Electron-Phonon Interactions on a Single-Branch Quantum Hall Edge

O. Heinonen[1,2,*] and Sebastian Eggert[1]

[1]*Institute of Theoretical Physics, Chalmers University of Technology and Göteborg University, S-412 96 Göteborg, Sweden*
[2]*Department of Applied Physics, Chalmers University of Technology and Göteborg University, S-412 96 Göteborg, Sweden*

We consider the effect of electron-phonon interactions on edge states in quantum Hall systems with a single edge branch. The presence of electron-phonon interactions modifies the single-particle propagator for general quantum Hall edges, and, in particular, destroys the Fermi liquid at integer filling. The effect of the electron-phonon interactions may be detected experimentally in the ac conductance or in the tunneling conductance between integer quantum Hall edges. [S0031-9007(96)00563-7]

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A two-dimensional electron gas subjected to a strong perpendicular magnetic field may exhibit the quantum Hall (QH) effect [1]. The effect occurs because the electron gas is incompressible at certain densities, which is due to an energy gap to bulk excitations. In the integer QH effect this gap is the kinetic energy gap (the cyclotron energy $\hbar\omega_c$) in a magnetic field, and in the fractional QH effect (FQHE) the gap arises because of the electron-electron interactions. As a result of the bulk energy gap, gapless excitations in the system can only exist at the edges. Such edge excitations are density modulations localized at the edges of the system, and all the low-energy properties of a QH system are determined by the edge excitations.

As was first demonstrated by Wen [2], the density operators for edge excitations obey a Kac-Moody algebra, similar in structure to that obeyed by Luttinger liquids [3]. Because time-reversal invariance is broken by the magnetic field, the excitations on one edge can only propagate in one direction [4] corresponding to "chiral Luttinger liquids" [2]. Wen calculated the single-particle propagator for the electron on the edge of a quantum Hall system at filling factor $\nu = 1/(2m + 1)$ and showed that it is given by [2] $G(x,t) \propto (x - v_F t)^{1/\nu}$. Here, x is a coordinate along the edge, and v_F is the edge velocity determined by the details of the potential confining the electron gas at the edge.

The appearance of the anomalous exponent in the single-particle propagator has important experimental implications. It was shown by Wen [5] and by Kane and Fisher [6] that the tunneling conductance between two counter-propagating edges of an FQHE system at bulk filling factor $\nu = 1/(2m + 1)$ depends on temperature as $T^{2(1/\nu-1)}$. The resonant tunneling conductance was calculated by Moon *et al.* [7], and Fendley, Ludwig, and Saleur [8], and measured by Milliken, Umbach, and Webb [9]. The experimentally measured tunneling conductance does indeed exhibit a $T^{2(1/\nu-1)}$ dependence, except for at the very lowest temperatures, where Coulomb interactions between the edges may modify the conductance [10].

In this Letter, we consider the effect of electron-phonon interactions on the QH edge states of spin-polarized QH systems with bulk filling factor $\nu = 1/(2m + 1)$ with a single branch of edge excitations on each edge. Electron-phonon interactions in regular Luttinger liquids have been considered previously [11–13]. Martin and Loss [12] showed that coupling the electron system to acoustic phonons destroys the Fermi surface, even in the absence of electron-electron interactions. However, this effect is only appreciable if the Fermi velocity v_F is of the same order as the renormalized sound velocity v_s [14]. This condition can be met in QH systems, where v_F is determined by the stiffness of the confining potential and the electron density, both of which may be tuned electrostatically by gates. We will show here that the electron-phonon interactions will modify the single-particle propagator even in a *chiral* Luttinger liquid. As a consequence, even the integer quantum Hall edge states will not be Fermi liquids in the presence of electron-phonon interactions.

The modification of the single-particle propagator by the electron-phonon interactions may be detected experimentally, and we will discuss two possibilities. First, the ac conductance will have resonances at (longitudinal) wave vectors q and frequencies ω related by $q = \omega/v_\alpha$, $q = \omega/v_\beta$, and $q = -\omega/v_\gamma$, where v_α, v_β, and v_γ are renormalized edge and sound velocities, which may in principle be resolved and detected in an experiment. On the other hand, the dc Hall conductance is *not* modified by the electron-phonon interactions. Second, the anomalous exponents in the single-particle propagator will modify the tunneling conductance and can in principle be measured, for example, in tunneling between two $\nu = 1$ edges in a bilayer system [15] with an overall filling of $\nu_{\text{tot}} = 2$. In the absence of electron-phonon interactions the edges of such a system are (chiral) Fermi liquids which propagate in the same direction. Coulomb interactions alone cannot change the temperature dependence [16] of the tunneling conductance from T^0. Therefore, *any* temperature behavior of the tunneling conductance at sufficiently low temperatures must be due to the electron-phonon interactions. The electron-phonon interaction also modifies the temperature dependence of the tunneling conductance between counterpropagating edge states at very low temperatures which has been measured by Milliken *et al.* [9].

The edge excitations in a quantum Hall system with filling fraction $\nu = 1/(2m + 1)$ can be described by density modulations of an effectively one-dimensional system [2]

$$H_e \equiv \frac{2\pi}{L} \frac{v_F}{\nu} \sum_{k>0} J_k J_{-k}, \qquad (1)$$

where the densities J_k obey commutation relations depending on the filling fraction ν: $[J_k, J_{k'}] = -L\nu k\delta_{k,-k'}/2\pi$. Here, k is the wave vector along the edge. This theory represents a single U(1) Kac-Moody algebra and it is well known how to "bosonize" it in terms of a chiral boson $\phi_R(x)$. The current density $j(x)$ is written as $j(x) = \sqrt{\frac{\nu}{\pi}} \frac{\partial \phi_R}{\partial x}$. Using $J_k = \int dx\, e^{ikx} j(x)$, we can immediately express the Hamiltonian in terms of the annihilation and creation operators of the boson mode expansion:

$$a_k^\dagger = \sqrt{\frac{2\pi}{L\nu k}} J_k, \qquad a_k = \sqrt{\frac{2\pi}{L\nu k}} J_{-k} \qquad (2)$$

(the zero modes are omitted). The full (spin-polarized) electron field can also be written in terms of the chiral boson and has been identified as [2]

$$\psi(x) \propto \exp(i\sqrt{1/4\nu\pi}\,\phi_R) \qquad (3)$$

(this is not to be confused with the quasiparticle field χ which can also be defined in terms of the chiral boson $\chi \propto e^{i\sqrt{\nu/4\pi}\,\phi_R}$, but carries fractional charge).

We now consider the interaction of such a system with phonons

$$H_{ph} = v_s \sum_{\mathbf{k}} |\mathbf{k}| b_{\mathbf{k}}^\dagger b_{\mathbf{k}}. \qquad (4)$$

One normal mode of the phonons is assumed to be along the quantum Hall edge. For phonon wavelengths much greater than the cross section of the electron edge branch (which is of the order of 100 Å) the normal modes perpendicular to the edge contribute in the adiabatic approximation only an overall constant, which can be ignored. For the relevant part of the interaction we can then write the crystal displacement $d(x)$ along this edge as

$$d(x) = \sum_k i(2L\rho v_s k)^{-1/2} e^{ikx}(b_k + b_{-k}^\dagger), \qquad (5)$$

where ρ is the linear mass density of the crystal and k a wave vector along the edge. The electron-phonon interaction then becomes

$$H_{e\text{-}p} = D \int dx\, \rho(x)\partial_x d(x)$$
$$= v_c \int dk\, k(a_k b_k^\dagger + a_k^\dagger b_k), \qquad (6)$$

where D is the deformation potential constant and the coupling $v_c = D\sqrt{\nu/\pi\rho v_s}$ is independent of k. For specific values of these parameters, we consider a QH edge in a GaAs heterojunction. We assume that the edge is along one of the cubic axes of GaAs so that the piezoelectric coupling vanishes and can be ignored. For electrostatic confinement by an electrode with potential V_g, v_F is of the order of $\omega_c \ell_B^2/\ell$, where $\ell_B = (\hbar c/eB)^2$ is the magnetic length and $\ell = V_g\epsilon/4\pi^2 n_0 e$ is the length scale of the electrostatic confining potential [17] (ϵ is the static dielectric constant and n_0 is the two-dimensional electron density). For a magnetic field strength of about 5 T and a density of $n_0 = 10^{15}$ m^{-2}, this gives a Fermi velocity approximately equal to the average sound velocity $v_s \approx 5 \times 10^3$ m/s in GaAs. Thus, it should be possible to optimize the effects of electron-phonon interactions in GaAs heterojunctions under ordinary conditions. The deformation potential constant D is approximately 7.4 eV [18]. Assuming an effective cross-sectional area of 10^{-14} m^{-2} of the GaAs phonon system in the direction perpendicular to the electron propagation, we then arrive at a coupling velocity $v_c/v_s \sim 0.1$.

At this point it is straightforward to diagonalize the complete Hamiltonian

$$H = \sum_{k>0} k\{v_F a_k^\dagger a_k + v_s[b_k^\dagger b_k + b_{-k}^\dagger b_{-k}] + v_c[a_k^\dagger(b_k + b_{-k}^\dagger) + \text{H.c.}]\} \qquad (7)$$

by using a generalized Bogoliubov transformation T

$$(a_k, b_k, b_{-k}^\dagger) = T\begin{pmatrix} \alpha_k \\ \beta_k \\ \gamma_{-k}^\dagger \end{pmatrix}, \qquad (8)$$

where T is given in terms of three variables ϕ, θ, η:

$$T = \begin{pmatrix} \cos\phi\cosh\theta & \sin\phi\cosh\theta\cosh\eta + \sinh\theta\sinh\eta & -\sin\phi\cosh\theta\sinh\eta - \sinh\theta\cosh\eta \\ -\sin\phi & \cos\phi\cosh\eta & -\cos\phi\sinh\eta \\ -\cos\phi\sinh\theta & -\sin\phi\sinh\theta\cosh\eta - \cosh\theta\sinh\eta & \sin\phi\sinh\theta\sinh\eta - \cosh\theta\cosh\eta \end{pmatrix}. \qquad (9)$$

The Hamiltonian is now written as

$$H = \sum_{k>0} k(\alpha_k, \beta_k, \gamma_{-k}^\dagger)A\begin{pmatrix} \alpha_k^\dagger \\ \beta_k^\dagger \\ \gamma_{-k} \end{pmatrix}, \qquad (10)$$

where the coupling matrix A is given by

$$A \equiv T^\dagger \begin{pmatrix} v_F & v_c & v_c \\ v_c & v_s & 0 \\ v_c & 0 & v_s \end{pmatrix} T. \qquad (11)$$

For the Hamiltonian to become diagonal, the off-diagonal elements of A are required to vanish, which determines the three angles ϕ, θ, η and in turn also the diagonal elements of A (i.e., the renormalized Fermi and sound velocities). Moreover, the boson representing the electron density now becomes according to Eq. (8)

$$a_k = T_{11}\alpha_k + T_{12}\beta_k + T_{13}\gamma^\dagger_{-k}. \quad (12)$$

Therefore, the electron field in Eq. (3) must now be expressed in terms of three independent boson fields, namely a right moving "charge" boson α_k and one right and one left moving "sound" boson β_k, γ_k. The electron propagator then becomes a product of three factors according to Eqs. (3) and (12)

$$G(x,t) \equiv \langle \psi^\dagger(x,t)\psi(0,0)\rangle$$

$$\propto \frac{1}{(x - v_\alpha t)^{T_{11}^2/\nu}} \frac{1}{(x - v_\beta t)^{T_{12}^2/\nu}} \frac{1}{(x + v_\gamma t)^{T_{13}^2/\nu}}, \quad (13)$$

where $v_\alpha = A_{11}$ is the renormalized Fermi velocity and $v_\beta = A_{22}$, $v_\gamma = A_{33}$ are the renormalized sound velocities. In Fig. 1 we have plotted the renormalized velocities v_i vs v_F ($i = \alpha, \beta, \gamma$) for a coupling of $v_c/v_s = 0.1$. The total "momentum" $v_\alpha + v_\beta - v_\gamma = v_F$ of the electron is conserved by the transformation T. The right propagating velocities v_α and v_β show a discontinuity at the resonance $v_F = v_s$ of equal magnitude which appears to be quadratic in the coupling v_c/v_s. The left propagating velocity v_γ is only slightly modified of the order of 1% from its original value v_s.

The fact that the electron propagator breaks up into three pieces, corresponding to the normal modes of the Hamiltonian Eq. (10), will have experimental consequences for transport properties. We first calculate the linear response to a scalar potential $\phi(x,y) = -Ey\cos(qx - \omega t)$, where we take $q, \omega > 0$. This potential gives an electric field $\mathbf{E}(x,y) = E[-qy\sin(qx - \omega t)\hat{\mathbf{x}} + \cos(qx - \omega t)\hat{\mathbf{y}}]$, where y can be taken to be constant. The perturbed charge density in response to the potential is then obtained as

$$\delta\rho(x,t) = \frac{e^2 Ey}{h} \nu q \cos(qx - \omega t)$$

$$\times \left[\frac{T_{11}^2}{v_\alpha q - \omega} + \frac{T_{12}^2}{v_\beta q - \omega} + \frac{T_{13}^2}{v_\gamma q + \omega} \right]. \quad (14)$$

By using the continuity equation $\partial\rho/\partial t + \partial j(x)/\partial x = 0$, we can then obtain the current response function to the applied potential as

$$\widetilde{\sigma}(q,\omega) = \frac{e^2\nu}{h} \omega \left[\frac{T_{11}^2}{v_\alpha q - \omega} \right.$$

$$\left. + \frac{T_{12}^2}{v_\beta q - \omega} + \frac{T_{13}^2}{v_\gamma q + \omega} \right]. \quad (15)$$

Experiments dictate that the dc Hall conductance $\sigma_H = \lim_{\omega\to 0} \lim_{q\to 0} \widetilde{\sigma}(q,\omega)$ must not be altered from its quan-
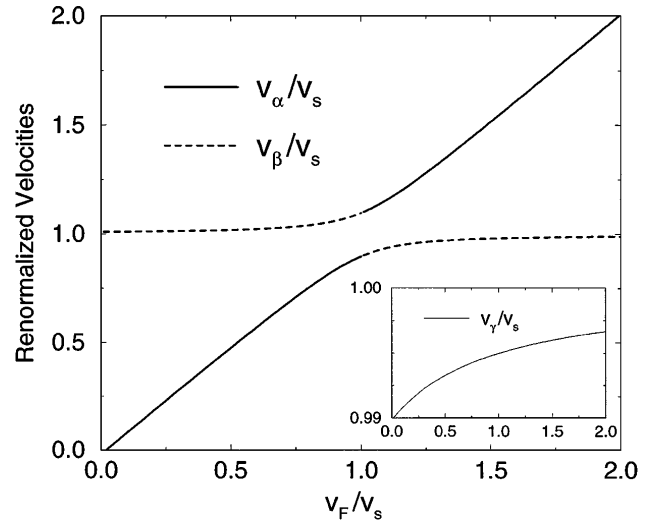

FIG. 1. The renormalized velocities v_α and v_β as a function of the Fermi velocity v_F. The sound velocity v_s sets the overall scale and the coupling has been chosen to be $v_c/v_s = 0.1$. The inset shows the renormalized velocity of the counterpropagating mode.

tized value $e^2\nu/h$, which is indeed the case according to Eq. (15) since the matrix elements obey the sum rule $T_{11}^2 + T_{12}^2 - T_{13}^2 = 1$. On the other hand, the ac conductance will exhibit resonance structures when $\omega = v_\alpha q$, $\omega = v_\beta q$, and $\omega = -v_\gamma q$ in response to a potential $\phi(x,y)$. Provided at least two of the "spectral weights" T_{11}^2, T_{12}^2, and T_{13}^3 are not too small, and the corresponding renormalized velocities are not too close, these resonances can then in principle be resolved and detected. In Fig. 2 we see that near $v_F/v_s \sim 1$, both T_{11}^2 and T_{12}^2 are close to 0.5, while v_α and v_β are on opposite sides of v_s (Fig. 1). With an experimentally reasonable value of $q \sim 10^5$ m^{-1} and $v_s \sim v_F \sim 10^3$ m/s, this gives a resonance at about 10^8 Hz, well within experimentally accessible range. Figure 2 also shows that the "total spectral weight" of the electron consists mostly of the forward propagating modes, which contribute almost equally at resonance $v_F = v_s$. This means that only very little charge is transported in the counterpropagating direction which was to be expected.

Since the single-particle propagator is changed by the electron-phonon interaction according to Eq. (13), the single-particle density of states and properties depending on it such as the tunneling conductance will also be affected by the coupling to the phonons. In particular, we consider the interlayer tunneling conductance between two edges of a bilayer system with integer filling in each layer, and a total filling factor of $\nu_{\text{tot}} = 2$. This can in principle be measured by attaching probes to the different layers separately in a system with large enough separation between the layers that the bulk tunneling probability vanishes. A gate at the edge can then be used to adjust the tunneling probability between the edges. At low enough voltage across the tunneling junction, tunneling through
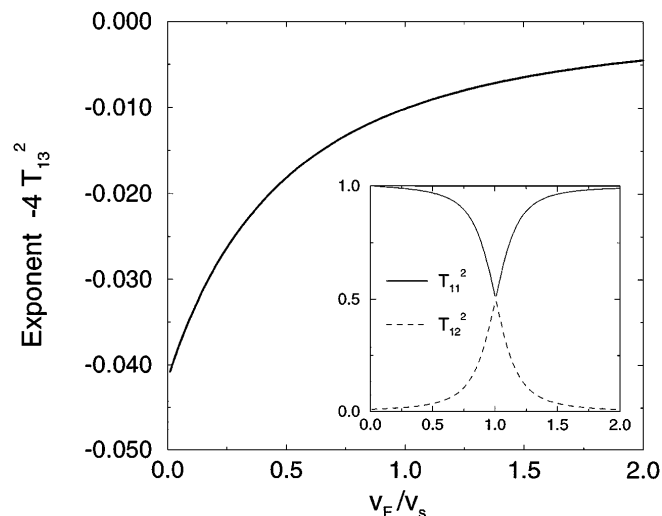
FIG. 2. The exponent of the temperature dependence of the tunneling conductance as a function of the Fermi velocity v_F/v_s for a coupling of $v_c/v_s = 0.1$. The inset shows the spectral weights of the forward propagating modes.

the bulk will be suppressed, and only the edge tunneling current appreciable. Note that in this arrangement the two edges propagate in the same direction.

The tunneling current is determined by the retarded response function [5] $X_{\mathrm{ret}}(t) = -i\theta(t)\langle[A(t), A^\dagger(0)]\rangle$, where $\Gamma A = \Gamma\psi_1(x = 0)\psi_2^\dagger(x = 0) + \mathrm{H.c.}$ is the tunneling operator, with Γ the tunneling amplitude, and $\psi_i(x)$, $i = 1, 2$, the electron field operator on the two edges. From Eq. (13) and following Wen [5] it is a straightforward exercise to determine the temperature dependence of the tunneling differential conductance, with the result that

$$\frac{dI_t}{dV_t}\bigg|_{V_t=0} \propto T^{2(T_{11}^2+T_{12}^2+T_{13}^2)-2} = T^{-4T_{13}^2}. \qquad (16)$$

Because of the fact that $T_{13}^2 \neq 0$ in the presence of electron-phonon interactions, the differential tunneling conductance will now depend on temperature, in fundamental contrast to the tunneling between two chiral Fermi liquids propagating in the same direction, which is temperature *independent* at low temperatures, even in the presence of electron-electron interactions between the two edges. From Fig. 2 we see that the magnitude of the exponent $-4T_{13}^2$ for the parameters chosen here is of the order of 10^{-2}, which is small (the exponent appears to be quadratic in the coupling constant v_c/v_s). However, the main point is that any measured temperature dependence at all will be due to the electron-phonon interactions.

In conclusion, we have shown that the electron-phonon interaction modifies the chiral Luttinger liquid on the quantum Hall edge. The single-particle propagator becomes a product of three separate modes, one of which is always counterpropagating. From this we have predicted direct experimental consequences for the ac conductivity of the quantum Hall bar and the temperature dependence of the tunneling conductance.

*Permanent address: Department of Physics, University of Central Florida, Orlando, FL 32816-2385.

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