Breakdown of topological protection under local periodic driving

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ABSTRACT

The bulk-edge correspondence guarantees that the interface between two topologically distinct insulators supports at least one topological edge state that facilitates scattering-free transport along the interface and is robust against static perturbations. Here, we address the question how spatially local dynamic perturbations of the interface affect the robustness of topological edge states and illuminate the generalization of the bulk-edge correspondence for Floquet systems for the special case of a static bulk. As model systems we consider two photonic implementations of the Su-Schriefer-Heeger model based on coupled plasmonic and dielectric waveguides. Time-periodic perturbations of the interface create Floquet replicas of the topological edge mode. Our experiments and Floquet analysis show that if the driving frequency is in the range for which the first Floquet replicas cross the static bands, the topological edge state couples to bulk states and the topological protection is destroyed. Otherwise the topological protection is conserved.

The topological properties of the bulk of a system can have a profound impact on the character of the modes at its boundary. Of particular interest are interfaces between two insulators with different topologies. According to the bulk-edge correspondence principle, such an interface hosts at least one edge state that is protected by topology, i.e., it allows for transport along the interface without scattering even in the presence of strong deformations in the static case. This intriguing property has been observed in a number of solid state, photonic, and cold atom systems.

A powerful tool for manipulating various quantum systems is time-periodic driving. The underlying principle is that driving of a system with frequency $\omega$ enables the hybridization of eigenstates of a static system, which are separated in energy by a multiple of $\hbar \omega$. As a result, new synthetically designed properties, inaccessible in equilibrium, can emerge. For instance, appropriately chosen driving regimes allow for coherent control of single-particle tunneling, tuning transport regimes from ballistic to localized, and inducing quantum phase transitions. Recently, it has also been shown that periodic driving can change the topological properties of a system. In particular, a system, trivial in equilibrium, can become a topological insulator under periodic driving due to breaking of time-reversal symmetry. In systems with time-periodic drive the bulk-edge correspondence needs to be generalized and anomalous edge modes can exist. In addition to the driving frequency and amplitude also the spatial extent of the driving is a valuable degree of freedom. As an example, by periodically driving individual lattice sites one can control the transmission across the modulated region, pump charge, and create new Floquet bound states.

In this article, we theoretically and experimentally study how local time-periodic perturbations affect a topologically protected edge state. Applying perturbations locally to the edge while keeping the bulk static allows us to illuminate the modified bulk-edge correspondence for special Floquet systems and to investigate the robustness of topological protection. As a simple yet topologically nontrivial system we analyze such perturbations in the Su-Schriefer-Heeger (SSH) model. Our theoretical predictions are confirmed by two independent experiments based on plasmonic and dielectric waveguide arrays (see Fig.1). Precise control of the system’s parameters as well as an uncomplicated detection technique make both types of coupled waveguides attractive for quantum simulations. In the static case, the SSH model describes a chain of identical lattice sites with alternating strong and weak bonds, that can be implemented by alternating short $d_1$ and long $d_2$ distances between adjacent waveguides, respectively. Depending on the choice of the unit cell, the SSH model exhibits two topologically distinct dimerizations. At each interface between two domains of different topology a topologically protected edge state occurs.
Figure 1. Sketches of the SSH chains with time-periodic perturbations of a single lattice site at the interface between two distinct dimerizations (top) and the corresponding experimental realizations (bottom). (a) The boundary is modulated in longitudinal direction. (b) The boundary is modulated in transverse direction. Here, $J_1$ ($J_2$) denotes the large (small) hopping amplitude in the bulk, $J_{0,1} (t)$ ($J_{-1,0} (t)$) is the periodically modulated hopping amplitude between the 0th and 1st lattice sites (0th and −1st lattice sites), $\omega$ is the driving frequency, $d_1$ ($d_2$) is the short (long) center-to-center distance, $A$ is the maximum deflection of the 0th waveguide from the center, and $P$ is the period of driving. The different frequency regimes are realized by varying the period $P$ while $A$ is always kept constant. Note that in the waveguide system the propagation distance $z$ corresponds to time $t$.

Spatially, this state is exponentially localized at the interface, while in the spectrum of the system it has a midgap position. An interface supporting a topological edge state can be created by repeating the weak bond twice. We apply local time-periodic perturbations associated with a single lattice site at the interface (site 0) by modulating the hopping amplitudes $J_{-1,0} (t)$ and $J_{0,1} (t)$ to its nearest neighbors and its local on-site potential $V_0 (t)$. Since in the waveguide model the propagation distance $z$ plays the role of time$^{28}$, bending the 0th waveguide sinusoidally we realize such perturbations. Two different modulations are considered: in longitudinal (Fig. 1(a)) and transverse direction (Fig. 1(b)). In contrast to previous studies$^{29,30}$, we do not drive the bulk of the SSH model to guarantee that the topological invariants stay unchanged.

Results and discussion

Theoretical analysis

We start with a theoretical analysis of our model based on the Floquet theory$^{31,32}$ (see Methods). Within this formalism a band structure can be unambiguously described in terms of so-called quasienergies, analogues of the eigen energies in a time-independent problem. The corresponding Floquet states belong to the extended Hilbert space, which is a direct product of the usual Hilbert space and the space of periodic functions with period $P = 2\pi/\omega$. In a Floquet picture our 1-dimensional time-periodic system can be displayed as a (1+1)-dimensional time-independent system$^{32}$. Figure 2 (a) shows the static (1+1)D lattice which is analogous to the SSH model with local harmonic perturbations of the topological defect. It consists of an infinite number of SSH chains labeled by the Floquet index $n$ with the overall potential shifted by $-n\omega$ (throughout the paper we set $\hbar = 1$). Periodic driving thus splits the band structure of the undriven system into infinitely many copies (Floquet replicas) spaced by $\omega$.$^{29,32}$ Fig. 2 (a) illustrates that local perturbations couple the chains only through the sites in the vicinity of the interface ($s = -1, 0, 1$) with the hopping amplitude $\Delta J/2$ determined by the modulation amplitude $A$. Hence, applying a local perturbation to the interface we selectively populate the Floquet replicas of the topological edge state while the bulk states stay almost unaffected.
Figure 2. Floquet analysis of the SSH model with local periodic perturbations at the interface. (a) (1+1)D time-independent analogue of the SSH model with local periodic perturbations at the interface. The Floquet index $n$ enumerates coupled SSH chains each with the overall potential $-n\omega$. Red and blue arrows denote the coupling between Floquet replicas ($n$) and sites ($s$) created by the harmonic driving of local couplings with amplitude $\Delta J$ and on-site potential with $\Delta V$ (see Methods for details). (b) Frequency-dependent quasienergy spectrum with assigned weights in case of the longitudinal perturbations at the interface. (c-j) Temporal evolution of the probability density (left) and corresponding momentum-resolved spectra (right) for: (c, d) undriven case, (e, f) low frequency ($\omega = 0.3J_1$), (g, h) intermediate frequency ($\omega = J_1$), (i, j) high frequency ($\omega = 2J_1$). The histograms at the right side from (c-i) show the distribution of the probability density at $t = 50J_1^{-1}$. Magenta arrows point to the 0th Floquet replica of the edge state, while green arrows indicate the location of its 1st Floquet replicas. Calculations for (b-j) were performed for longitudinally modulated SSH model with $2M = 100$ dimers, $J_1 = 1$, $J_2/J_1 = 0.5$, and $\Delta J = 0.3J_1$. As initial conditions we solely excited the 0th lattice site.
In the following we present results for the model with the longitudinal modulation of the topological defect. In this case, the couplings to the left $J_{-1,0}(t)$ and right $J_{0,1}(t)$ nearest neighbors of the $0^{th}$ site change with a phase shift of $\pi$. We choose $J_{-1,0} = J_2 + \Delta J \sin \omega t$, $J_{0,1} = J_2 - \Delta J \sin \omega t$ and $V_0(t) = 0$. The corresponding quasienergy spectrum is presented in Fig. 2 (b). Color coding indicates the spectral weight of each Floquet state (see Methods Eq. (11)). One notices that the number of edge states within the gap of bulk states differs from the static case and edge states become visible above the higher and below the lower band. This is in full agreement with the edge-state counting rules of Floquet Hamiltonians. We note that no anomalous edge states occur here since there is no periodic drive of the bulk.

As reference, we consider the static system ($\omega = 0$). In Fig. 2 (c) we plot the corresponding temporal evolution of the probability density $|\Psi(s,t)|^2$ for the single-site input at the $0^{th}$ lattice site. Here, the excited bulk modes are spreading ballistically while the topological edge state shows itself as a fraction of the probability density localized at the interface. The momentum distribution of the probability density $|\tilde{\Psi}(k,E)|^2$ (Fig. 2 (d)) features two cosine-shaped bands and a horizontal line in the middle of the band gap, a manifestation of the topological edge state.

In the low frequency regime ($\omega < |J_1 - J_2|$), the first $(n = \pm 1)$ replicas of the zero-energy mode lay inside the band gap (see green arrows in Fig. 2 (f)). We note that for all the modulation amplitudes accessible in the experiments the effect of higher $(|n| > 1)$ replicas is negligible. Fig. 2 (e) shows that $|\Psi(s,t)|^2$ stays localized at the $0^{th}$ lattice site. This picture completely changes in the intermediate frequency regime ($|J_1 - J_2| < \omega < |J_1 + J_2|$), when the first replicas of the topological edge state enter the energy interval of the static bulk states. The population of the zero-energy state drops drastically, while the bulk bands gain more weight. The periodic driving couples the topological edge state and the bulk states, destroying the topological protection; i.e. the probability density delocalizes (Fig. 2 (g)) and the momentum distribution also shows the pronounced coupling (green arrows in Fig. 2 (h)). Finally, in the high frequency regime ($\omega > |J_1 + J_2|$) the $1^{st}$ Floquet replicas lay outside of the band. Consequently, the probability density is again localized and the population of the topological edge state is restored (Figs. 2 (i), 2 (j)). Note that the periodic intensity modulation at the interface in Figs. 2 (e) and (i) results from beating of the topological edge state and its Floquet replicas. The asymmetry of the probability density distribution $|\Psi(s,t)|^2$ with respect to the interface in Figs. 2 (e), (g), and (i) results from the $\pi$ phase shift of the longitudinal coupling modulation.

Analogous calculations for the transversal perturbations (couplings are modulated in phase) show qualitatively the same behavior. In this case the left-right symmetry is conserved which leads to symmetric distribution of $|\Psi(s,t)|^2$ around the $0^{th}$

![Figure 3. Real- (left) and corresponding Fourier-space (right) leakage radiation micrographs of the DLSPPW arrays, analogous to the SSH model with a topological defect at $x = 0$. The geometric parameters of all arrays are chosen such that $J_2/J_1 = 0.5$. (a) and (b) correspond to the static case. In (c-h) the defect is modulated in longitudinal direction ($\Delta J \approx 0.25 J_1$) with different frequencies: (c,d) low frequency regime ($\omega = 0.49 J_1$), (e,f) intermediate frequency regime ($\omega = 0.84 J_1$), (g,h) high frequency regime ($\omega = 4.9 J_1$). The histograms at the right side from the real-space images show the intensity distribution after the propagation distance of $z = 100 \mu m$. Fourier-space the magenta arrows highlight the $0^{th}$ Floquet replica of the edge state, while green ones point to the location of its first Floquet replicas.](image-url)
Adding a periodic local on-site potential variation for the 0th site also does not change the picture if the corresponding amplitude $\Delta V$ is smaller or in the order of $\Delta J$.

**Experiments**

We provide experimental evidence of these effects using two photonic systems: arrays of dielectric loaded surface plasmon polariton waveguides (DLSPPWs) with longitudinal modulation (Fig. 1(a)) and dielectric waveguide arrays with transversal modulation (Fig. 1(b)). The details about the respective fabrication methods and the optical experiments are given in Methods.

We first consider longitudinal modulation in DLSPPW arrays. In these experiments, leakage radiation microscopy (see Methods) gives direct access to the full real-space intensity distributions as well as the momentum resolved spectra in Fourier space (see Fig. 3). For all measurements, surface plasmon polaritons (SPPs) were excited at a single waveguide in the center ($x = 0$) which represents the interface.

The case of the static SSH model is shown in Figs. 3 (a) and 3 (b). In real space (Fig. 3 (a)), the excitation of the topologically protected mode results in localization of SPPs at the interface. The decaying intensity along the $z$-axis is due to radiation losses and absorption. However, this does not affect the topological properties of the system. The momentum resolved spectrum of the static SSH model reveals the midgap position of this mode (see Fig. 3 (b)). We note that the asymmetry of the bulk bands arises from non-vanishing next-nearest neighbor coupling.

As predicted by Floquet theory, SPP localization at the interface is also observed for modulation at low (Figs. 3 (c) and 3 (d)) and high (Figs. 3 (g) and 3 (h)) frequencies. In these cases, the 1st Floquet replicas do not overlap with the bulk bands. In contrast, in the intermediate frequency regime (Figs. 3 (e) and 3 (f)), the energy of the 1st Floquet replicas coincides with the static bulk states and delocalization of SPPs into the bulk is observed (see histogram in Fig. 3 (e)). Hence, we see clear experimental evidence of the destruction of a topologically protected edge mode by local driving in agreement with theoretical predictions above.

Dielectric waveguide arrays fabricated by direct 3D laser writing are ideally suited for a transversal modulation of the interface (see Methods). Figure 4 displays the intensities at the output facet of the array. In Fig. 4 (a) we see that light is localized around the defect at the central site at $x = 0$ for the low and high frequency regime (topmost and bottom panel). In contrast, the light couples to the bulk modes for intermediate frequencies.

To exclude any influence of fabricational deviations of distinct samples, the switching between different frequency regimes can also be done in one sample by changing the wavelength of the light. This changes the hoppings, and therefore the ratio of $\omega / J_1$, the width of the band gap and the maximum energy of the bulk bands. Hence, the position of the Floquet replicas relative to the bulk bands can be controlled. For a wavelength of 680 nm the first Floquet replicas lie outside the bulk bands, corresponding to the high frequency regime. We see that the light is localized around the site at $x = 0$. With increasing wavelength the energy of the replicas moves into the bulk band, and we again observe coupling to the bulk modes and spreading of the light, starting at a wavelength of 750 nm. This confirms that the observed effects are not due to fabricational deviations.

![Figure 4](image-url)

**Figure 4. Measurements in 3D printed dielectric waveguides for transverse defect modulation.** Shown are the intensities in the waveguides at the output facet after a propagation in the array of ~ 24 hops with $J_1$. (a) Measurement for several structures with different period (frequency of modulation $\omega$) with otherwise the same parameters ($J_2 / J_1 = 0.48$) at a wavelength of $\lambda = 710$ nm. For small frequencies, the light is localized around the defect (at $x = 0$). When the frequency is increased, light couples to the bulk states ($0.56 \leq \omega / J_1 \leq 1.12$) and localizes in the defect again for large frequencies. (b) In a structure with fixed period of the defect modulation the wavelength is tuned. Light is delocalized/couples strongly to the bulk, when the first Floquet mode hits the bulk band, starting at 750 nm. Note that $J_2 / J_1$ changes with the wavelength: $J_2 / J_1 = 0.47$ (680 nm), 0.48 (710 nm), 0.52 (750 nm), 0.53 (780 nm) and 0.55 (810 nm).
between different samples.

In conclusion, we have shown that driving a defect locally in a system with non-trivial (bulk) topology results in a breakdown of the edge state’s topological protection for modulation energies in the range of the bulk bands. This was demonstrated in calculations using Floquet theory and proven by measurements in plasmonic and dielectric waveguide arrays for longitudinal and transversal modulation of the defect. Such model systems serve to control the localization and the steering of light via an external parameter. Our work gives insight into Floquet engineering of photonic systems and to what extent topological protection is preserved in the periodically driven case.

**Methods**

**Floquet analysis**

Our theoretical analysis is based on the Floquet theory. It provides a general framework for treating systems, which are governed by time-periodic Hamiltonians $H(t + P) = H(t)$ with a period $P = 2\pi/\omega$ and states that there is a complete set of solutions of the Schrödinger equation $i\frac{d}{dt}|\psi(t)\rangle = H(t)|\psi(t)\rangle$ of the form\(^\text{32}\)

$$|\psi_\alpha(t)\rangle = \exp(-ie_\alpha t)|u_\alpha(t)\rangle. \quad (1)$$

Here $e_\alpha$ is called quasienergy and $|u_\alpha(t)\rangle$ is the associated Floquet mode. Note that we set $\hbar = 1$. The quasienergies are defined up to multiplies of $\omega$, and the Floquet modes are $P$-periodic functions $|u_\alpha(t + P)\rangle = |u_\alpha(t)\rangle$.

After the substitution of the Floquet ansatz (1) into the Schrödinger equation, we directly obtain

$$\left(H(t) - i\frac{d}{dt}\right)|u_\alpha(t)\rangle = e_\alpha|u_\alpha(t)\rangle. \quad (2)$$

Using spectral decomposition of the Hamiltonian and the Floquet modes

$$H(t) = \sum_{n = -\infty}^{\infty} e^{-in\omega t} H_n, \quad |u_\alpha(t)\rangle = \sum_{n = -\infty}^{\infty} e^{-in\omega t} |u^n_\alpha\rangle, \quad (3)$$

we arrive at the time-independent Floquet equation

$$(H_0 - n\omega)|u^n_\alpha\rangle + \sum_{m \neq n} H_m |u^n_m\rangle = e_\alpha |u^n_\alpha\rangle, \quad \forall n \in \mathbb{Z}. \quad (4)$$

Consider the systems sketched in Fig. 1, where $J_{-1,0}(t) = J_2 + \Delta J \sin(\omega t)$, $J_{0,1}(t) = J_2 + \Delta J \sin(\omega t + \phi)$, and $V_0(t) = -\Delta V + \Delta V \cos(\omega t)$. For the longitudinal modulation (Fig. 1 (a)) we have a phase factor of $\phi = \pi$ and $\Delta V = 0$, while for the transversal modulation (Fig. 1 (b)) $\phi = 0$ and $\Delta V \neq 0$ holds.

Assuming $4M + 1$ lattice sites ($M$ dimers to either side of the defect and one unpaired site in the middle), the corresponding Hamiltonian can be written as a sum of a time-independent and time-periodic part

$$H = H_0 + H_P(t), \quad (5)$$

where

$$H_0 = \sum_{s = -M+1}^{0} (J_1 a_{2s}^\dagger a_{2s-1} + J_2 a_{2s-1}^\dagger a_{2s}) + \sum_{s = 0}^{M-1} (J_2 a_{2s}^\dagger a_{2s+1} + J_1 a_{2s+1}^\dagger a_{2s+2}) + \text{h.c.} - \Delta V a^\dagger \alpha_0 \quad (6)$$

and

$$H_P(t) = \Delta J \sin(\omega t) a_{-1}^\dagger a_0 + \Delta J \sin(\omega t + \phi) a^\dagger \alpha_1 + \text{h.c.} + \Delta V \cos(\omega t) a^\dagger \alpha_0. \quad (7)$$

We introduced by $a^\dagger_s$ the creation operator at lattice site $s$. In our further calculations we express $H_0$ and $H_P(t)$ by $(4M + 1) \times (4M + 1)$ matrices

$$H_0 = \begin{pmatrix}
\ddots & J_2 & J_1 & J_2 \\
J_2 & 0 & J_1 & J_2 \\
J_1 & -\Delta V & J_2 & 0 \\
J_2 & 0 & J_2 & 0 \\
& J_1 & 0 & J_2 \\
& J_2 & \ddots & \ddots
\end{pmatrix}, \quad H_P(t) = H_1 e^{-i\alpha t} + H_{-1} e^{i\alpha t}, \quad (8)$$

$$P = 2\pi/\omega$$
Arrays of DLSPPWs were fabricated by negative-tone gray-scale electron beam lithography where the Fourier components are determined by two numbers \( n \) and \( s \), where \( s \) is the site index within each chain. Due to local perturbations the chains are coupled to each other only through the sites in the vicinity of the topological defect \( (s = -1, 0, 1) \). The harmonic variation of the hopping \( J_{-1,0}(t) \) and \( J_{0,1}(t) \) thus induces the bonds between the sites \( |n, 0| \) and \( |n + 1, 1| \), \( \forall n \) with the hopping amplitude \( \Delta J/2 \). If one adds a harmonic on-site potential variation at the 0th lattice site with the amplitude \( \Delta V \), it creates bonds between the central sites \( |n, 0| \) and \( |n \pm 1, 0| \) \( \forall n \) with the hopping term \( \Delta V/2 \).

A sufficiently large truncated version of equation (10) yields eigenvectors and eigenvalues that converge well. Since the quasienergies are periodic in the frequency domain with a period \( \omega \), we restrict ourselves to the eigenvalues only from the first Floquet Brillouin zone, i.e. we choose \( \varepsilon \in [-\omega/2, \omega/2] \). The corresponding eigenvectors contain the Fourier components of the Floquet modes \( |\phi_n^\alpha\rangle \) where each of them is associated with the energy \( \varepsilon_n^\alpha = \varepsilon_n + n\omega \). The spectral weight of a state with an energy \( \varepsilon_n^\alpha \) is calculated as the overlap of the corresponding Floquet mode at \( t = 0 \) with the initial conditions \( |\Psi(0)) = u_0(0) \) (where \( |0\rangle \) denotes the vacuum state) weighted by the norm of the \( n \)th Fourier component:

\[
\begin{align*}
    w(\varepsilon_n^\alpha) &= |\langle u_{\alpha}(0)|\Psi(0)\rangle|^2 \langle u^\dagger_{\alpha}|u^\dagger_{\alpha}\rangle,
    &|u_{\alpha}(0)\rangle = \sum_n |u_n^\alpha\rangle
\end{align*}
\]

The complete solution of the Schrödinger equation is reconstructed as follows:

\[
|\Psi(t)\rangle = \sum_{\alpha} C_{\alpha} \sum_n \exp(-i\varepsilon_n^\alpha t)|u_n^\alpha\rangle,
\]

where the constants \( C_{\alpha} \) are determined by \( C_{\alpha} = \langle u_{\alpha}(0)|\Psi(0)\rangle \). The 2D Fourier transform \( |\Psi(k,E)\rangle \) yields the momentum representation of the wave function \( |\Psi(s,t)\rangle \). The particular solutions for different driving regimes are presented and discussed in the main body of the letter.

**Dielectric loaded surface plasmon polariton waveguides**

Arrays of DLSPPWs were fabricated by negative-tone gray-scale electron beam lithography. As shown in Fig. 1 (a) DLSPPWs consist of poly(methyl methacrylate) (PMMA) ridges deposited on top of a 60 nm thick gold film evaporated on a glass substrate. Additionally, 5 nm of Cr was used as an adhesion layer. The width and the height of each waveguide were designed to be 250 nm and 110 nm, respectively, to guarantee single-mode operation at the working light wavelength of \( \lambda = 980 \text{nm} \). In order to keep the heights of the waveguides constant, the proximity effect in lithographic process was compensated by equalizing the background dose. The waveguide geometry was controlled after fabrication by atomic force microscopy. In all the samples the short distance was \( d_1 = 0.7 \mu\text{m} \) and the long distance was \( d_2 = 1.1 \mu\text{m} \). These separations correspond to coupling constants \( J_1 = 0.16 \mu\text{m}^{-1} \) and
We estimated the amplitude of the waveguide period. Therefore, we define the period of modulation $P$ to be half the waveguide period (see Fig. 1(b) bottom).

$$J_{0,1}(t) = J_1 \cdot p_1 \exp \left( -p_2 \cdot A \sin(\omega t) \right),$$  

(13)

where $p_1 = 0.49$ and $p_2 = 1.75 \, \mu m^{-1}$ are fitting parameters and $\omega$ is the modulation frequency. For all the samples the maximum deflection of the central waveguide was chosen to be $A=0.3 \, \mu m$, being a good trade-off between bending losses and the strength of dynamic effects. It corresponds to the coupling variation of $\Delta J \approx 0.25J_1$ (for linear approximation of the exponent in (13)). Varying the period $P$ from 8 $\mu m$ up to 80 $\mu m$ we realized different frequency regimes. Due to strong confinement of the SPPs we can neglect the variation of the effective refractive index due to curvature of the waveguide, i.e. we can set the on-site potential $V_0(t) = 0$.

SPPs were excited by focusing a TM-polarized laser beam (NA of the focusing objective is 0.4) onto the grating coupler, which was fabricated on top of the central waveguide. The propagation of SPPs in the array was monitored by real- and Fourier-space leakage radiation microscopy (LRM)\textsuperscript{24,33}. The leakage radiation as well as the transmitted laser beam were both collected by a high NA oil immersion objective (Nikon 1.4 NA, 60x Plan-Apo). The transmitted laser was filtered out by placing a knife edge at the intermediate back focal plane (BFP) of the oil immersion objective. The remaining radiation was imaged onto an sCMOS camera (Andor Zyla). Real-space SPP intensity distributions were recorded at the real image plane while the momentum-space intensity distribution was obtained by imaging the BFP of the oil immersion objective.

**Dielectric waveguides**

The dielectric waveguide arrays were fabricated in a two-step process\textsuperscript{25}. First, the inverse of the waveguide structure was 3D-printed by two-photon lithography in a negative tone photoresist (IP-Dip, Nanoscribe). After development the hollow structure was then infiltrated with SU8-2 (MicroChem) to create the waveguides. A softbake was done to solidify the SU8. The resulting refractive indices of the outside material and the waveguide core were $n_0 = 1.54$ and $n_{\text{core}} = 1.59$ respectively. The radius of the waveguides $r$ as well as the small distance $d_1$ and large distance $d_2$ were measured by scanning electron microscopy. For all samples, we fixed these parameters to be $r=(0.52 \pm 0.03) \, \mu m$, $d_1=(1.41 \pm 0.02) \, \mu m$ and $d_2=(1.69 \pm 0.01) \, \mu m$. For transverse modulation of the defect the couplings from site 0 to its left and right neighbors are equal, $J_{-1,0} = J_{0,1}$. $J_{0,1}$ scales exponentially as

$$J_{0,1}(t) \approx \exp \left( -p \sqrt{d_2^2 + \frac{A^2}{2}(1 - \cos(\omega t))} \right)$$

(14)

The parameter $p$ depends on the refractive index contrast, used wavelength etc.; $A$ is the maximum deflection of the waveguide and $\omega$ the frequency of the modulation. In the experiments presented in Fig. 4 (a) $J_{0,1}$ varied from 0.48$J_1$ to 0.13$J_1$, while for those in Fig. 4 (b) the variation depended on the wavelength: from 0.47$J_1$ to 0.01$J_1$ (680 nm), from 0.48$J_1$ to 0.01$J_1$ (710 nm), from 0.52$J_1$ to 0.02$J_1$ (750 nm), from 0.53$J_1$ to 0.02$J_1$ (780 nm) and from 0.55$J_1$ to 0.03$J_1$ (810 nm).

In the dielectric waveguides, we also have to take into account an additional local on-site potential at site 0 of

$$V_0(t) = -\Delta V + \Delta V \cos(\omega t)$$

(15)

This is due to the fact that one can rewrite a curved waveguide in terms of a straight waveguide with changed refractive index\textsuperscript{34}. We estimated the amplitude $\Delta V$ to be proportional to

$$\Delta V = 2r n_{\text{core}} A (\omega/2)^2 \pi/\lambda$$

(16)

with the waveguide diameter $2r$. This additional local on-site potential at site 0 shifts the energy of the edge state by the amount of $\Delta V$. As there is no difference if the waveguide bends up or down, the on-site potential and couplings vary with twice the waveguide period. Therefore, we define the period of modulation $P$ to be half the waveguide period (see Fig. 1(b) bottom).

For the measurements in Fig. 4 (a), five arrays with defects with different period $P = (979 \pm 14) \, \mu m$, $(783 \pm 11) \, \mu m$, $(588 \pm 11) \, \mu m$, $(392 \pm 6) \, \mu m$, $(200 \pm 3) \, \mu m$ were fabricated in one sample. The amplitude of modulation was fixed to be $A=(1.36 \pm 0.04) \, \mu m$. A different sample was used for the measurements shown in Fig. 4 (b). Here, $A=(2.63 \pm 0.08) \, \mu m$ and $2P=(302 \pm 2) \, \mu m$.

To conduct the measurements, the beam from a tunable white light laser (SuperK EVO, NKT photonics) was sent through a VARIO (NKT photonics) filter box to select a certain wavelength (bandwidth 10 nm). The beam was then expanded and focused through an objective (Zeiss, NA 0.4, 20x) into the defect waveguide at site 0 at the input facet. We observed the intensity distribution in the sample at the (opposite) output facet by imaging it through an identical objective and a lens onto a CMOS-camera (Thorlabs). This corresponds to a propagation of 833 $\mu m$ in $z$ or about 24 hops with $J_1$. 

$J_1 = 0.08 \, \mu m^{-1}$, respectively. The propagation constant of a single DLSPPW is $\beta = 6.65 \, \mu m^{-1}$. These parameters were chosen to ensure sufficient coupling between the adjacent waveguides and to introduce perceptible dimerization to see topological effects. The position of the central waveguide was modulated sinusoidally resulting in

$$J_{0,1}(t) = J_1 \cdot p_1 \exp \left( -p_2 \cdot A \sin(\omega t) \right),$$

(13)
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**Author contributions statement**

Z.C. fabricated the plasmonic samples, conducted the corresponding optical experiments, and did the Floquet analysis. S.L. conceived the plasmonic experiment. C.J. conceived together with G.v.F. the dielectric experiment, fabricated the dielectric waveguide samples, did the corresponding measurements and contributed to the development of the theoretical understanding of the effects. Z.C. and C.J. wrote the first draft of the paper. C.D. developed together with S.E. the Floquet analysis and contributed to the theoretical understanding of the effects. M.F. and F.L. contributed to the theoretical understanding of the effects. All authors discussed the results and reviewed the manuscript.

**Additional information**

The authors declare that they have no competing financial interests.