig. 1 [3]. Atomic coherences generated by the intense driving fields reduce the absorption of a pair of appropriately phased probe fields. The build-up of the coherence between the two lower levels, which plays the most important role, is equivalent to optical pumping into a certain coherent superposition of these levels. The pair of probe fields does not couple to this state if the relative phase is chosen correctly and hence the population is trapped in the coherent superposition state [3]. A small population in the upper lasing level, smaller than the population in each of the lower states, provided for instance by pumping out of one particular sublevel of the ground state manifold, then leads to amplification. It should be noted, that the presence of other coherences makes amplification possible even if the population in the coupled superposition state(s) is larger than the population in the upper lasing level [5].

Since the coherence between the lower levels depends on the relative phase of the two driving fields, diffusion of this phase leads to a decay of the coherence and therefore to a decay of the trapping state [6,7]. That is why it is reasonable to expect that LWI gain will be decreased in the presence of phase diffusion. The lower level coherence also depends on the relative amplitude of the two driving fields and hence amplitude fluctuations will influence the laser gain too. Unlike the phase fluctuations, the amplitude fluctuations are superimposed on

[1] Permanent address: Institut für Theoretische Physik, Ludwig-Maximilians-Universität, 80333 München, Germany.

[2] Also: Max-Planck-Institut für Quantenoptik, 85748 Garching, Germany.

[3] The phenomenon of coherent population trapping has been extensively studied in the past. For early work see for instance Ref. [4].

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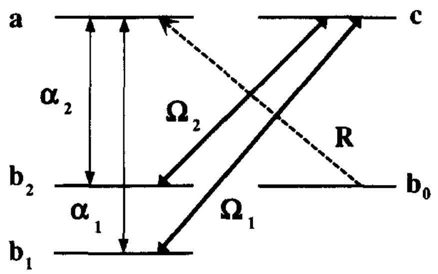

Fig. 1. Double-Λ configuration. Strong bichromatic fields of Rabi-frequency Ω_1 and Ω_2 generate coherence between levels b_1 and b_2 (and other coherences) allowing non-inversion amplification of appropriately phased probe fields α_1 and α_2. Collisions induce population exchange between lower levels with rate γ_c.

the large mean values of the driving fields (they are believed to be far above the threshold), and are neglected for the present discussion.

The analysis shows that in addition to the gain reduction due to the decrease of population trapping [6,7] there is another effect which gives rise to a reduction of the gain. In the bichromatic scheme non-inversion amplification occurs only under two-photon resonance conditions, that is if the oscillation frequency of the coherence exactly matches the beat frequency of the two probe fields. The diffusion of the relative phase of the two driving fields, on the other hand, is equivalent to a fluctuating beat frequency which leads to a fluctuating oscillation frequency of the coherence. Therefore the two-photon resonance condition is only fulfilled in the mean and there will be additional contributions to the absorption. Unlike the absorption contributions due to the reduced population trapping, they are not directly proportional to the number of atoms and cannot be compensated by increasing the driving field intensities.

The semiclassical properties of the double-Λ-scheme of Fig. 1 can be described with a set of density matrix equations in a rotating frame. Since we are interested here only in the linear gain of the system, we may treat the interaction with the single-mode probe fields, characterized by the coherent amplitudes α_1 and α_2, in a perturbative way.

In the zeroth order in the probe fields, but in all orders in the Rabi-frequencies Ω_1 and Ω_2 of the driving fields we have the set of equations:

$$\frac{d}{dt}\rho_{cc} = -\Gamma\rho_{cc} + i(\Omega_1^*\rho_{cb_1} - c.c.) + i(\Omega_2^*\rho_{cb_2} - c.c.), \tag{1a}$$

$$\frac{d}{dt}\rho_{b_{1,2}b_{1,2}} = \gamma_{1,2}\rho_{cc} + \gamma'_{1,2}\rho_{aa} - 2\gamma_c\rho_{b_{1,2}b_{1,2}} + \gamma_c\rho_{b_0b_0} + \gamma_c\rho_{b_{2,1}b_{2,1}} - i(\Omega_{1,2}^*\rho_{cb_{1,2}} - c.c.), \tag{1b}$$

$$\frac{d}{dt}\rho_{b_0b_0} = \gamma_0\rho_{cc} + \gamma'_0\rho_{aa} - 2\gamma_c\rho_{b_0b_0} + \gamma_c\rho_{b_1b_1} + \gamma_c\rho_{b_2b_2} + R(\rho_{aa} - \rho_{b_0b_0}), \tag{1c}$$

$$\frac{d}{dt}\rho_{cb_{1,2}} = -\tfrac{1}{2}(\Gamma + 2\gamma_c)\rho_{cb_{1,2}} + i\Omega_{1,2}(\rho_{cc} - \rho_{b_{1,2}b_{1,2}}) - i\Omega_{2,1}\rho_{b_{2,1}b_{1,2}}, \tag{1d}$$

$$\frac{d}{dt}\rho_{b_1b_2} = -2\gamma_c\rho_{b_1b_2} + i\Omega_2\rho_{cb_1}^* - i\Omega_1^*\rho_{cb_2}, \tag{1e}$$

$$\frac{d}{dt}\rho_{aa} = -\Gamma'\rho_{aa} + R(\rho_{b_0b_0} - \rho_{aa}), \tag{1f}$$

where

$$\Gamma = \gamma_1 + \gamma_2 + \gamma_0, \quad \Gamma' = \gamma'_1 + \gamma'_2 + \gamma'_0, \tag{2}$$

γ_μ being the radiative decay rates out of level c and γ'_μ are the corresponding decay rates out of level a to the level b_μ. Note, that we have included a collisional population exchange between the lower levels with rate γ_c.

In order to make the discussion more transparent we have assumed resonant driving fields. R describes the pumping rate from b_0 which provides the population in the upper level a of the probe transitions.

We now analyze the influence of phase diffusion of the driving fields. For this we assume independent, freely diffusing phases of the driving fields [8]

$$\Omega_{1,2} = \Omega_{1,2}^0 \exp(\mathrm{i}\phi_{1,2}), \tag{3}$$

with

$$\dot{\phi}_{1,2} = \mu_{1,2}(t), \tag{4}$$

$$\langle \mu_{1,2}(t) \rangle = 0, \qquad \langle \mu_l(t)\mu_n(t') \rangle = \Delta\nu_l\, \delta_{ln}\delta(t-t'). \tag{5}$$

Eqs. (1) are therefore stochastic equations with multiplicative white noise, which gives rise to noise-induced drift terms that alter the semiclassical evolution of the system. In order to make these effects transparent we introduce the new variables $\tilde{\rho}_{\mathrm{cb}_{1,2}} = \rho_{\mathrm{cb}_{1,2}} \exp(-\mathrm{i}\phi_{1,2})$, $\tilde{\rho}_{\mathrm{b_1b_2}} = \rho_{\mathrm{b_1b_2}} \exp[\mathrm{i}(\phi_1 - \phi_2)]$. The corresponding equations of motion read:

$$\frac{\mathrm{d}}{\mathrm{d}t}\tilde{\rho}_{\mathrm{cb}_{1,2}} = \left(\frac{\mathrm{d}}{\mathrm{d}t}\rho_{\mathrm{cb}_{1,2}}\right) \exp(-\mathrm{i}\phi_{1,2}) - \mathrm{i}\mu_{1,2}(t)\tilde{\rho}_{\mathrm{cb}_{1,2}}, \tag{6a}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\tilde{\rho}_{\mathrm{b_1b_2}} = \left(\frac{\mathrm{d}}{\mathrm{d}t}\rho_{\mathrm{b_1b_2}}\right) \exp[\mathrm{i}(\phi_1 - \phi_2)] + \mathrm{i}[\mu_1(t) - \mu_2(t)]\tilde{\rho}_{\mathrm{b_1b_2}}. \tag{6b}$$

We can perform the average with respect to the phase fluctuations. Noting that for multiplicative white noise we have [8]

$$\begin{aligned}\langle \mu_1(t)\tilde{\rho}_{\mathrm{cb}_1}(t)\rangle &= \langle \mu_1(t)\tilde{\rho}_{\mathrm{cb}_1}(t-\epsilon)\rangle + \int_{t-\epsilon}^{t} \mathrm{d}\tau\, \langle \mu_1(t)\tilde{\rho}_{\mathrm{cb}_1}(\tau)\rangle \\ &= \langle \mu_1(t)\tilde{\rho}_{\mathrm{cb}_1}(t-\epsilon)\rangle - \mathrm{i}\int_{t-\epsilon}^{t} \mathrm{d}\tau\, \langle \mu_1(t)\mu_1(\tau)\rangle\langle\tilde{\rho}_{\mathrm{cb}_1}(\tau-\epsilon)\rangle \\ &\quad + (-\mathrm{i})^2 \int_{t-\epsilon}^{t} \mathrm{d}\tau \int_{\tau-\epsilon}^{\tau} \mathrm{d}\tau'\, \langle \mu_1(t)\mu_1(\tau)\mu(\tau')\rangle\langle\tilde{\rho}_{\mathrm{cb}_1}(\tau'-\epsilon)\rangle + \ldots = -\tfrac{\mathrm{i}}{2}\Delta\nu_1\langle\tilde{\rho}_{\mathrm{cb}_1}(t-\epsilon)\rangle + \mathrm{O}(\epsilon),\end{aligned} \tag{7}$$

we find in the limit $\epsilon \to 0$

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle\tilde{\rho}_{\mathrm{cb}_{1,2}}\rangle = -\tfrac{1}{2}(\Gamma + 2\gamma_{\mathrm{c}} + \Delta\nu_{1,2})\langle\tilde{\rho}_{\mathrm{cb}_{1,2}}\rangle + \mathrm{i}\Omega_{1,2}^0(\langle\rho_{\mathrm{cc}}\rangle - \langle\rho_{\mathrm{b_{1,2}b_{1,2}}}\rangle) - \mathrm{i}\Omega_{2,1}^0\langle\tilde{\rho}_{\mathrm{b_{2,1}b_{1,2}}}\rangle, \tag{8a}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle\tilde{\rho}_{\mathrm{b_1b_2}}\rangle = -[2\gamma_{\mathrm{c}} + \tfrac{1}{2}(\Delta\nu_1 + \Delta\nu_2)]\langle\tilde{\rho}_{\mathrm{b_1b_2}}\rangle + \mathrm{i}\Omega_2^0\langle\tilde{\rho}_{\mathrm{cb}_1}^*\rangle - \mathrm{i}\Omega_1^{0*}\langle\tilde{\rho}_{\mathrm{cb}_2}\rangle. \tag{8b}$$

Eq. (8) shows, that the phase diffusion leads to additional coherence decay terms. The linewidth of the fields adds to the decay rates of the corresponding optical coherence and the diffusion coefficient of the relative phase, which is here the sum of the two linewidth, adds to the decay of the lower level coherence [6,7].

We solve Eqs. (1a)–(1e) and (8) in steady-state. For simplification we assume $\Omega_1^0 = \Omega_2^0 = \Omega = \Omega^*$, $\gamma_1 = \gamma_2 = \gamma_0 = \gamma$ and the same for the values with primes. We find for the degree of coherence between b_1 and b_2 and the population in b_0, c and a:

$$\frac{\langle\rho_{\mathrm{b_1b_2}}\rangle}{\langle\rho_{\mathrm{bb}}\rangle} = -\frac{4\Omega^2\Gamma}{D}, \tag{9a}$$

$$\frac{\langle\rho_{b_0b_0}\rangle}{\langle\rho_{bb}\rangle} = \frac{(R+\Gamma')[4\Omega^2(2\gamma_c\Gamma_c+\Gamma\gamma_c)+\Gamma_c^2\Gamma(\Gamma+\Gamma_c)]}{D(\gamma_c R+\gamma' R+\Gamma\gamma_c)}, \tag{9b}$$

$$\frac{\langle\rho_{cc}\rangle}{\langle\rho_{bb}\rangle} = \frac{8\Omega^2\Gamma_c}{D}, \tag{9c}$$

$$\frac{\langle\rho_{aa}\rangle}{\langle\rho_{bb}\rangle} = \frac{R[4\Omega^2(2\gamma_c\Gamma_c+\Gamma\gamma_c)+\Gamma_c^2\Gamma(\Gamma+\Gamma_c)]}{D(\gamma_c R+\gamma' R+\Gamma\gamma_c)}, \tag{9d}$$

where $\Gamma_c = 2\gamma_c + \frac{1}{2}(\Delta\nu_1 + \Delta\nu_2)$, and $D = 4\Omega^2(\Gamma + 2\Gamma_c) + \Gamma\Gamma_c(\Gamma + \Gamma_c)$. In Eqs. (9) we have neglected the linewidth contributions to the usually much larger optical decay rates. Eq. (9a) shows a reduction of the degree of coherence and therefore a reduction of coherent population trapping with increasing Γ_c, an effect discussed in Refs. [6,7]. Associated with this is an enhanced optical pumping into level b_0.

For $\Gamma_c \ll \Gamma$ the decrease of coherence can be compensated by increased driving field strength. The enhanced population in b_0 is rather of advantage for the practical implementation of cw-laser operation due to the following reasons. Suppose that the pump fields are perfectly coherent and there are no collisions. As one can see from Eqs. (9), the populations in b_0 and therefore in a are zero. In other words all the population will be optically pumped into the uncoupled combination of states b_1 and b_2 regardless of the initial conditions, and therefore the system has zero gain in the steady state. That is continuous-wave LWI is not possible in such a system. However if phase diffusion or collisions are present, ρ_{aa} is no longer zero. This leads (if the coherence $\rho_{b_1b_2}$ is not completely destroyed) to positive gain in the steady state and allows continuous-wave LWI.

We are now in a position to discuss the influence of the pump-laser phase diffusion on the LWI dynamics in first order of the probe fields. In this case the equations of motion for the atomic variables read

$$\frac{d}{dt}\rho_{ab_{1,2}} = -\gamma_{ab_{1,2}}\rho_{ab_{1,2}} + ig_{1,2}\alpha_{1,2}(\rho_{aa} - \rho_{b_{1,2}b_{1,2}}) - ig_{2,1}\alpha_{2,1}\rho_{b_{2,1}b_{1,2}} + i\Omega_{1,2}\rho_{ac}, \tag{10a}$$

$$\frac{d}{dt}\rho_{ac} = -\gamma_{ac}\rho_{ac} - ig_1\alpha_1\rho^*_{cb_1} - ig_2\alpha_2\rho^*_{cb_2} + i\Omega_1^*\rho_{ab_1} + i\Omega_2^*\rho_{ab_2}. \tag{10b}$$

Here $g_\mu = (\wp_\mu/\hbar)(\sqrt{\hbar\nu_\mu/2\varepsilon_0 V})$ are the atom–field coupling strengths, $\wp_\mu$ and ν_μ being the dipole matrix element and the transition frequency (= field frequency) of the corresponding probe transition. V is the interaction volume. In the following we assume $g_1 = g_2 = g$. The relaxation rates are $\gamma_{ab_{1,2}} = \gamma_c + \Gamma'/2$ and $\gamma_{ac} = \gamma_d + (\Gamma + \Gamma')/2$. We assumed here that there is no population exchange between levels a and c but included a dephasing rate γ_d between the two levels. Eqs. (10) have stochastic character because of the stochastic nature of $\Omega_{1,2}$ and $\rho_{b_1b_2}$. In order to see the effect of the pump-field phase diffusion on the gain one has to consider the evolution of the mean photon numbers (not the coherent amplitudes) of both fields.

$$\frac{d}{dt}\langle\alpha_\mu^*\alpha_\mu\rangle = -igN\langle\alpha_\mu^*\rho_{ab_\mu}\rangle + \text{c.c.} \tag{11}$$

Here N is the number of atoms in the interaction volume. As will be seen later on, the field intensities are coupled to the real part of cross-correlation function $\langle\alpha_1^*\alpha_2\exp(i\psi)\rangle$, $\psi = \phi_2 - \phi_1$, for which one finds the equation of motion

$$\frac{d}{dt}\langle\alpha_1^*\alpha_2\exp(i\psi)\rangle = -\tfrac{1}{2}(\Delta\nu_1+\Delta\nu_2)\langle\alpha_1^*\alpha_2\exp(i\psi)\rangle - igN\langle\alpha_1^*\rho_{ab_2}\exp(i\psi)\rangle + igN\langle\rho^*_{ab_1}\alpha_2\exp(i\psi)\rangle, \tag{12}$$

where the first term on the r.h.s. is a noise induced drift term similar to that appearing in Eq. (8b). We proceed by evaluating the expressions on the r.h.s. of Eqs. (11) and (12) in the adiabatic limit. We find for example

$$0 = \frac{d}{dt}\langle\rho_{ab_1}\alpha_1^*\rangle = -\gamma_{ab}\langle\rho_{ab_1}\alpha_1^*\rangle + ig\langle\alpha_1^*\alpha_1(\rho_{aa} - \rho_{bb})\rangle$$
$$- ig\langle\alpha_1^*\alpha_2\exp(i\psi)\tilde\rho^*_{b_1b_2}\rangle + i\Omega\langle\alpha_1^*\rho_{ac}\exp(i\phi_1)\rangle + igN\langle\rho^*_{ab_1}\rho_{ab_1}\rangle. \tag{13}$$

The last term on the r.h.s. is of higher order in the coupling strength and will be disregarded. Eq.(13) shows the above mentioned coupling of the field intensities to the cross-correlation function. As will be explained in the following the zeroth order density matrix elements ρ_{aa}, ρ_{bb}, and $\tilde{\rho}_{b_1b_2}$ can be factorized out in the second and third term. Let us for example consider the term $\langle n_{12}\tilde{\rho}_{b_1b_2}\rangle$, with $n_{12} = \alpha_1^*\alpha_2 \exp(i\psi)$. The time evolution of n_{12} is given by

$$\frac{d}{dt}n_{12}(t) = -i\mu_{12}(t)n_{12}(t) + \text{higher order terms in } g, \tag{14}$$

where $\mu_{12} = \mu_1 - \mu_2$. Formally integrating Eq.(14) yields the iterative expression

$$\langle n_{12}(t)\tilde{\rho}_{b_1b_2}(t)\rangle = \langle n_{12}(t-\epsilon)\tilde{\rho}_{b_1b_2}(t)\rangle - i\int_{t-\epsilon}^{t} d\tau \langle n_{12}(\tau-\epsilon)\rangle\langle \mu_{12}(\tau)\tilde{\rho}_{b_1b_2}(\tau)\rangle$$
$$+ (-i)^2 \int_{t-\epsilon}^{t} d\tau \int_{\tau-\epsilon}^{\tau} d\tau' \langle n_{12}(\tau'-\epsilon)\rangle\langle \mu_{12}(\tau)\mu_{12}(\tau')\tilde{\rho}_{b_1b_2}(\tau)\rangle + \cdots + \text{higher order terms in } g. \tag{15}$$

Making use of $\langle \mu_{12}(\tau)\tilde{\rho}_{b_1b_2}(\tau)\rangle = \frac{1}{2}(\Delta\nu_1 + \Delta\nu_2)\langle\tilde{\rho}_{b_1b_2}(\tau)\rangle$, one can see, that all but the first term vanish or are proportional to powers of ϵ larger than one. Hence only the first term survives in the limit $\epsilon \to 0$. Similar arguments apply to the second term on the r.h.s. of Eq.(13), and therefore the zeroth order matrix elements can be factorized and $\langle \rho_{ab_1}\alpha_1^*\rangle$ can be expressed in terms of the field intensity $\langle\alpha_1^*\alpha_1\rangle$, the cross-correlation function $\langle\alpha_1^*\alpha_2\exp(i\psi)\rangle$, and $\langle\alpha_1^*\rho_{ac}\exp(i\phi_1)\rangle$. In a similar way one can derive a set of algebraic equations for the other quantities on the r.h.s. of Eqs.(11) and (12) as well as for $\langle\alpha_1^*\rho_{ac}\exp(i\phi_1)\rangle$ and $\langle\alpha_2^*\rho_{ac}\exp(i\phi_2)\rangle$. Inserting the solutions into the equations of motion of the field intensities and the cross-correlation function eventually yields

$$\frac{d}{dt}\langle\alpha_1^*\alpha_1\rangle = (A+B)\langle\alpha_1^*\alpha_1\rangle - (\bar{A}+B)\mathrm{Re}[\langle\alpha_1^*\alpha_2\exp(i\psi)\rangle], \tag{16a}$$

$$\frac{d}{dt}\langle\alpha_2^*\alpha_2\rangle = (A+B)\langle\alpha_2^*\alpha_2\rangle - (\bar{A}+B)\mathrm{Re}[\langle\alpha_1^*\alpha_2\exp(i\psi)\rangle], \tag{16b}$$

$$\frac{d}{dt}\mathrm{Re}[\langle\alpha_1^*\alpha_2\exp(i\psi)\rangle] = (A+B-\Delta)\mathrm{Re}[\langle\alpha_1^*\alpha_2\exp(i\psi)\rangle] - \tfrac{1}{2}\bar{A}\langle\alpha_1^*\alpha_1\rangle - \tfrac{1}{2}\bar{A}\langle\alpha_2^*\alpha_2\rangle, \tag{16c}$$

where $\Delta = \frac{1}{2}(\Delta\nu_1 + \Delta\nu_2)$ and

$$A = \frac{2g^2N}{D}\left[h\left(\gamma_{ab} + \frac{\Omega^2}{\gamma_{ac}}\right)(\langle\rho_{aa}\rangle - \langle\rho_{bb}\rangle) + \frac{\Omega^2}{\gamma_{ac}}\langle\tilde{\rho}_{b_1b_2}\rangle\right],$$

$$\bar{A} = \frac{2g^2N}{D}\left[\frac{\Omega^2}{\gamma_{ac}}(\langle\rho_{aa}\rangle - \langle\rho_{bb}\rangle) + \left(\gamma_{ab} + \frac{\Omega^2}{\gamma_{ac}}\right)\langle\tilde{\rho}_{b_1b_2}\rangle\right],$$

$$B = \frac{2g^2N}{D}\frac{2\gamma_{ab}\Omega^2}{\Gamma\gamma_{ac}}(\langle\rho_{bb}\rangle + \langle\tilde{\rho}_{b_1b_2}\rangle - \langle\rho_{cc}\rangle), \qquad D = \gamma_{ab}\left(\gamma_{ab} + \frac{2\Omega^2}{\gamma_{ac}}\right), \tag{17}$$

where we have again neglected the linewidth contributions to the optical decay rates. The eigenvalues and eigensolutions of Eqs. (16) are

$$\lambda_1 = A + B, \qquad f_1 = \bar{n}_1 - \bar{n}_2, \tag{18a}$$

$$\lambda_2 = A + B - \frac{\Delta}{2} - \left(\frac{\Delta^2}{4} + \bar{A}(\bar{A}+B)\right)^{1/2}, \qquad f_2 = (\bar{A}+B)(\bar{n}_1 + \bar{n}_2) + (\lambda_1 - \lambda_2)\bar{n}_{12}, \tag{18b}$$

$$\lambda_3 = A + B - \frac{\Delta}{2} + \left(\frac{\Delta^2}{4} + \bar{A}(\bar{A} + B)\right)^{1/2}, \qquad f_3 = (\bar{A} + B)(\bar{n}_1 + \bar{n}_2) + (\lambda_1 - \lambda_3)\bar{n}_{12}, \tag{18c}$$

where $\bar{n}_\mu = \langle\alpha_\mu^*\alpha_\mu\rangle$ and $\bar{n}_{12} = \langle n_{12}\rangle$. In order to simplify the further discussion we disregard the contributions due to coherences other than the one between b_1 and b_2, in particular due to the a–c coherence. This corresponds for example to the case of strong collisional dephasing of the a–c coherence, such that $\Omega^2/\gamma_{ac} \ll \gamma_{ab}$. We would like to emphasize though that all the following conclusions remain qualitatively the same if the a–c coherence is taken into account. Under these conditions the eigenvalues take on the simple form

$$\lambda_1 = \frac{2g^2N}{\gamma_{ab}}(\langle\rho_{aa}\rangle - \langle\rho_{bb}\rangle), \tag{19a}$$

$$\lambda_2 = \frac{2g^2N}{\gamma_{ab}}(\langle\rho_{aa}\rangle - \langle\rho_{bb}\rangle) - \tfrac{1}{4}(\Delta\nu_1 + \Delta\nu_2) - \left[\tfrac{1}{16}(\Delta\nu_1 + \Delta\nu_2)^2 + \left(\frac{2g^2N}{\gamma_{ab}}\langle\tilde{\rho}_{b_1b_2}\rangle\right)^2\right]^{1/2}, \tag{19b}$$

$$\lambda_3 = \frac{2g^2N}{\gamma_{ab}}(\langle\rho_{aa}\rangle - \langle\rho_{bb}\rangle) - \tfrac{1}{4}(\Delta\nu_1 + \Delta\nu_2) + \left[\tfrac{1}{16}(\Delta\nu_1 + \Delta\nu_2)^2 + \left(\frac{2g^2N}{\gamma_{ab}}\langle\tilde{\rho}_{b_1b_2}\rangle\right)^2\right]^{1/2}. \tag{19c}$$

Under non-inversion conditions the only positive eigenvalue is λ_3, which therefore determines the gain. The diffusion of the relative phase of the driving fields leads to a decrease of $|\rho_{b_1b_2}|$, as per Eq. (9a) and hence to an enhanced loss term in Eq. (19c) [6,7]. In addition to this there is a loss contribution, which is not simply proportional to the number of atoms. The physical origin of these additional losses are the imposed fluctuations of the oscillation frequency of the lower-level coherence leading to a partial violation of the two-photon-resonance condition. In the limit $N \to 0$, λ_3 of course vanishes regardless of the magnitude of the phase diffusion.

In order to estimate the size of the additional absorption contribution, we have to compare $g^2N\langle\rho_{aa}\rangle/\gamma_{ab}$ with $\Delta\nu_1 + \Delta\nu_2$. We note, that for optical wavelength g^2N/γ_{ab} is of order $1s^{-1}$ times the number density in cm^{-3}. The additional loss contributions due to the finite linewidth of the driving fields therefore becomes important only for systems with rather low densities of atoms in the excited state, which is however the case of interest for non-inversion amplification.

It is interesting to note, that λ_1 is also eigenvalue of the equations of motion of the mean coherent amplitudes. Its negativity indicates a locking of the two amplitudes leading to strongly correlated output fields [4].

In conclusion, we have found that the gain in a non-degenerate double-Λ laser under non-inversion conditions is affected by the phase diffusion of driving fields in two ways.

First the phase diffusion leads to a decay of the coherent trapping state. If the linewidth is small compared to the optical decay rates, this effect can however be compensated by increased pump-field intensities. Associated with the reduced population trapping is an enhanced optical pumping into the uncoupled ground state b_0, which can be useful for a practical implementation of continuous wave LWI.

We also found another linewidth effect on the laser gain, which is particularly important for small densities of excited atoms. The diffusion of the relative phase of the two driving fields leads to a fluctuating oscillation frequency of the coherence. Therefore the two-photon resonance condition required for non-inversion amplification of a bichromatic probe field is only fulfilled in the mean and the gain is reduced [5].

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[4] This effect is related to the phenomenon of pulse matching recently discussed by Harris [9].

[5] The effects discussed in this paper are of course not present in a degenerate scheme, where a single driving field coherent or incoherent couples *both* lower levels. In such a case there are a priori no fluctuations in the beat note between the "two" driving fields. Furthermore the lower-level coherence undergoes a forced "oscillation" with frequency zero and hence a single probe field is always in "two-photon" resonance.

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